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INVESTIGATION OF THE INFLUENCE OF PRELIMINARY BUCKLING OF CYLINDRICAL SHELL REINFORCED BY A CROSS SYSTEM OF RIBS AND FILLED WITH MEDIUM ON CRITICAL STRESSES OF GENERAL STABILITY LOSS

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Abstract- Influence of preliminary buckling of a shell reinforced by a regular cross system of ribs and filled with medium on critical load parameters of general stability loss is investigated. The investigation is based on the problem statement used the mixed energy method and nonlinear equation of combined deformations.

Keywords: Damaged Shell, Ribbing, Vibrations, Viscous-Elastic Medium, Elastic Matrix.

I. INTRODUCTION

Cylindrical shells reinforced by a regular cross system of ribs are important structural elements of rockets, submarines, motor vehicles and etc. Investigation of the behavior of such structures with regard to external factors is of special importance in the field of contact problems in the theory of ribbed shells. In papers [1-3], the stability under longitudinal compression was considered without taking into account the preliminary buckling of ribbed cylindrical shells filled with medium.

II. PROBLEM STATEMENT

Total energy of the system is written in the form [4]:

$$\Pi = Э + A \tag{1}$$

where

$$Э = \frac{Eh^3}{24(1-\nu^2)R^2}\int_0^{\xi_1}\int_0^{2\pi}\left\{\left(\frac{\partial^2 w}{\partial\xi^2}+\frac{\partial^2 w}{\partial\theta^2}\right)^2 - 2(1-\nu)\left[\frac{\partial^2 w}{\partial\xi^2}\frac{\partial^2 w}{\partial\theta^2}-\left(\frac{\partial^2 w}{\partial\xi\partial\theta}\right)^2\right]\right\}d\xi d\theta +$$

$$+\frac{h}{2Er^2}\int_0^{\xi_1}\int_0^{2\pi}\left\{\left(\frac{\partial^2\varphi}{\partial\xi^2}+\frac{\partial^2\varphi}{\partial\theta^2}\right)^2 - 2(1-\nu)\left[\frac{\partial^2\varphi}{\partial\xi^2}\frac{\partial^2\varphi}{\partial\theta^2}-\left(\frac{\partial^2\varphi}{\partial\xi\partial\theta}\right)^2\right]\right\}d\xi d\theta +$$

$$+\frac{1}{2}\sum_{i=1}^{k}\left\{E_c F_c L_1 \varepsilon_c^2\Big|_{\theta=\theta_i} + \frac{1}{r^3}\int_0^{\xi_1}\left[E_c I_{yc}\left(\frac{\partial^2 w}{\partial\xi^2}\right)^2 + G_c I_{kp.c}\left(\frac{\partial^2 w}{\partial\xi\partial\theta}\right)^2\right]\Bigg|_{\theta=\theta_i} d\xi\right\} +$$

$$+\sigma_x h\int_0^{\xi_1}\int_0^{2\pi}\left\{\frac{1}{E}\left(\frac{\partial^2\varphi}{\partial\theta^2}-\nu\frac{\partial^2\varphi}{\partial\xi^2}\right)-\frac{1}{2}\left(\frac{\partial w}{\partial\xi}\right)^2\right\}d\xi d\theta - \frac{N_c}{2r}\sum_{i=1}^{k}\int_0^{\xi_1}\left(\frac{\partial w}{\partial\xi}\right)^2\Bigg|_{\theta=\theta_i} d\xi +$$

$$+\frac{1}{2R^3}\sum_{j=1}^{k_1}\int_0^{2\pi}\left[E_s I_{xs}\left(\frac{\partial^2 w}{\partial\theta^2}+w\right)^2 + G_s I_{\kappa p.s}\frac{\partial^2 w}{\partial\xi\partial\theta}\right]\Bigg|_{\xi=\xi_j} d\theta$$

and $\xi = \frac{x}{r}$, $\theta = \frac{y}{r}$; E_c, G_c, E_s, G_s are elastic modulus and shear modulus of the material of longitudinal ribs; k, k_1 are numbers of longitudinal and lateral ribs, respectively; σ_x is axial compression stresses; u, v, w are components of the displacement vector of the shell; h and R are the thickness and radius of the shell, respectively; E, ν are Young's modulus and Poisson ratio of the shell material; $\xi_1 = \frac{L_1}{r}$, L_1 is the shell length, $F_c, I_{yc}, I_{kp.c}$ are the areas and the moments of inertia of cross-section of longitudinal bar with respect to the ox and oz axes, as well as the moment of inertia under torsion, w_0 is initial deflection, ε_c is mean shortening of longitudinal bar, N_c is mean force in longitudinal bar, i.e.

