Abstract- This paper describes a new stochastic heuristic algorithm in engineering problem optimization especially in power system applications. An improved particle swarm optimization (PSO), called Adaptive particle swarm optimization (APSO), mixed with simulated annealing (SA) that will be named APSO-SA is introduced. This algorithm uses a novel PSO algorithm (APSO) to increase convergence rate and incorporate the ability of SA to avoid being trapped in local optimum. The APSO-SA algorithm efficiency is verified using some benchmark functions. This paper presents the application of APSO-SA to find optimal location, type and size of flexible AC transmission system devices. Two types of FACTS devices, Thyristor Controlled Series Capacitor (TCSC) and Static VAR Compensator (SVC) are considered. The main objectives of presented method are increasing the voltage stability index and over load factor, decreasing the cost of investment and total real power losses in the power system. In this regard, two cases namely, single-type devices (same type of FACTS devices) and multi-type devices (combination of TCSC, SVC) are considered. Using the proposed method, the locations, type and sizes of FACTS devices are obtained for reaching the optimal objective function. APSO-SA is used to solve the above non–linear programming problem for better accuracy and fast convergence. The presented method expands the search space, improves performance and accelerates to the speed convergence, in comparison with the standard PSO algorithm. The optimization results are compared with standard PSO method. This comparison confirms the efficiency and validity of the proposed method. The proposed approach is examined and tested on IEEE 14-bus systems by MATLAB software. Numerical results demonstrate that the APSO-SA is fast and has much less computational cost.

Keywords: FACTS Devices, Optimal Location, PSO Algorithm, SA Algorithm, APSO-SA Algorithm.

I. INTRODUCTION

Flexible AC Transmission System (FACTS) has received much attention in the last decades. It uses high current power electronic devices to control the voltage, power flow, stability, etc. of a transmission system. FACTS devices can be connected to a transmission line in various ways, such as in series, shunt, or a combination of series and shunt. The term and definition of various FACTS devices are described in references [1, 2]. FACTS devices are very effective and capable of increasing the power transfer capability of a line, insofar as thermal limits permit, while maintaining the same degree of stability [3, 4].

In recent years, with the deregulation of the electricity market, due to the competition between utilities the number of unplanned delivered power increases. If these exchanges are not controlled, some lines may become overloaded. These devices control the power flow in the network, reduce the flow in overloaded lines, thereby resulting in increase loadability, low system losses, improved stability of network and reduced cost of production [1, 5, 6]. It is important to find the location, type and size of these devices because of their significant costs. Studies and realizations have shown their capabilities in steady state or dynamic conditions.

The reference [7] provides an idea regarding the optimal locations of FACTS devices, without considering the investment cost of FACTS device and their impact on the generation cost. The optimal location with considering the generation cost of the power plants and investment cost of the FACTS devices studied in [8]. The reference [9] discusses about optimal location problem by power loss reduction. The main objective of this paper is to develop an algorithm to find and choose the optimal location, type and size of FACTS devices based on the Economic saving function, which obtained by energy loss reduction. In this paper presents the PSO and APSO-SA methods for ascertaining optimal location of FACTS devices to achieve minimum cost of FACTS devices, total real power losses in the power system and to improve system loadability (SL), while satisfying the power system constraints, for single and multi-type FACTS devices. In the single type case variables for the optimization of each device are its location in the network and its setting. In the case of multi-type devices, the type of device is taken as additional variable for optimization.
This paper is organized as follows. Following the introduction, mathematical models is described in section 2. Then in section 3, objective function is described. In section 4, the proposed method for optimal location of FACTS devices is discussed in detail and section 5, implemented algorithm is described. The simulation results are given in section 6. Finally, a brief conclusion appears in section 7.

II. MATHEMATICAL MODELS

A. Steady State Models of FACTS Devices

For static applications, FACTS devices can be modeled by Power Injection Model (PIM) [7, 8, 10, 11]. The injection model describes the FACTS as a device that injects a certain amount of active and reactive power to a node, so that the FACTS device is represented as PQ elements. The PIM doesn’t destroy the symmetrical characteristic of the admittance matrix and allows efficient and convenient integration of FACTS devices in to existing power system analytical tools. This is the main advantage of PIM.

TCSC:

Figure 1 shows the model of transmission line with TCSC connected between buses \(i\) and \(j\). The transmission line is represented by its lumped \(\pi\) equivalent parameters.

![Figure 1](image1.png)

During the steady state condition, the TCSC can act as capacitive or inductive mode, respectively to decrease or increase the impedance of branch. The TCSC is modeled with variable series reactance. Its value is function of the reactance of line, \(X_L\), where the device is located. The upper and lower limit of TCSC reactance is given in (1).

\[
-0.8X_L \leq X_{\text{TCSC}} \leq 0.2X_L \quad \text{p.u.}
\]  

The corresponding power injection model of TCSC incorporated within the transmission line is shown in Figure 2 [12-14]. The difference of line admittance, before and after installation of TCSC is given in (2).

