International Journal on
"Technical and Physical Problems of Engineering"
(IJTPE)
Published by International Organization of IOTPE

ISSN 2077-3528
IJTPE Journal
www.iotpe.com
ijtpe@iotpe.com

September 2012	Issue 12	Volume 4	Number 3	Pages 32-40

SINGULAR SPECTRAL ANALYSIS APPLIED FOR SHORT TERM ELECTRICITY PRICE FORECASTING

K. Afshar [1] N. Bigdeli [1] H. Shayeghi [2]

1. Electrical Engineering Department, Imam Khomeini International University, Qazvin, Iran
afshar@ikiu.ac.ir, bigdeli@ikiu.ac.ir
2. Technical Engineering Department, University of Mohaghegh Ardabili, Ardabil, Iran, hshayeghi@gmail.com

Abstract- In this paper, the data analysis and short term price forecasting in Iran electricity market as a market with pay-as-bid payment mechanism has been considered. The proposed method is a modified singular spectral analysis (SSA) method. SSA decomposes a time series into its principal components i.e. its trend and oscillation components, which are then used for time series forecasting effectively. The employed data are the experimental time series from Iran electricity market in its real size and is long enough to make it possible to take properties such as non-stationarity of market into account. The obtained results, discussed comprehensively, show that the method has a good ability in characterizing and prediction of the desired price time series.

Keywords: Deregulation, Electricity Market, Pay-as-Bid, Principal Component Analysis (PCA).

I. INTRODUCTION

By the transition of traditional power markets towards restructured ones, both costumers and producers require predicting some market indices to adjust their activities in the market. These indices being load and price indices are the essential factors for generating biding curves which in turn play a major role in the risk management as well as profit maximization of the generating and distribution companies (GenCo's and DisCo's) [1-2].

The pricing mechanism in an electricity market can be either uniform pricing (UP) or pay-as-bid (PAB) pricing. In each market, the maximum accepted bid block sets the market-clearing price (MCP). Under the UP structure, MCP is paid to every winning block. In the PAB structure, every winning block gets its bid price as its income [3-4]. Therefore, proper bidding strategy is critical in profit maximizing in the electricity markets and so the bid blocks should be generated regarding the market price indices. Usually, there is limited information about market indices. Therefore, producers and consumers rely on price forecast information to prepare their corresponding bidding strategies [5-6]. In UP markets with no congestion in the transmission network, MCP is the one important price index to forecast [7].

In the literature, there are many efforts for forecasting of MCP. For example, time series models [1], jump diffusion/mean reversion models [8], artificial neural networks [6, 9-10], GARCH models [11-12], wavelet transform [13] and ARIMA models [14] have been proposed for this purpose. However, if congestion exists in the network, some other price indices such as zonal market-clearing price (ZMCP) or locational marginal price (LMP) should be considered instead of MCP [15].

The PAB markets, on the other hand, experience a different situation. While in the literature, there are fewer efforts for price forecasting in this area, there are different price indices which may be important to forecast for a proper bidding strategy. The most important factors are the hourly Weighted Average Price (WAP), the hourly minimum and maximum accepted price. WAP in a PAB electricity market is defined as the average of the accepted price blocks in the market, where each price has been weighted by the amount of its underlying accepted energy in the market, for each hour. The hourly Minimum/Maximum Accepted Price is the lowest/highest value of the accepted price stages in the market, for the desired hour. In an un-congested market, WAP is used for profit computation and risk analysis for bidding at a special price. Minimum/maximum accepted price, on other hand, is related to the foremost/upmost price blocks in the bidding curves [2, 16].

Iran electricity market is a pool-based PAB electricity markets in which, the cap of affordable prices is predetermined by market regulator. Due to the congestion status of the market, the maximum accepted price is the cap price and so is constant [16]. Therefore, the topmost price index in this market is the hourly WAP is announced daily and used for profit computation and risk analysis for bidding at a special price. The characteristics and the predictability of WAP of Iran electricity market has been closely examined via various time series analysis methods by the authors in series of papers [2, 17-18]. Besides, a neural network model has been developed in [2] for forecasting WAP. Based on these analysis results, this time series performs a non-stationary seasonal chaotic behavior which also preserves almost the same average predictability properties in different time intervals [17]. These properties imply an important characteristic in this WAP time series: it is driven by a recurrent formula. By this expression, we mean that the WAP time series in each sample time (a desired hour) can be formulated in terms of its previous values. Therefore, it seems reasonable that one employs a recurrent- driven forecasting method which is takes the special characteristics of the time series into account.