$$\varepsilon_c = \frac{r}{L_1}\int_0^{L_1}\varepsilon_x d\xi, \quad N_c = \frac{F_c r}{L_1}\int_0^{L_1}\sigma_{xc} d\xi$$

The influence of medium on the shell is determined as external surface loads applied to the shell and is calculated as a work performed by these loads when the system changes from strained state to the initial unstrained state. It is represented in the form:

$$A = -R^2\int_0^{\xi_2}\int_0^{2\pi} q_z w d\xi d\theta \tag{2}$$

The Pasternak model [5] is used to determine q_z. The essence of this model is that the influence of medium on the shell on the contact surface is determined by the relation

$$q_z = \left(\tilde{q} + \tilde{q}_0\nabla^2\right)w = Kw \tag{3}$$

where ∇^2 is Laplace's two-dimensional operator on the contact surface. Strain continuity equation for $w_0 = 0$ is written in the form of relation (4).

$$\Delta\Delta\varphi = E\left\{\left(\frac{\partial^2 w}{\partial\xi\partial\theta}\right)^2 - \frac{\partial^2 w}{\partial\xi^2}\frac{\partial^2 w}{\partial\theta^2} - r\frac{\partial^2 w}{\partial\xi^2}\right\} \tag{4}$$

III. METHOD OF SOLUTION

It is accepted that the end faces of the shell are simply supported. It is assumed that before buckling of the shell, the given axial compression stresses are uniformly distributed on the area of cross-section of the end face. Compatibility of longitudinal displacements of the shell and ribs is provided only at the points on the shell edges.

We specify the shell deflection in the form of the sum

$$w = w_1 + w_2 = f_1 \sin d_{m_1}\xi \sin n_1\theta + f_2 \sin d_{m_2}\xi \sin n_2\theta \quad (5)$$

where f_1, f_2 are varying parameters, $d_{m_s} = \frac{\pi m_s}{\xi_1}$ $(s = 1,2)$, m_1 and m_2, $2n_1$ and $2n_2$ are numbers of half-waves in the longitudinal and peripheral directions. The preliminary buckling forms correspond to the first summand of (5), and forms of the deflection of general stability loss correspond to the second summand.

After buckling of the shell in the form of $w_1 = f_1 \sin d_{m_1}\xi \sin n_1\theta$, the shell behavior with increasing of load is considered as behavior of a nonlinear system, and the expression for deflection doesn't change with the increase of the amplitude. Stability loss conditions are of the form

$$\frac{\partial \Pi(w_1)}{\partial f_1} = 0, \quad \frac{\partial \Pi(w)}{\partial f_2} = 0 \quad (6)$$

Using the strain compatibility equation (4), we find the stress function

$$\varphi = E\left\{\frac{f_1^2}{32}\left(\frac{n_1^2}{d_{m_1}^2}\cos 2d_{m_1}\xi + \frac{d_{m_1}^2}{n_1^2}\cos 2n_1\theta\right) - \right.$$
$$- f_1 \frac{d_{m_1}^2 r}{\left(d_{m_1}^2 + n_1^2\right)^2}\sin d_{m_1}\xi \sin n_1\theta -$$
$$- f_2 \frac{d_{m_2}^2 r}{\left(d_{m_2}^2 + n_2^2\right)^2}\sin d_{m_2}\xi \sin n_2\theta -$$
$$-\frac{1}{4} f_1 f_2 \left[\frac{\left(d_{m_1}n_2 - d_{m_2}n_1\right)^2}{\left(b_1^2 + b_3^2\right)^2}\cos b_3\xi \cos b_1\theta - \right. \quad (7)$$
$$- \frac{\left(d_{m_1}n_2 + d_{m_2}n_1\right)^2}{\left(b_2^2 + b_3^2\right)^2}\cos b_3\xi \cos b_2\theta -$$
$$- \frac{\left(d_{m_1}n_2 - d_{m_2}n_1\right)^2}{\left(b_1^2 + b_4^2\right)^2}\cos b_4\xi \cos b_1\theta +$$
$$\left.\left. + \frac{\left(d_{m_1}n_2 + d_{m_2}n_1\right)^2}{\left(b_2^2 + b_4^2\right)^2}\cos b_4\xi \cos b_2\theta \right] - \frac{r^2\sigma_x}{2E}\theta^2\right\}$$