![Figure 2](image2.png)

\[
\Delta Y_{ij} = Y'_{ij} - Y_{ij} = (g'_{ij} + jb'_{ij}) - (g_{ij} + jb_{ij})
\]

where,

\[
g_{ij} = \frac{r_{ij}}{\sqrt{x_{ij}^2 + x_{ij}^2}} \quad , \quad b_{ij} = \frac{-x_{ij}}{\sqrt{x_{ij}^2 + x_{ij}^2}}
\]

\[
g'_{ij} = \frac{r_{ij}}{\sqrt{(x_{ij} + x_{\text{TCSC}})^2 + (x_{ij} + x_{\text{TCSC}})^2}} \quad , \quad b'_{ij} = \frac{-(x_{ij} + x_{\text{TCSC}})}{\sqrt{(x_{ij} + x_{\text{TCSC}})^2 + (x_{ij} + x_{\text{TCSC}})^2}}
\]

When TCSC is installed in the line between buses \(i\) and \(j\), the reformed admittance matrix is obtained from (4).

![Figure 3](image3.png)

\[
Y'_{bus} = Y_{bus} + \begin{bmatrix}
0 & 0 & 0 & \cdots & 0 & 0 & 0 \\
0 & \Delta Y_{ij} & 0 & \cdots & 0 & 0 & 0 \\
0 & 0 & \Delta Y_{ij} & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

\[
V_{bus} = \begin{bmatrix}
V_i \\
V_j \\
\end{bmatrix}
\]

\[
I_{bus} = \begin{bmatrix}
I_{bus}^X \\
I_{bus}^Y \\
\end{bmatrix}
\]

SVC:

The main purpose of SVC is voltage controlling at weak points in the network. Figure 3 shows the single line diagram of compensated transmission line with SVC at bus \(j\) and its power injection model is represented in Figure 4. In this study, the SVC is treated as a variable capacitance, where \(I_{\text{SVC}}\) is the complex injected current of SVC at node \(j\) [14]. It can be expressed as follows:

\[
I_{\text{SVC}} = jB_{\text{SVC}} \cdot V_j
\]

![Figure 4](image4.png)

\[
\text{The SVC can behavior as capacitive or inductive mode to absorb or inject reactive power, respectively. The SVC can be represented by a shunt variable susceptance inserted in the bus or at the midpoint of the transmission line. The SVC is a voltage controlling device and its susceptance must be determined for regulation of bus voltage at the desired value. The SVC nominal values are corresponding to power system. In this paper, we considered as below:}
\]

\[
-100 \leq Q_{\text{SVC}} \leq +100 \quad \text{Mvar}
\]
When the SVC is installed at node \( j \), the reformed admittance matrix can be expressed as (7).

\[
\begin{bmatrix}
0 & 0 & 0 & \ldots & 0 & 0 & 0 \\
0 & 0 & 0 & \ldots & 0 & 0 & 0 \\
0 & 0 & Y_{\text{SVC}} & \ldots & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & \ldots & 0 & 0 & 0 \\
0 & 0 & 0 & \ldots & 0 & 0 & 0 \\
0 & 0 & 0 & \ldots & 0 & 0 & 0 \\
\end{bmatrix}
\]

B. Power System Model

The power flow equations with FACTS devices are given as below:

\[
P_{\text{gen}} - P_{\text{load}} = \sum_{j=1}^{n} \left| V \right|^2 \left[ G_{\text{FACTS}} \cos \delta_{ij} + B_{\text{FACTS}} \sin \delta_{ij} \right] = 0
\]

\[
Q_{\text{gen}} - Q_{\text{load}} = \sum_{j=1}^{n} \left| V \right|^2 \left[ G_{\text{FACTS}} \sin \delta_{ij} - B_{\text{FACTS}} \cos \delta_{ij} \right] = 0
\]

\[
\left| V \right|_{\min} \leq \left| V \right| \leq \left| V \right|_{\max}
\]

\[
\delta_{ij} \leq \delta_{ij}^{\max}
\]

where, 

- \( P_{\text{gen}}, Q_{\text{gen}} \): Generated real and reactive power at bus \( i \);
- \( P_{\text{load}}, Q_{\text{load}} \): Real and reactive power of load at bus \( i \);
- \( n \): Number of buses;
- \( G_{\text{FACTS}}, B_{\text{FACTS}} \): Real part of \((i, j)\)th element of network admittance matrix included FACTS devices;
- \( \delta_{ij} \): Difference of phase angle between buses \( i \) and \( j \);
- \( \left| V \right|_{\min}, \left| V \right|_{\max} \): Maximum and minimum voltage magnitude at bus \( i \).

III. APSO-SA ALGORITHM

In recent years, many optimization algorithms are introduced. Some of these algorithms are traditional optimization algorithms. Traditional optimization algorithms use exact methods to find the best solution. The idea is that if a problem can be solved, then the algorithm should find the global best solution. As the search space increases the cost of these algorithms increases. Therefore, when the search space complexity increases the exact algorithms can be slow to find global optimum.

