

## INRUSH CURRENT ANALYZING IN THE TRANSFORMERS BASED ON PREISACH MODEL

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**Abstract-** Inrush current of transformers at the time of switching has many destructive effects on the power systems, so, many researches are being done to estimate the value of it accurately. This phenomenon depends on the non-linear and complicated magnetic characteristics of the transformer core. Unfortunately there is no unique and comprehensive modeling approaches for this problem. In this paper, we propose a plenary model for transformers applying the Preisach model which is a capable one for representing the magnetic behavior of the ferromagnetic materials. The proposed model is able to analyze the transient events such as, transient inrush current and harmonical performance of the transformers. The suggested algorithm minimizes the problems associated with applying the Preisach model into a transformer connected to any external circuit properly.

**Keywords:** Inrush Current, Preisach Model, Remnant Flux, Transformer.

### I. INTRODUCTION

Long ago the inrush current of transformers endamaged power networks and were taken into consideration [1]. Spreading investigations have been done about the origin of consisting and removing or reducing it like: changing the winding configuration transformers in the form of SPS or SPSP [2], creating a virtual air gap by means of a coil which is feeded by a DC current that cause to saturate the part of a transformer core at the time of Plugging transformers (AGW or Air Gap Winging) [3]. Using a DC reactor in a series mode with the transformer. Through a diode bridge at the time of switching the reactor will be charged and after that will be issued from the circuit, therefore, the inrush current will reduce and it would not influence the transformer operation [4].

The reason of occurring the inrush current is the magnetic behavior of the transformer core. Because of the complexity and being non-linear and multi-value of the magnetic behavior of ferromagnetic materials, most of the studyings and estimating the values of the inrush current are based on experiments. In addition, these investigations just show some information about the peak of the inrush

current and the factors influence it which is not capable to model the transient behavior of transformer.

In this paper, according to the Preisach model that is a comprehensive model in expressing the behavior of the ferromagnetic materials, we model the behavior of a transformer core and farraginous them with electrical relationships.

Classical Preisach model (CPM) of the ferromagnetic materials was first introduced by Ferenc Preisach and afterward it became a basic for all Preisach models. Filip et. al. in 1994 described the Preisach model under classical conditions that there was a logical adaptation between the classical model of Preisach and the experiments conducted on the silicon laminations. Recently, many efforts have ever made by Mayergoyz for developing the Preisach model in both scalar and vector modes (see [5]). This model has been used for various calculations of electrical and mechanical systems. For instance, article [6] indirectly used this model to calculate the local iron loss of a synchronous machine. But, not many experiments for applying the Preisach model directly to the real systems like transformers, have been conducted. Therefore the problems occurred in applying the Preisach model to a transformer coupled to the external circuits have been remained unsolved. These problems are due to the difficulties such as being multi-inputs, too many constraints, non-linearity, multi-value nature and time-consuming computation process.

Also, in this paper comprehensive algorithm for calculating the transformer performances in both steady and transient state is given, too. Using this computerized model, almost all of the transformer phenomena such as inrush currents and ferroresonance phenomenon can be studied with some accuracy.

### II. PREISACH MODEL

In the recent years, Preisach model has been developed in both scalar and vector modes for describing the hysteresis phenomenon. In the steady state, the magnetization of a ferromagnetic material in a periodic sinusoidal or non-sinusoidal magnetic field can be easily calculated by a delay component. This delay component

is simply shown in Figure 1 in which the relation between an input variable  $u(t)$  and an output variable  $f(t)$  is as:

$$\begin{aligned} f(t) &= 1 & \text{if } u(t) &\geq a \\ f(t) &= -1 & \text{if } u(t) &\leq b \\ f(t) &= \text{unchanged} & \text{if } b < u(t) < a \end{aligned} \quad (1)$$