where

$b_1 = n_2 - n_1,\ b_2 = n_2 + n_1$

$b_3 = d_{m_2} - d_{m_1},\ b_4 = d_{m_2} + d_{m_1}$

After substitution of the expressions for w and φ into (1)-(3) we get

$$\Pi = \frac{\pi E h^3 L_1}{48\left(1-\nu^2\right)r^2}\left[f_1^2\left(d_{m_1}^2 + n_1^2\right)^2 + f_2^2\left(d_{m_2}^2 + n_2^2\right)^2\right] + \quad (8)$$
$$+ \frac{\pi E h L_1}{2r^3}\left\{\frac{f_1^4}{64}\left(d_{m_1}^4 + n_1^4\right) + \right.$$
$$+ \frac{f_1^2}{2}\frac{d_{m_1}^4 r^2}{\left(d_{m_1}^2 + n_1^2\right)^2} + \frac{1}{2} f_2^2 \frac{d_{m_2}^4 r^2}{\left(d_{m_2}^2 + n_2^2\right)^2} +$$
$$+ \frac{f_1^2 f_2^2}{32}\left[\left(d_{m_1}n_2 - d_{m_2}n_1\right)^4 \cdot \left(\frac{1}{\left(b_1^2 + b_3^2\right)^2} + \frac{1}{\left(b_2^2 + b_4^2\right)^2}\right) + \right.$$
$$\left.\left. + \left(d_{m_1}n_2 + d_{m_2}n_1\right)^4\left(\frac{1}{\left(b_2^2 + b_3^2\right)^2} + \frac{1}{\left(b_1^2 + b_4^2\right)^2}\right)\right]\right\} +$$
$$+ \frac{E_c F_c}{4}\left(\frac{k L_1 d_{m_1}^4}{32 r^4} f_1^4 + 2\frac{k L_1 \sigma_x^2}{E^2}\right) + \frac{E_c I_{yc} L_1}{4r^4} f_2^2 d_{m_2}^4 \sum_{i=1}^{k}\sin^2 n_1\theta_i +$$
$$+ \frac{G_c I_{kp.c} L_1}{4}\left[f_1^2 d_{m_1}^2 n_1^2 \sum_{i=1}^{k}\cos^2 n_1\theta_i + f_2^2 d_{m_2}^2 n_2^2 \sum_{i=1}^{k}\cos^2 n_2\theta_i\right] +$$
$$+ \frac{\pi E_s I_{xs}}{2r^3} f_2^2\left(n_2^2 - 1\right)^2 \sum_{j=1}^{k_1}\sin^2 d_{m_2}\xi_j +$$
$$+ \frac{\pi G_s I_{kp.s}}{2r^3}\left(f_1^2 d_{m_1}^2 n_1^2 \sum_{j=1}^{k_1}\cos^2 d_{m_1}\xi_j + f_2^2 d_{m_2}^2 n_2^2 \sum_{j=1}^{k_1}\cos^2 d_{m_2}\xi_j\right) -$$
$$- \frac{1}{E}2\pi r h L_1 \sigma_x^2 - \frac{\pi h L_1 \sigma_x}{4r}\left(f_1^2 d_{m_1}^2 + f_2^2 d_{m_2}^2\right) -$$
$$- \frac{F_c L_1 \sigma_x d_{m_2}^2}{4r^2} f_2^2 \sum_{i=1}^{k}\sin^2 n_2\theta_i \left(1 + \frac{d_{m_1}^2}{8r^2} f_1^2\right) +$$