There are several stochastic algorithms such as: Genetic algorithms (GA) (Holland, 1975), Guided Local Search (GLS) (Voudouris, 1997), Tabu Search (TS) (Glover, 1989, 1990), Variable Neighbourhood Search (VNS) (Mladenovic and Hansen, 1997), Iterated Local Search (ILS) (Stützle, 1999), Simulated Annealing (SA) (Kirkpatrick et al. 1983), Greedy Randomized Adaptive Search Procedure (GRASP) (Feo and Resende, 1995), Memetic Algorithms (MA) (Moscato, 1989), Scatter Search (SS) (Cung et al. 1997), Ant Colony Optimization (ACO) (Marco Dorigo et al. 1999), Particle Swarm Optimization (PSO) (Kennedi and Eberhart 1995) and Shuffled Frog Leaping algorithm (SFL) (Eusuff, Lansey 2003). Each of these algorithms has its characteristics. Particle swarm optimization (PSO) and simulated annealing (SA) are two efficient and well known stochastic algorithms.

A. The Standard PSO Algorithm

A particle swarm optimizer is a population based stochastic optimization algorithm modeled based on the simulation of the social behavior of bird flocks. PSO is a population-based search process where individuals initialized with a population of random solutions, referred to as particles, are grouped into a swarm. Each particle in the swarm represents a candidate solution to the optimization problem, and if the solution is combination of variables, the particle can correspondingly be a vector of variables. In a PSO system each particle is “flown” through the multidimensional search space, adjusting its position in the search space according to its own experience and that of neighboring particles. The particle therefore makes use of the best position encountered by itself and that of its neighbors to position itself toward and optimal solution. The performance of each particle is evaluated using a predefined fitness function, which encapsulates the characteristics of the optimization problem.

Generally, a numerical optimization problem can be described as follows [15]:

\[
\min \quad F(X), \quad X = [x_1, x_2, \ldots, x_N]^T
\]

s.t. \( x_i \in [a_i, b_i], \quad i = 1, 2, \ldots, N \) (12)

The core operation of PSO is the updating formula of the particles, i.e. the velocity updating equation and position updating equation. The global optimizing model proposed by Shi and Eberhart (1999) is as follows:

\[
v_{id}(t+1) = \omega v_{id}(t) + C_1 r_1(v_{id}(t) - x_{id}(t)) + C_2 r_2(p_{pd} - x_{id}(t))
\]

where, \( v_{id} \) is the velocity of particle \( i \)th in dimension \( d \)th (is the particle position), \( \omega \) is the inertia weight factor, \( C_1 \) and \( C_2 \) are two positive constant parameters called acceleration coefficients. Rand1 and Rand2 are the random functions in the range \([0, 1]\), \( p_{pd} \) is the best position of the \( i \)th particle in dimension \( d \)th and \( p_{pd} \) is the best position among all particles in the swarm.

B. Simulated Annealing

Simulated Annealing (Metropolis et al. 1956, Kirkpatrick et al. 1983) [16, 17] is a metastrategy local search method that attempts to avoid producing the poor local minimum inherent in the steepest ascent method. It is a metaheuristic algorithm used to navigate through the space of solutions containing many local minimum and has been applied to many combinatorial optimization problems. The main idea behind Simulated Annealing is an analogy with the way in which liquids freeze and crystallize. When liquids are at a high temperature their molecules can move freely in relation to each other. As
the liquid’s temperature is lowered, this freedom of movement is lost and the liquid begins to solidify. If the liquid is cooled slowly enough, the molecules may become arranged in a crystalline structure. The molecules making up the crystalline structure will be in a minimum energy state. If the liquid is cooled very rapidly it does not form such a crystalline structure, but instead forms a solid whose molecules will not be in a minimum energy state.

The fundamental idea of Simulated Annealing is therefore that the moves made by an iterative improvement algorithm are like the rearrangements of the molecules in a liquid that occur as it is cooled and that the energy of those molecules corresponds to the cost function which is being optimized by the iterative improvement algorithm. Thus, the simulated annealing algorithm aims to achieve a global optimum by slowly converging to a final solution, making downwards moves with occasional “upwards” moves (the probability of these occurring decreasing with the “temperature”) and thus hopefully ending up in a global optimum. This is in contrast to the greedy approach of only considering the move which results in the largest possible decrease (if minimizing) in the objective function, which resembles a rapid cooling of a liquid to a solid, and thus according to the hypothesis, resulting in a local optimum rather than a global optimum.

In SA algorithm, the improvements are obtained by choosing another solution \((x')\) that belongs to the neighborhood of the current solution \((x_0)\). When the current solution changes from \(x_0\) to \(x'\), the objective function will also change, namely, \(\Delta = f(x') - f(x_0)\), where \(f(x)\) is the value of the objective function at \(x\). For the minimization problem, if \(\Delta \geq 0\), the new solution will be accepted. If \(\Delta < 0\), the new solution will be accepted with the probability \(\exp(-\Delta / T)\), where \(T\) is the temperature (this is simply implemented by choosing a random number in the range from 0 to 1) and comparing this with the probability; if it is less, the new solution will be accepted otherwise it will be rejected. Generally, the algorithm starts from a high temperature, and then the temperature is gradually decreases. At each temperature, the search will be performed for a certain number of iterations, which is called the temperature length. When the termination condition is satisfied, the algorithm will stop.