Let us introduce the operator  $\hat{\gamma}_{ab}$  as it operates on input  $H(t)$  and gives  $f(t)$ . If we suppose that there is infinite number of these delay functions with the same operators for a random point of the magnetic material, so the output of this set will be as:

$$M(t) = \iint_{a \geq b} p(a,b) \cdot \hat{\gamma}_{ab} \cdot H(t) \, dadb \quad (2)$$

Where  $p(a,b)$  is named the density function. Preisach operator has a local memory with specific values of maximum and minimum. For a homogeneous magnetic material, the field intensity in which saturation occurs is denoted by  $H_s$ . Therefore if  $a > H_s$  or  $b < -H_s$  then  $p(a,b) = 0$  and the Preisach triangle shown in Figure 2 is defined as:

$$S^\Delta(t) = \{(a,b) \mid a \geq b, b \geq -H_s, a \leq H_s\} \quad (3)$$

For each point  $(a,b) \in S$ , there is an operator similar to  $\hat{\gamma}_{ab}$  and for a given time  $t$ , the  $S$  plot is divided into 2 parts:

$$\begin{aligned} S^+(t) &= \{(a,b) \in S \mid \text{output of } \hat{\gamma}_{ab} \text{ at } t \text{ is } +1\} \\ S^-(t) &= \{(a,b) \in S \mid \text{output of } \hat{\gamma}_{ab} \text{ at } t \text{ is } -1\} \end{aligned} \quad (4)$$

At any time  $t$ ,  $S(t) = S^+(t) \cup S^-(t)$  and the equation (2) can be rewritten as:

$$M(t) = \iint_{S^+(t)} p(a,b) \, dadb - \iint_{S^-(t)} p(a,b) \, dadb \quad (5)$$

At a time  $t$ , the Preisach plane is divided into the high switching field  $S^+$  and low switching field  $S^-$ . While the input  $H$  increases, a vertical line sweeps the Preisach plane from left to right and when  $H$  decreases, a horizontal line sweeps the plane up to down. If  $H$  with a limited number of local extremum varies between  $-H_s$  and  $H_s$ , obviously  $M$  would change between  $M_s$  and  $-M_s$ . Therefore, we can easily prove that the density function should satisfy the equation given as:

$$\iint_{S(t)} p(a,b) \, dadb = 2M_s \quad (6)$$

So the equation 5 can be rewritten as:

$$M(t) = -M_s + 2 \iint_{S^+(t)} p(a,b) \, dadb \quad (7)$$

By calculating  $M$  using equation 7, we can compute the flux density  $B$  of a given point by [5]:

$$B(t) = \mu_0 \{H(t) + M(t)\} \quad (8)$$

where  $\mu_0 = 4\pi \times 10^{-7}$ .

### III. SIMULATION OF THE PREISACH MODEL

While ferromagnetic material is exposed to a periodic symmetric field with a specified peak value, the magnetization and the steady-state flux density of each point of the material can be calculated by one-dimensional Preisach model. In fact, the number of broken lines shown in Figure 2 at the boundary of  $S^+$  and  $S^-$ , illustrates the number of local extremum of magnetic field, since last absolute extremum until present time. Knowing the vertices of the broken lines up to the time  $t$ , the value of  $M$  and then  $B$  can be determined for the time  $t + \Delta t$  by a computer simulation. The program examines whether the value of  $H(t + \Delta t)$  is more or less than  $H(t)$  and then calculates the values of  $M$  and  $B$  using equations 7 and 8. After these calculations, new vertices of cursives in the border of  $S^+$  and  $S^-$  for the time  $t + \Delta t$  can be determined.

#### A. Density Function

The steel lamination employed as a core for the transformer of the present paper is LOSIL-630. Four descriptive parameters of two-variable density function of the material have been already determined precisely using the experimental results by reference [6]. Parameters of the multi-equation function are given in this reference.

General form of the density function is:

$$2p(a,b) = \frac{m_{ss}}{\pi\sigma_1\sigma_2} \exp\left(-\frac{(a+b)^2}{4\sigma_1^2} - \frac{(a-b-2u_c)^2}{4\sigma_2^2}\right)$$

Where four parameters  $\sigma_1$ ,  $\sigma_2$ ,  $u_c$  and  $m_{ss}$  are different for various materials and areas of the Preisach triangle. As mentioned earlier, these variables are determined using the experimental  $B-H$  and iron loss per kilogram characteristics of the material given by manufacturers and the best curve fitting approach. For more details see reference [6].