$$+ \frac{2r^4\sigma_x^2}{E} + \pi r^2\left[\tilde{q} - \tilde{q}_0\left(n_1^2 + d_{m_1}^2\right)S_1\right] +$$
$$+ \pi r^2\left[\tilde{q} - \tilde{q}_0\left(n_2^2 + d_{m_2}^2\right)S_1\right] =$$
$$= \frac{\pi E h^5 L_1}{48\left(1-\nu^2\right)r^3}\{A_1\tilde{f}_1^4 + \left(A_2 - A_3\eta + A_8\right)\tilde{f}_1^2 +$$
$$+ \left[\left(A_4 + A_5\right)\tilde{f}_1^2 + A_6 - A_7\eta + A_9\right]\tilde{f}_2^2\} + C_0$$

where $\tilde{f}_1 = \frac{f_1}{h}, \tilde{f}_2 = \frac{f_2}{h}$; C_0 is an addend independent of f_1 and f_2, $\eta = \frac{\sigma_x r}{Eh}$. The coefficients A_i $(i = 1,2,...,9)$ are calculated by the following formulas

$$A_1 = \frac{3}{8}\left(1-\nu^2\right)\left[\left(1 + 2\overline{\gamma}_c^{(1)}\right)d_{m_1}^4 + n_1^4\right]$$

$$A_2 = \left(d_{m_1}^2 + n_1^2\right)^2 + \frac{1-\nu^2}{a^2}\frac{d_{m_1}^4}{\left(d_{m_1}^2 + n_1^2\right)^2} +$$
$$+ \frac{2}{a^2}\left(\mu_c^{(1)} + \mu_s^{(1)}\right)d_{m_1}^2 n_1^2$$

$$A_3 = \frac{1-\nu^2}{a^2} d_{m_1}^2 h^*$$

$$A_4 = \frac{3}{4}\delta\left(1-\nu^2\right)\left\{\left(d_{m_1}n_2 - d_{m_2}n_1\right)^4 \cdot \right.$$

$$\cdot\left[\frac{1}{\left(b_1^2+b_3^2\right)^2} + \frac{1}{\left(b_2^2+b_4^2\right)^2}\right] +$$

$$\left. +\left(d_{m_1}n_2 + d_{m_2}n_1\right)^4\left[\frac{1}{\left(b_2^2+b_3^2\right)^2} + \frac{1}{\left(b_1^2+b_4^2\right)^2}\right]\right\}$$

$$A_5 = -3\left(1-\nu^2\right)d_{m_1}^2 d_{m_2}^2 \bar{\gamma}_c^{(1)}\sigma_1,$$

$$A_6 = \left(d_{m_2}^2 + n_2^2\right)^2 + \frac{1-\nu^2}{a^2}\frac{d_{m_1}^4}{\left(d_{m_2}^2+n_2^2\right)^2} +$$

$$+\frac{2}{a^2}\left[\eta_c^{(1)}\sigma_1 d_{m_2}^4 + \mu_c^{(1)}\sigma_2 d_{m_2}^2 + \right.$$

$$\left. +\eta_{s1}^{(2)}\sigma_3\left(n_2^2-1\right)^2 + \mu_s^{(2)}\sigma_4 d_{m_1}^2 n_2^2\right]$$

$$A_7 = \frac{1-\nu^2}{a^2}\left(1+2\bar{\gamma}_c^{(1)}\right)d_{m_2}^2 h^*$$

$$A_8 = \frac{48\left(1-\nu^2\right)r^5}{Eh^5 L_1}\left[\tilde{q} - \tilde{q}_0\left(n_1^2 + d_{m_1}^2\right)S_1\right]$$

$$A_9 = \frac{48\left(1-\nu^2\right)r^5}{Eh^5 L_1}\left[\tilde{q} - \tilde{q}_0\left(n_2^2 + d_{m_2}^2\right)S_2\right]$$

$$S_1 = \frac{1}{2} - \frac{\sin 2d_{m_1}\xi_1}{4d_{m_1}}, \quad S_2 = \frac{1}{2} - \frac{\sin 2d_{m_2}\xi_1}{4d_{m_2}}$$

where

$$\delta = 1, \quad \sigma_1 = \frac{1}{k}\sum_{i=1}^{k}\sin^2 n_2\theta_i, \quad \sigma_2 = \frac{1}{k}\sum_{i=1}^{k}\cos^2 n_2\theta_i$$

$$\sigma_3 = \frac{1}{k_1+1}\sum_{j=1}^{k_1}\cos^2 d_{m_2}\xi_j, \quad a^2 = \frac{h^2}{12r^2}$$