The most significant character of SA is the probabilistic jumping property, i.e. a worse solution has a probability to be accepted as the new solution. Moreover, by adjusting the temperature, such a jumping probability can be controlled. In particular, the probability is rather high when temperature is high and decreases as the temperature decreases; and when the temperature tends to zero the probability approaches to zero so that only better solution can be accepted. It has been theoretically proved that under certain conditions SA is globally convergent in probability 1.

C. APSO-SA Algorithm

Slow convergence of PSO before providing an accurate solution is a drawback, closely related to its lack of any adaptive accelerators in the velocity updating formulae. In Equation (13), \(C_i\) and \(C_j\) determine the step size of the particles movements through the \(p_{gd}\) and \(p_{id}\) respectively. In the original PSO, these step sizes are constant and for the all particles are same. For doing more sensitive and faster movements, new step sizes can be modified, which they should accelerate the convergence rate.

In each iteration, the value of objective function is a criterion that presents the relative improvement of this movement in respect to the previous iteration movement. Thus the difference between the values of objective function in the different iterations can select as the accelerators. Adding two additional coefficients to the original step sizes in the Equation (13), it causes to adaptive movements. Therefore, velocity updating formula turns to the following form.

\[
\begin{align*}
\dot{v}_{id}(t+1) &= W\text{ }\dot{v}_{id}(t) + C_1\text{ }\text{rand}\text{ }\ast (f(p_{gd}(t)) - f(x_{id}(t))) \ast \\
&\ast (p_{id} - x_{id}(t)) + C_2\text{ }\text{rand}\text{ }\ast (f(p_{gd}(t)) - f(x_{id}(t))) \\
&\ast (p_{id} - x_{id}(t)) \tag{15}
\end{align*}
\]

where, \(f(p_{gd}(t))\) is the best fitness function that is found by rth particle and \(f(p_{id}(t))\) is the best fitness function that is found by swarm up to now and other parameters are chosen the same as section III.A. Globally optimize an objective function in a given search domain consists in finding its global optimum without being trapped in any local optimum. When strongly multi-modal problems are being optimized, PSO algorithm usually suffers from the premature suboptimal convergence (simply premature convergence or stagnation) which occurs when some poor particles attract the swarm, due to a local optimum or bad initialization, preventing further exploration of the search space. According to [18], although PSO finds good solutions much faster than other evolutionary algorithms, it usually can not improve the quality of the solutions as the number of iterations is increased. The rational behind this problem is that particles converge to a single point, which is on the line between the global best and personal best positions. This point is not guaranteed to be even a local optimum. Proofs can be found in [19].

Another reason for this problem is the fast rate of information flow between particles, resulting in the creation of similar particles (with a loss in diversity) which increases the possibility of being trapped in local minima [20]. This feature prevents standard PSO from being really of practical interest for a lot of applications. In general, any mechanism that can increase diversity will help in preventing premature convergence. In fact, to overcome this issue, a "hybrid" method can be proposed. By combining APSO with SA algorithm, we can get a new mixed optimization approach, called APSO-SA.

Using of jumping property of SA can help to more diversification that by it the algorithm escapes from local optimum. Fast and adaptive properties of APSO will help to rapid convergence, when SA combines with APSO. As
mentioned in section 3.2. SA accepts worse solutions with a probability of \( \exp(-\Delta/T) \). When algorithm becomes trapped in a local optimum valley it can jump of valley with a probability led to more diversity. So, with employing both of SA and APSO algorithms, to develop a new mixed algorithm APSO-SA, we can get full use of the strong quick convergence ability of APSO and the strong local search ability of SA and offsets the weaknesses of each other. In fact, APSO-SA has rapid convergence but not premature convergence.

In APSO-SA algorithm, we name every point which is found by equation (16), the temporary point \( x_{gd}(p) \) \((x_{gd}(p) = x_{gd}(t+1))\). If \( x_{gd}(p) \) is better than \( x_{gd}(t) \), it will be accepted and if it is worse than \( x_{gd}(t) \), we will accept it with probability of \( \exp(-\Delta/T) \), \((\Delta = f(x_{gd}(p)) - f(x_{gd}(t))).\) This process is performed for all particles. When a temporary point rejected, that we name it a detoured particle \( x_{gd}(d) \), it is given back in the opposite direction of the previous movement. These descriptions are formulated by the following equations.

\[
x_{gd}(p) = x_{gd}(t) + v_{gd}(t) \\
\Delta = f(x_{gd}(p)) - f(x_{gd}(t)) \\
\text{if } \Delta < 0 \text{ then } x_{gd}(t+1) = x_{gd}(p) \quad (16) \\
\text{if } \Delta \geq 0 \text{ then } x_{gd}(d) = x_{gd}(p) + \alpha * v_{gd}(t) \quad x_{gd}(t+1) = x_{gd}(d) \\
\text{where,} \\
\alpha = \begin{cases} +1 & \text{probability } = e^{-\Delta/T} \\
-1 & \text{other wise} \end{cases} \\
(17)
\]

In general, the proposed APSO-SA algorithm works as follows. First, the algorithm parameters such as number of particles, initial particles and velocities, constants of \( C_1 \) and \( C_2 \), \( T_0 \) and annealing schedule and any other parameters are initialized. Then the algorithm starts with the initial swarm as initial solutions. Computing new velocities using APSO algorithm, temporary positions are calculated. For each particle, if \( \Delta < 0 \) then the solution will be accepted as a better solution, otherwise worse solution will be accepted with probability of \( \exp(-\Delta/T) \), and detoured particle is turned back to the opposite direction of the traveled route, equations 16 and 17. This procedure causes diversification and escaping from local optimum. This process is iterated for all the particles in the swarm. Afterwards, the annealing schedule is performed. If one of the termination conditions is satisfied then the algorithm stops else the proposed procedure is iterated.