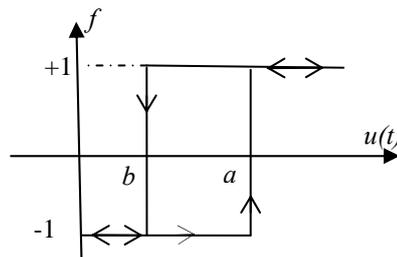


Figure 1. Delay component

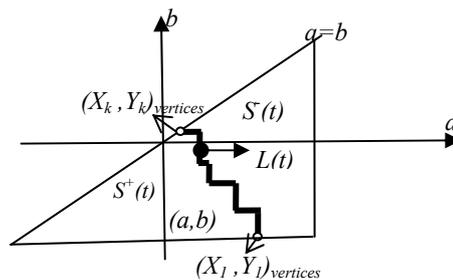


Figure 2. Preisach triangle

**B. Initial conditions**

By changing the peculiarities of the Preisach triangle vertices, we can consider the various magnetic initial conditions for the transformer core. Figures 3 and 4 show the Preisach diagram for the different initial conditions (various remnant flux density values). As it shown in the figures, because of having different vertices, the integral on the  $S^+(t)$  area is different for each figure. This feature enables us to reach different residues by varying the peculiarities of vertices and by applying it to the mentioned model we can peruse the effect of different residues on the transient behavior of transformers.

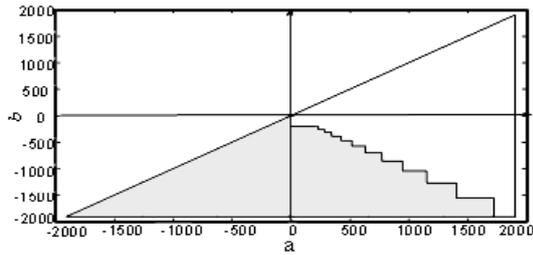


Figure 3. Initial Triangle Preisach while remnant flux density of core is -0.7065 Tesla

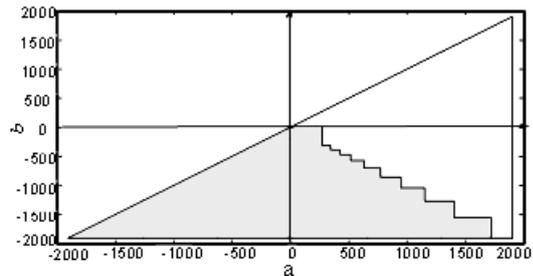


Figure 4. Initial Triangle Preisach while remnant flux density of core is 0.71313 Tesla

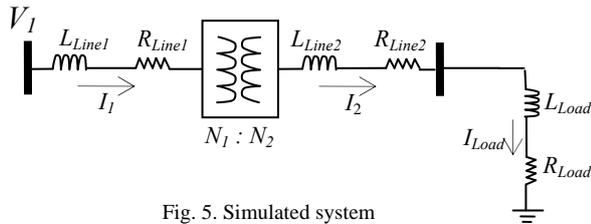


Fig. 5. Simulated system

**IV. DISCRETE MODEL OF TRANSFORMER**

According to the reference [7], the values of inrush current in three-phase transformers can be calculated by applying a coefficient ( $1/\sqrt{3}$  for primary delta connection and  $2/3$  for primary star connection) to the single-phase values of it. So, for simplifying the modeling, we consider the network as a single-phase.