$$\sigma_4 = \frac{1}{k_1+1}\sum_{j=1}^{k_1}\sin^2 d_{m_2}\xi, \quad \bar{\gamma}_c^{(1)} = \frac{F_c k}{2\pi rh}$$

$$\eta_c^{(1)} = \frac{E_c\left(J_{yc} + h_c^2 F_c\right)k}{2\pi r^3 hE}\left(1-\nu^2\right)$$

$$\bar{\mu}_s^{(1)} = \frac{J_{kp.s}}{Lhr^2}, \quad \mu_c^{(1)} = \frac{G_c\left(1-\nu^2\right)J_{kp.c}k}{2\pi r^3 hE}$$

$$\eta_{s1}^{(2)} = \frac{E_s\left(I_{xs} + h_s^2 F_s\right)\left(k_1+1\right)\left(1-\nu^2\right)}{EhL_1 r^2}$$

From conditions (6) we get the set of equations

$$2A_1\tilde{f}_1^2 + \left(A_2 - A_3\eta + A_8\right) = 0$$

$$\left(A_4 + A_5\right)\tilde{f}_1^2 + A_6 - A_7\eta + A_9 = 0$$

and from it we find the parameter of critical stresses of general stability loss taking into account the preliminary buckling of the casing

$$\eta = \frac{A_6 + A_9 - \dfrac{\left(A_2+A_8\right)\left(A_4+A_5\right)}{2A_1}}{A_7 - \dfrac{A_3\left(A_4+A_5\right)}{2A_1}} \tag{9}$$

and relative deflection of the shell in the state preceding the general stability loss:

$$\tilde{f}_1^2 = -\frac{A_2 + A_8 - A_3\eta}{2A_1} \tag{10}$$

It should be noted that when $A_8 = A_9 = 0$ equations (9) and (10) transform to the formulas given in [1] and corresponding to shells without medium. The parameter η should be compared with η_p that corresponds to the least critical load with the bending of longitudinal ribs without taking into account the shell buckling. While calculating the critical load parameter, the minimization of η is carried out with respect to the form parameters m_2 and n_2. The domain of each of these parameters is constructed in the vicinity of those values of m_1 and n_1 that correspond to the minimum of η_p.

IV. RESULTS AND CONCLUSIONS

The results of the calculation of critical load parameter η taking into account the preliminary buckling of the shell are given in Table 1. The domain of reinforcement parameters is chosen so that the condition $\eta_p = \alpha\eta_{ob}$ ($\alpha \in [2;3]$), η_{ob} is critical stresses parameters of the shell) is satisfied. The following parameters are common for these shells: $h^* = 1/40;\ \xi_1 = 2;$ the number of lateral ribs $k_1 = 3;$ longitudinal ribs of cross-section with the ratio of height to width $\psi_1 = 14$. Ratio of the area of lateral section of longitudinal ribs to the cross-section of the shell varies $\bar{\gamma}_c^{(1)} = 0,6;1$. The number of longitudinal ribs also varies: $k = 16, 24, 32$. Annular ribs had the following characteristics: $\frac{\eta_{s1}^{(2)}}{a^2(k_1+1)} = 8,4$; $\frac{\mu_s^{(2)}}{a^2(k_1+1)} = 0,054$. The following values were accepted for the filler: $\tilde{q}/\tilde{q}_0 = 3;\ \tilde{q}_0/E = 0,002$

As it is seen from Table 1, for the considered shells the ratio $\eta/\eta_p \in [0,82;1]$. The ratio η/η_p increases with the growth of the number of longitudinal ribs at the same their total cross-section area. The numbers m_2 and n_2 coincide with the numbers m and n corresponding to η_p or differ insignificantly from them. The form of stability loss with the bending of ribs may respond to symmetric buckling with respect to the mean surface.

Table 1. Results of the calculation of critical load parameter η taking into account the preliminary buckling of the shell

$\bar{\gamma}_c^{(1)}$	k	η_{ob}	η_p	η	$\tilde{f}_1$	η/η_p
0.6	16	0.685	2.005	1.651	1.961	0.82
	24	0.734	1.563	1.383	1.513	0.88
	32	0.758	1.344	1.342	1.731	1.00
1	16	0.798	1.886	1.753	1.481	0.93
	24	0.891	1.842	1.743	1.403	0.95
	32	0.759	1.719	1.701	1.751	0.99

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