The general pseudo-code for APSO-SA algorithm is given in Appendix A.

1. The term \( f(p_{gd}(t)) - f(x_{gd}(t)) \) and \( f(p_{gd}(t)) - f(x_{gd}(t)) \) are named local and global adaptive coefficients, respectively. In the each iteration, the former term defines the movement step size in the direction of best position which is found by \( i \)th particle in dimension \( i \)th and the later term defines movement step size in the direction of the best optimum point which ever have been found by the swarm, adaptively. In other words, the adaptive coefficients decrease or increase the movement step size relative to being close or far from the optimum point, respectively. By means of this method, velocity can be updated adaptively instead of being fixed or changed linearly. Therefore, using the adaptive coefficients, the convergence rate of the algorithm will be increased that it is performed by the proportional large or short steps. Here, this fast version of the PSO algorithm is called Fast PSO (APSO).

2. Stochastic optimization approaches have problem dependent performance. This dependency usually results from the parameter setting of each algorithm. Thus using different parameter settings for APSO algorithm, which is a stochastic optimization algorithm, result in high performance variances. In general, no single parameter setting exists which can be applied to all problems. Therefore, all parameters of APSO should be determined optimally, by trial and error.

3. There are three stopping criteria. The first criterion is related to the maximal number of iterations of the algorithm, the second one is when no improvement has been made for a certain number of iterations in the best solution and the third one is when a satisfactory solution is found. The adaptive version of PSO is proposed for continuous variable functions. Moreover, the main idea of fasting can be applied to the discrete form of the PSO [21].

5. Increasing the value of the inertia weight, \( w \), will increase the speed of the particles resulting in more exploration (global search) and less exploitation (local search). On the other hand, decreasing the value of \( w \) will decrease the speed of the particle resulting in more exploitation and less exploration. Thus, an iteration-dependent weight factor often outperforms a fixed factor. The most common functional form for this weight factor is linear, and changes with step \( i \) as follows:

\[
W_{(i+1)} = W_{\text{max}} - \frac{W_{\text{max}} - W_{\text{min}}}{N_{\text{iter}}} \times i \\
(18)
\]

where \( N_{\text{iter}} \) is the maximum number of iterations and \( W_{\text{max}} \) and \( W_{\text{min}} \) are selected to be 0.9 and 0.1, respectively.

6. The initial temperature \( T_0 \) and the annealing way play important roles in SA and may affect the performance of APSO-SA. In the following simulations, the initial temperature is set by the following empirical formula [22]:

\[
T_0 = \frac{f(p_{gd})}{\ln(0.2)} \\
(19)
\]

The \( p_{gd} \) is the best position between the all particles in swarm. As for the annealing way, exponential annealing function, i.e. \( T(t+1) = \theta * T(t) \), is employed, where \( 0 < \theta < 1 \) denotes the annealing rate.

7. Stop condition typically can be happen, when no improvement has been made for a certain number of iteration or the maximum number of iteration has been
reached or when $T_0$ be smaller than the smallest typical temperature ($T_{\text{min}}$).

8. Lastly, the proposed APSO is still a general optimization algorithm that can be applied to any real world continuous optimization problems.

In this paper, we will apply such an approach for a multi objective function and we will compare the obtained results from APSO-SA with standard PSO algorithm for IEEE 14-bus system.

IV. IMPLEMENTATION OF SUGGESTED ALGORITHM

For implemented algorithm we used two cases:

A. Single-Type case

In this case, the goal of optimization is to find the best location in the power system. Therefore, a configuration is represented with two variables as below:

1. The first variable corresponds to the location of device and it contains the numbers of the nodes or branches where the FACTS device (TCSC or SVC) is located. The possible values are identified in Table 1.

2. The second variable indicates the size of FACTS device. Its value will be normalized in the range of 0 to 1. According to the FACTS devices model, those real value, $z_{\text{real}}$, can be calculated as below:

$$z_{\text{real}} = z_{\text{min},F} + (z_{\text{max},F} - z_{\text{min},F}) z_F$$

where, $z_{\text{min},F}$ and $z_{\text{max},F}$ are respectively minimum and maximum setting value of device and $z_F$ is its normalized value.

Figure 5a shows single line diagram of IEEE 14 bus system with FACTS device. Figure 5b illustrates the configuration of coded solution for single type FACTS devices.

B. Multi-Type Case

In this case, the goal of optimization is to find the best location for two FACTS devices (TCSC and SVC). Hence, a configuration is represented with three variables as below [7]:

1. The first element corresponds to the location of the devices and it contains the numbers of the elements (nodes and branches) where the FACTS devices are located.

2. The second string indicates the type of devices. A value is assigned to each type of FACTS devices: 0 for no devices, 1 for TCSC and 2 for SVC.