Simulated system in this paper consists of a small experimental transformer that primary and secondary are connected via transmission lines to a power supply and load as shown in Figure 5. Design details of the transformer are given in Table 1. Resistances and inductances of the transmission lines are:  $R_{Line1} = 8.3e-03$ ,  $L_{Line1} = 8.3e-06$ ,  $R_{Line2} = 0.05$  and

$L_{Line2} \cong 0$  per unit. The system equations in a discrete form can be written as:

$$V_1(t) = (R_{T1} + R_{Line1}) I_1(t) + (L_{T1} + L_{Line1}) \frac{dI_1(t)}{dt} + N_1 A_c \frac{dB(t)}{dt} \tag{9}$$

$$N_2 A_c \frac{dB(t)}{dt} = (R_{T2} + R_{Load}) I_2(t) + (L_{T2} + L_{Load}) \frac{dI_2(t)}{dt} \tag{10}$$

$$H(t) = \frac{N_1}{l_c} I_1(t) - \frac{N_2}{l_c} I_2(t) \tag{11}$$

$$B(t) = \text{Preisach}(H(t), H(t - \Delta t)) \tag{12}$$

Where  $A_c$  and  $l_c$  are the cross-section and average length of the core,  $N_1$  and  $N_2$  are the numbers of turns of primary and secondary windings,  $R_{T1}$  and  $L_{T1}$  are the resistance and the leakage inductance of the primary side while  $R_{T2}$  and  $L_{T2}$  are the resistance and the leakage inductance of the secondary respectively.

However for solving the equations, we use the Rang-Koutah method so the discrete equations are rewritten as follows:

$$\left( \frac{V_1(t + \Delta t) + V_1(t)}{2} \right) \Delta t - (R_{T1} + R_{Line1}) \left( \frac{I_1(t + \Delta t) + I_1(t)}{2} \right) \Delta t - (L_{T1} + L_{Line1}) (I_1(t + \Delta t) - I_1(t)) - N_1 A_c (B(t + \Delta t) - B(t)) = 0 \tag{13}$$

$$N_2 A_c (B(t + \Delta t) - B(t)) - (R_{Load} + R_{T2}) \left( \frac{I_2(t + \Delta t) + I_2(t)}{2} \right) \Delta t - (L_{T2} + L_{Load}) (I_2(t + \Delta t) - I_2(t)) = 0 \tag{14}$$

$$l_c H(t + \Delta t) - N_1 I_1(t + \Delta t) + N_2 I_2(t + \Delta t) = 0 \tag{15}$$

$$B(t + \Delta t) = \text{Preisach}(H(t + \Delta t), H(t)) \tag{16}$$

Having the system parameters and the input voltage of the power supply, the final goal of solving the equations set (13 to 16) can be the instantaneous values of the currents.

The Preisach model is the most time consuming blocks of the whole simulations. Therefore, to reduce the computation times and to avoid divergence problem of the iterative methods, instead of moving on 't' axis with specified time step  $\Delta t$ , we can move on the H axis with intelligent step size of  $\Delta H$  and solve the equations by assuming a known value for  $H(t + \Delta t)$ . At any time 't', the values of  $I_1(t)$ ,  $I_2(t)$ ,  $H(t)$  and  $B(t)$  are assumed known. Applying an initial guess for the value of  $H(t + \Delta t)$  (a value closed to the value of  $H(t)$ ) the value of  $B(t + \Delta t)$  is evaluated by the Preisach model. After that, the unknown variables, comprising  $\Delta t$ ,  $I_1(t + \Delta t)$  and  $I_2(t + \Delta t)$  are computed using equations 13 to 15. The calculated value for  $\Delta t$  has to be positive, real and small, otherwise, the value of  $H(t + \Delta t)$  should be revised properly. The way of revising the value of  $H(t + \Delta t)$  is a key point. The flowchart of the proposed algorithm is illustrated in Figure 6.

In the algorithm, to determine the ascending or descending trend of  $H$ , a variable named *check* is defined. If the value of *check* is 1 then  $H$  is increasing and commonly except in the returning point we should add a positive value such as  $\Delta H$  to  $H$  in order to get a proper value for  $H(t + \Delta t)$  otherwise, if the value of *check* is zero we must subtract  $\Delta H$  from  $H$  and get a value for  $H(t + \Delta t)$ .

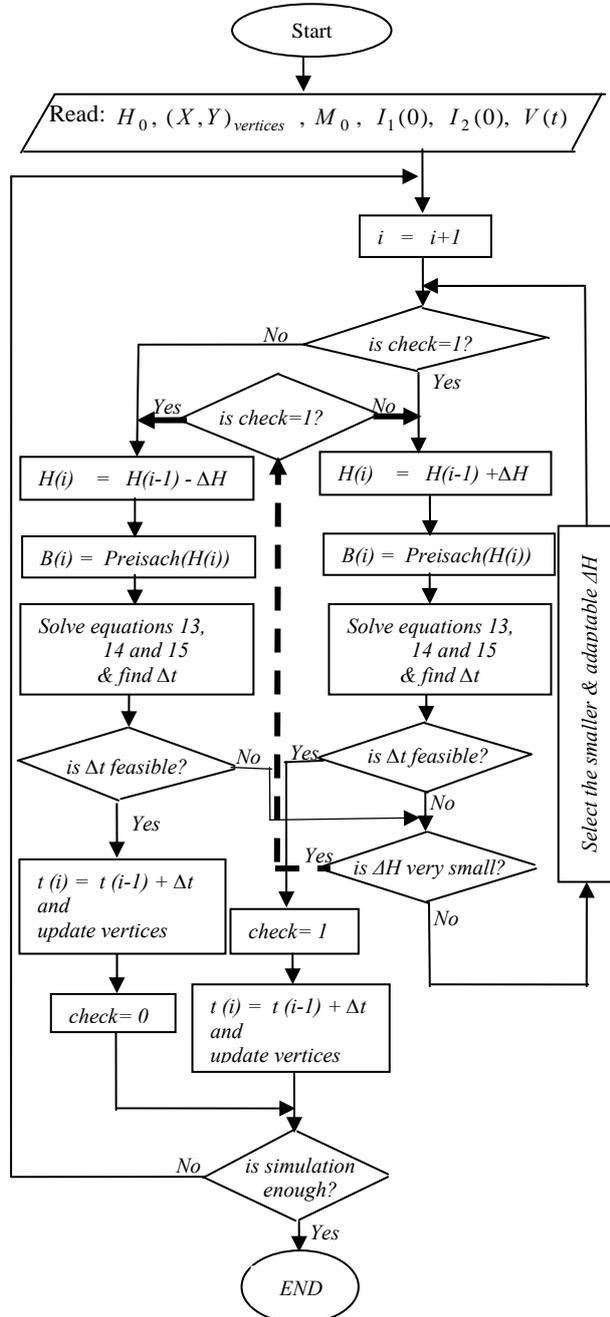


Figure 6. Simulation Algorithm

**V. SIMULATION RESULTS**

The system introduced previously is simulated under various loads. As an example, the simulation results for an inductive load with  $R_{Load} = 0.5 p.u.$  and  $L_{Load} = 1.25e - 04 p.u.$  are presented. A voltage source as

$V_1(t) = 220\sqrt{2} \sin(100\pi t + \varphi_0)$  is applied to the beginning terminals of the transmission line.

Figure 12 shows the variation of the peak value of the inrush current per the various initial phase voltage ( $\varphi_0$ ). Figures 7 to 11 display the transient wave form of current and the  $B-H$  curve for two critical values for initial phase voltages. Figure 13 shows the effect of remnant flux on the inrush current. For instance, for two values of magnetic residues of a core,  $B-H$  curves and transient wave forms currents of transformers in figures 14 to 16 are illustrated.