3. The third element shows the size of FACTS devices. It may take a discrete values normalized to be in range of 0 to 1.

Figure 6 illustrates the configuration of coded solution for multi type FACTS devices with three coded strings.

Table 1. Numbering of the power system elements

<table>
<thead>
<tr>
<th>Values</th>
<th>Elements</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>bus 1</td>
</tr>
<tr>
<td>$n_b$</td>
<td>bus $n_b$</td>
</tr>
<tr>
<td>$n_b+1$</td>
<td>branch 1</td>
</tr>
<tr>
<td>$n_b+n_s$</td>
<td>branch $n_s$</td>
</tr>
</tbody>
</table>

2. The second variable indicates the size of FACTS device. Its value will be normalized in the range of 0 to 1. According to the FACTS devices model, those real value, $z_{\text{real}}$, can be calculated as below:

$$z_{\text{real}} = z_{\text{min},F} + (z_{\text{max},F} - z_{\text{min},F}) z_F$$

where, $z_{\text{min},F}$ and $z_{\text{max},F}$ are respectively minimum and maximum setting value of device and $z_F$ is its normalized value.

Figure 5a shows single line diagram of IEEE 14 bus system with FACTS device. Figure 5b illustrates the configuration of coded solution for single type FACTS devices.

V. PROPOSED OBJECTIVE FUNCTION

The main goal of optimization is to minimize the installation costs of FACTS devices and real power losses in power systems and to improve system load ability. The objective function is defined as sum of three terms with individual criteria. The first one is related to installation cost, the second part of the objective function concerns total real power losses in the power system and the third term corresponds to increasing load ability.

$$IC + P_{\text{Lk}} + [J - 1]$$

The optimal installation cost of FACTS device in US$ is given as below:

$$IC = C \ast S$$

where, $C$ is the installation cost of FACTS devices in US$/KVAR$. The installation cost of TCSC and SVC are taken from Siemens database and reported in [23]. The installation cost of various FACTS devices are given by:

$$C_{\text{TCSC}} = 1.5 S^2 - 713 S + 153750$$
$$C_{\text{SVC}} = 0.3 S^2 - 305 S + 127380$$
where, the $S$ is the operating range of FACTS devices in MVAR.

$$S = |Q_1| - |Q_2|$$  \hspace{1cm} (24)

where, the $Q_1$ and $Q_2$ are the reactive power flow in the line after and before installing FACTS device in MVAR, respectively.

The exact loss formula of a power system with $N$ buses is [24]:

$$P_{kk} = \sum_{j=1}^{N} \sum_{k=1}^{N} \left[ \alpha_{jk} (P_j P_k + Q_j Q_k) + \beta_{jk} (Q_j P_k - P_j Q_k) \right] \hspace{1cm} (25)$$

where $P_j, P_k$ and $Q_j, Q_k$, respectively, are the real and reactive powers injected at buses $j$ and $k$. The $\alpha_{jk}$ and $\beta_{jk}$ are the loss coefficients defined by:

$$\alpha_{jk} = \frac{r_{jk}}{V_j V_k} \cos(\delta_j - \delta_k) \hspace{1cm} (26)$$

$$\beta_{jk} = \frac{r_{jk}}{V_j V_k} \sin(\delta_j - \delta_k) \hspace{1cm} (27)$$

where, the $r_{jk}$ is the real part of the $jk$ element of impedance matrix ($[Z_{mm}]$). If single type FACTS devices are used, the total loss can be written as follows [24].

$$P_{kk} = P_{kk} - [P_p + P_c] \hspace{1cm} (28)$$

More than one device used at time, the total loss can be expressed as:

$$P_{kk} = P_{kk} - \sum [P_p + P_c]$$ \hspace{1cm} (29)

where $P_p, P_c$ are injected real powers by installed FACTS devices.

In (30) $J$ includes the indicating violation factor of line power flow limits and bus voltage limits [25]:

$$J = \prod_{\text{Line}} \left[ \frac{\overline{OVL}_{\text{bus}}}{\overline{OVL}_{\text{bus}}} \right] \prod_{\text{bus}} \left[ \frac{\overline{VS}_{\text{bus}}}{\overline{VS}_{\text{bus}}} \right] \hspace{1cm} (30)$$

where, the $OVL$ and $VS$ denote the line over load factor and bus voltage stability index, respectively.

The injected reactive power by SVC at connected bus is $Q_{SVC}$ and reactance of the line where TCSC is installed.

$$\alpha_{jk} = \frac{r_{jk}}{V_j V_k} \cos(\delta_j - \delta_k) \hspace{1cm} (26)$$

$$\beta_{jk} = \frac{r_{jk}}{V_j V_k} \sin(\delta_j - \delta_k) \hspace{1cm} (27)$$

where, the $r_{jk}$ is the real part of the $jk$ element of impedance matrix ($[Z_{mm}]$). If single type FACTS devices are used, the total loss can be written as follows [24].

$$P_{kk} = P_{kk} - [P_p + P_c] \hspace{1cm} (28)$$

More than one device used at time, the total loss can be expressed as:

$$P_{kk} = P_{kk} - \sum [P_p + P_c]$$ \hspace{1cm} (29)

where $P_p, P_c$ are injected real powers by installed FACTS devices.

In (30) $J$ includes the indicating violation factor of line power flow limits and bus voltage limits [25]:

$$J = \prod_{\text{Line}} \left[ \frac{\overline{OVL}_{\text{bus}}}{\overline{OVL}_{\text{bus}}} \right] \prod_{\text{bus}} \left[ \frac{\overline{VS}_{\text{bus}}}{\overline{VS}_{\text{bus}}} \right] \hspace{1cm} (30)$$

where, the $OVL$ and $VS$ denote the line over load factor and bus voltage stability index, respectively.

Objective function is optimized with the following constraints:

1. Line flow and bus voltage constraints.
2. FACTS device’s constraints

(i) $-0.8X_L \leq X_{TCSC} \leq 0.2X_L \hspace{1cm} (33)$

A. Constraints

Objective function is optimized with the following constraints:

1. Line flow and bus voltage constraints. This constraint is defined by Equation (30).
2. FACTS device’s constraints

(i) $-100 \text{MVAR} \leq Q_{SVC} \leq 100 \text{MVAR} \hspace{1cm} (34)$

where, the $X_{TCSC}$ and $X_L$ are injected reactance by TCSC and reactance of the line where TCSC is installed. The injected reactive power by SVC at connected bus is $Q_{SVC}$.

3. Power flow constraints

$$g(V, \theta) = 0 \hspace{1cm} (35)$$

where,

$$g(V, \theta) = \begin{cases} P(V, \theta) - P_{\text{net}}^{\text{bus}} & V_{\text{bus}} \geq 0.9 \text{pu} \\ \frac{Q(V, \theta) - Q_{\text{net}}^{\text{bus}}}{P_{\text{bus}}(V, \theta) - P_{\text{net}}^{\text{bus}}} & V_{\text{bus}} < 0.9 \text{pu} \end{cases} \hspace{1cm} (36)$$

where, $P$, and $Q$ are the calculated real and reactive power for $PQ$ bus, $P_{\text{net}}$, is the calculated real power for $PQ$ bus, $Q_{\text{net}}$ and $Q_{\text{net}}$ are the specified real and reactive power for $PQ$ bus, $V$ is the voltage magnitude at different buses and $\theta$ is the voltage phase angle at different buses.

B. Finding MSL

After the maximum numbers of iterations the value of $J$ is checked for the $p_{gd}$ particle. If it is equal to 1 then using that $p_{gd}$ particle, the current value of SL can be met out without violating line flow and bus voltage limit constraints and the $p_{gd}$ particle is saved with its cost of installation and SL. Then SL is increased by 1% and again the PSO or APSO-SA algorithms are run. If the value of $J$ for the $p_{gd}$ particle is not equal to 1 then the $p_{gd}$ particle is unable to meet the current SL and the $p_{gd}$ particle with $J = 1$, obtained in the previous run is considered as the best optimal settings and the SL corresponding to that $p_{gd}$ particle is considered as the MSL. The step by step procedure to find optimal installation cost of FACTS devices and the MSL is shown in Figure 7.

VI. SIMULATION RESULTS

The solutions for optimal location of FACTS devices to minimize the objective function for IEEE 14-bus system was obtained and discussed below. The test system data are taken from [26]. The location, setting of FACTS devices and optimal objective function value, total real power losses of power system and maximum system loadability (MSL) are obtained using the PSO and APSO-SA techniques for single-and multi-type devices and it is given in Table 2. In this table, the $P_{p_{gd}}$, $Q_{p_{gd}}$ and $P_{p_{gd}}$, $Q_{p_{gd}}$ are real and reactive power flow in the line $p = q$, before and after placing FACTS device, respectively. Convergence speed of the objective function with using the PSO and APSO-SA techniques for single and multi-type devices are shown in figures 9 to 11, respectively.

In the case of TCSC, it can be seen that the TCSC must be installed in line (5-6) with using PSO and APSO-SA techniques. It is observed that the improvement in
objective function value and total real power losses of the power system, but increasing installation cost of TCSC with using APSO-SA algorithm, in comparison with the standard PSO algorithm.

In the case of SVC, it is observed that placing SVC in a bus 13 with using PSO and APSO-SA techniques causes improvements in objective function value, total real power losses in the power system and MSL, but increased installation cost of SVC with using APSO-SA algorithm, in comparison to the standard PSO algorithm.

In the case of multi-type devices, improvement in objective function value, total real power losses of in the power system, installation cost and MSL with using APSO-SA algorithm, in comparison with the standard PSO algorithm was observed.

It is observed in Figs.8-10, speed convergence sensible in the case of multi-type in comparison with case of single-type.