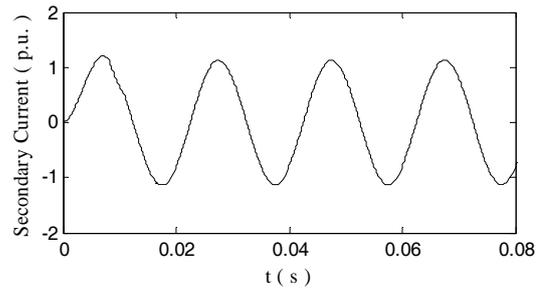


Figure 7. Transient current of secondary while  $\varphi_0 = 0$  and remnant flux of core is zero

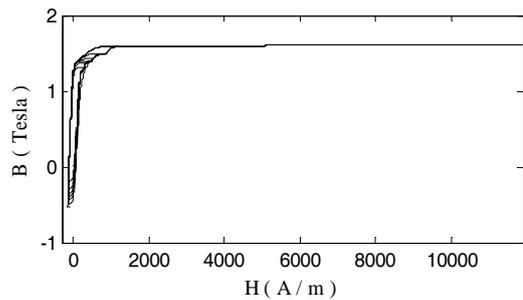


Figure 8. Transient  $B-H$  curve while  $\varphi_0 = 0$  and remnant flux of core is zero

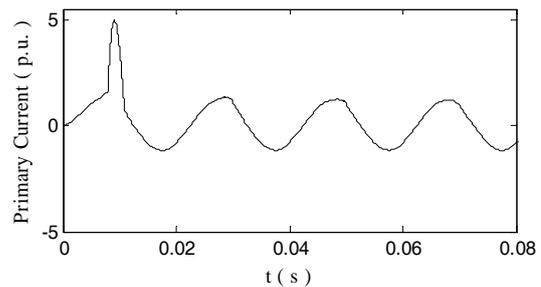


Figure 9. Transient current of primary while  $\varphi_0 = 0$  and remnant flux of core is zero

Table 1. Transformer and line details

Primary winding resistance ( $R_{T1}$ )	0.07 p.u	
Primary leakage inductance ( $L_{T1}$ )	8.5e-05 p.u.	
Number of the primary winding turns	433 turns	
Secondary winding resistance ( $R_{T2}$ )	0.25 p.u.	
Secondary leakage inductance ( $L_{T2}$ )	2.3e-3 p.u.	
Number of the secondary winding turns	77 turns	
Core dimensions	Design type	EI 150N
	$A_c$	24 cm <sup>2</sup>
	$L_c$	35 cm

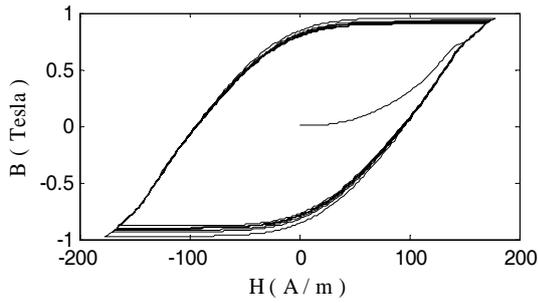


Figure 10. Transient  $B-H$  curve while  $\varphi_0 = \pi/2$  and remnant flux of core is zero

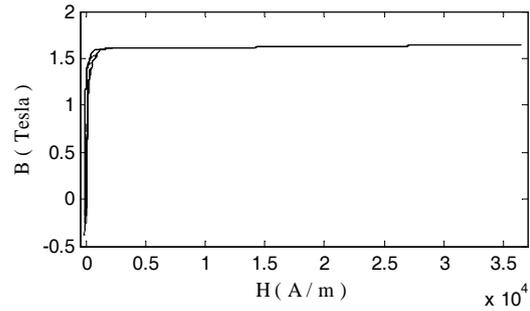


Figure 14. Transient  $B-H$  curve while remnant flux density of core is 0.71313 Tesla and  $\varphi_0 = 0$

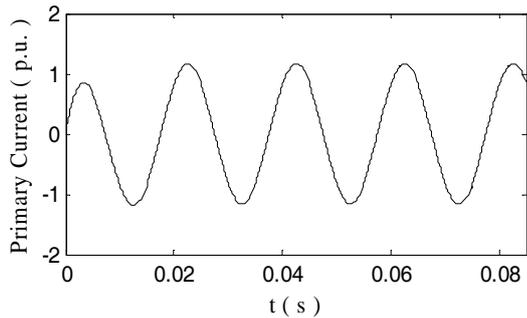


Figure 11. Transient current of primary while  $\varphi_0 = \pi/2$  and remnant flux of core is zero