VII. CONCLUSIONS

In this paper, the optimal location of FACTS devices are found to minimize the cost of installation and total real power losses of power system and improve system loadability, for single and multi-type FACTS devices using PSO and APSO-SA techniques. Simulations were performed on IEEE 14 bus system. Optimizations were performed on the parameters namely location of FACTS devices, their settings in the line for single-type FACTS devices. In the case of multi-type FACTS devices, the type of device to be placed is also considered as a variable in the optimization. In both single-and multi-type devices, it is observed that:

1. The APSO-SA algorithm improves acceleration of the convergence speed, in comparison with the standard PSO algorithm.
2. The APSO-SA algorithm expands the search space, in comparison with the standard PSO algorithm.
3. Decreasing the real power losses of power system with optimal location FACTS devices.
4. System loadability can be improved further after optimal placing of FACTS devices.
5. In the case of multi-type FACTS devices improve objective function, in comparison with the case of single-type.
6. Real and reactive power flow improved after placing FACTS devices when compared with before placing them.

APPENDIX

The pseudo code for APSO-SA algorithm

Initialize APSO-SA parameters.
(APSO procedure)
Loop:
REPEAT
For each particle $i$;
Evaluate the objective function of the particle $i$, i.e. $f(x_{i,0})$;
Update the global and local best positions and their objective function values;
Calculate the velocity by Equation (15);
(SA procedure)
Calculate the temporary position $x_{i,d}(p)$ by Equation (16);
Using the equation (17) calculate the new position;
Perform annealing schedule;
END of Loop
If stop condition is true then stop else go to Loop;

REFERENCES

Start

1. Preparation of database using line, bus, generator and load data, Get type of FACTS devices to be installed, number of particles and max. iterations.

2. Generation population of particles with the variables in normalized form

3. Check for proper arrangement of FACTS devices, compute line flows and voltage at buses for each particle, by denormalizing the variables, updating line data/bus data and running load flow using Newton Raphson method

4. Compute the fitness function using the eqn.(21) for each particle

5. Find the particle having minimum value of fitness function in the population, denote it by pbest

6. Compute the fitness function using the eqn.(21) for each particle

7. Create new population using the eqns (13) and (14) or using the eqns (15) and (16)

8. Increment SL by 1%

9. Stop

Figure 7. The Flowchart of PSO or APSO-SA algorithm for optimal location of FACTS devices

Table 2. Line flows before and after installing single- and multi-type FACTS devices, optimal setting and optimal objective function value, cost of installation, total real power losses and MSL with PSO and APSO-SA algorithm

<table>
<thead>
<tr>
<th>Case</th>
<th>Type of device used</th>
<th>From Bus</th>
<th>To Bus</th>
<th>$P_{ref}$ (pu)</th>
<th>$Q_{ref}$ (pu)</th>
<th>$P_{pqa}$ (pu)</th>
<th>$Q_{pqa}$ (pu)</th>
<th>$P_{pqb}$ (pu)</th>
<th>$Q_{pqb}$ (pu)</th>
<th>Device Setting (pu)</th>
<th>$F$</th>
<th>$P/(LK)(b)$ (118MW)</th>
<th>$P/(LK)(a)$ (MW)</th>
<th>$IC$ ($*10^{-6}$ US$)$</th>
<th>MSL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single Type</td>
<td>TCSC (PSO)</td>
<td>5</td>
<td>6</td>
<td>1.385</td>
<td>1.026</td>
<td>1.45</td>
<td>1.01</td>
<td>-0.47</td>
<td>0.258</td>
<td>59.208</td>
<td>0.0034</td>
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<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>TCSC (APSO-SA)</td>
<td>5</td>
<td>6</td>
<td>1.385</td>
<td>1.026</td>
<td>1.5</td>
<td>0.979</td>
<td>-0.61</td>
<td>0.237</td>
<td>58.8</td>
<td>0.0059</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>SVC (PSO)</td>
<td>13</td>
<td></td>
<td>-0.135</td>
<td>-0.058</td>
<td>-0.161</td>
<td>-0.037</td>
<td>-0.34</td>
<td>0.21</td>
<td>59.62</td>
<td>0.031</td>
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<td>-0.058</td>
<td>-0.164</td>
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<td>0.032</td>
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<td></td>
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<tr>
<td>Multi Type</td>
<td>TCSC (PSO)</td>
<td>5</td>
<td>6</td>
<td>1.385</td>
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<tr>
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<td>-0.153</td>
<td>-0.058</td>
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<td>53.95</td>
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<tr>
<td></td>
<td>TCSC (APSO-SA)</td>
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<tr>
<td></td>
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<td>1.026</td>
<td>-0.058</td>
<td>1.385</td>
<td></td>
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</tbody>
</table>


BIOGRAPHIES

Naser Mahdavi Tabatabaei was born in Tehran, Iran, 1967. He received the B.Sc. and the M.Sc. degrees from University of Tabriz (Tabriz, Iran) and the Ph.D. degree from Iran University of Science and Technology (Tehran, Iran), all in Power Electrical Engineering, in 1989, 1992, and 1997, respectively. Currently, he is a Professor of Power Electrical Engineering at International Ecoenergy Academy, International Science and Education Center and International Organization on TPE (IOTPE). He is also an academic member of Power Electrical Engineering at Seraj Higher Education Institute and teaches Power System Analysis, Power System Operation, and Reactive Power Control. He is the secretaries of International Conference and Journal on TPE (ICTPE, IJTPE). His research interests are in the area of Power Quality, Energy Management Systems, ICT in Power Engineering and Virtual E-learning Educational Systems. He is a member of the Iranian Association of Electrical and Electronic Engineers (IAEEE).

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