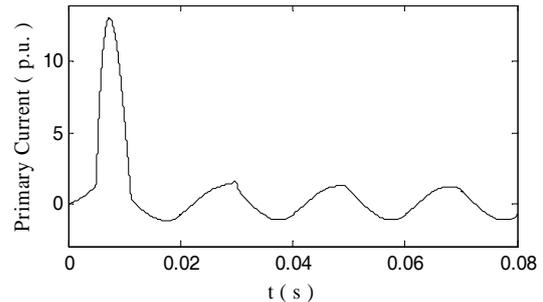


Figure 15. Transient current of primary while remnant flux density of core is 0.71313 Tesla and  $\varphi_0 = 0$

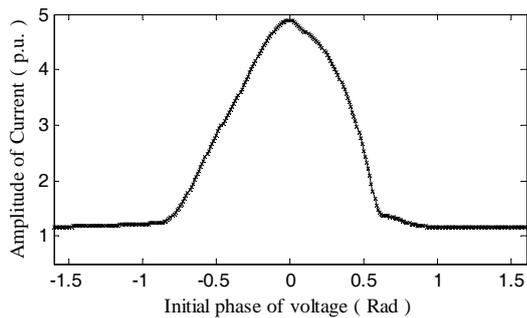


Figure 12. Amplitude of inrush current for diverse value of  $\varphi_0$  while remnant flux of core is zero

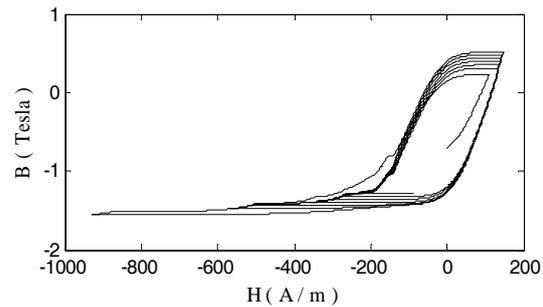


Figure 16. Transient  $B-H$  curve while remnant flux density of core is -0.7065 Tesla and  $\varphi_0 = 0$

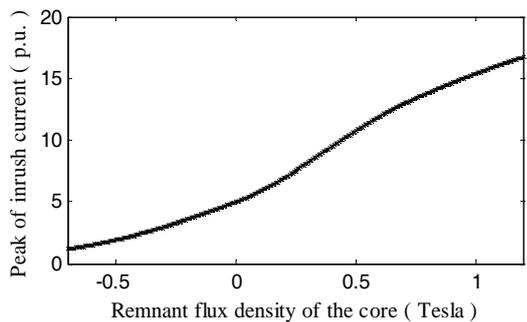


Figure 13. Amplitude of inrush current for diverse value of remnant flux of core while  $\varphi_0$  is zero

## VI. CONCLUSIONS

According to the simulation results, it is obvious that the suggested model is able to represent the transient behavior and the inrush current in a transformer. This model also shows the effects of initial electric and magnetic conditions. When the initial phase of voltage is

zero, the worst transient condition occurs and when the initial phase is  $\pi/2$  no transient state and inrush current exist. Both initial magnetic conditions- positive residue and negative residue- affect the inrush current. If the remnant flux has same direction with the created flux at the first instant after switching transformer, in the positive half cycle the inrush current occurs otherwise causes to reduce the inrush current and in the negative half cycle the inrush current will be maximized. The load of transformer will not affect this phenomenon and in all states the primary current of transformer is severely transient behavior but the secondary one is not in this situation. When there is no inrush current the  $B-H$  curve of the transformer core is completely asymmetric around the origin and gradually goes to be full symmetric.

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## **BIOGRAPHIES**



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