Motivated by the above discussions, in this paper, a modified singular spectrum analysis method (SSA) [19-20] is used for forecasting WAP. SSA technique, is a novel and powerful technique of time series analysis and forecasting incorporating the elements of classical time series analysis, multivariate statistics, multivariate geometry, dynamical systems and signal processing [19, 21]. The aim of SSA is to make a decomposition of the original series into the sum of a small number of independent and interpretable components such as a slowly varying trend, oscillatory components and a structure-less noise. Some of these components are then used for time series forecasting. The employed method modifies the basic SSA method for two purposes: 1) to compensate the seasonality of the price time series regarding its trend, 2) to use the real data for short term price forecasting to avoid cumulative error. The proposed approach is then applied to forecast the WAP time series of Iran electricity market. The results are representative of the effectiveness of the method in spite of sever volatilities in WAP. This paper is organized as follows. In Section 2, the Iran electricity market is briefly introduced. The SSA methodology is described in Section 3 and the singular spectral analysis results of WAP time series is presented in Section 4. Forecasting with SSA and the proposed modified SSA method are described in section 5. The forecasting results and conclusions are brought in Sections 6 and 7, respectively.

II. THE IRAN ELECTRICITY MARKET

In this section, the Iran's electricity market is briefly introduced. For more details one may refer to [22]. Iran's electricity market was launched at 23 October 2003. In this market, 24 power generating companies (including 450 units) and 43 distribution companies participate in wholesale energy trade, every day [23]. The heart of the market is a mandatory pool. All producers and consumers have to send their bids before 10 am every day to the market. Once the electricity purchase and sale offers have been received, checked and accepted, they are matched by market operator, who calculates the maximum, minimum accepted price and the weighted average price (WAP) of each hour, and shares out production and demand amongst parties involved in the auction.

It should be noted that in Iran's electricity market the cap of affordable prices for all peak and off-peak hours is predetermined homogeneously by the market regulatory board. Therefore, no regulation concerning peak and off-peak prices is required in comparing/analyzing prices in these hours. Besides, this limitation confines the expected spikes in the market price indices to some extent. In the next stage, results of auction are sent to system operator, who draws up the provisional feasible daily schedule at 2 pm. System operator is responsible for secure, safe and reliable operation of interconnected grid [22].

Payments for generators are divided to two parts: the capacity payment and the energy payment. All the available capacities in the market receive a certain hourly fixed payment, which is set annually by the market regulatory board. Energy payment is the other part of generators payments. The payment mechanism of energy in the Iran's electricity market is PAB [22]. Therefore, an exact estimate of price indices is necessary for producers to offer their bids through proper bidding strategies. Hourly WAP is the most important price index which is announced publically in Iran's electricity market. The other important available index is the hourly required load (RL) time series. The term hourly required load is the total energy which is exchanged in the market for each hour. The cross correlation and joint predictability of WAP in terms of RL has been examined in [17]. From, these results, although WAP is correlated with RL, its nature is completely different with RL. Therefore, in this paper, just WAP is employed for the price index prediction via the proposed modified SSA method. As it will be seen, WAP shows self sufficient for the forecasting purpose due to its recurrence property.

III. THE SSA METHODOLOGY

Consider the real-valued nonzero time series $Y_T = (y_1, ..., y_T)$ of sufficient length *T*. The main purpose of SSA is to decompose the original series into a sum of series, so that each component in this sum can be identified as either a trend, periodic or quasi-periodic component (perhaps, amplitude-modulated), or noise. This is followed by reconstruction of original series [19].

The SSA technique, described in [20], consists of two complementary stages: decomposition and reconstruction. Each of these stages includes two separate steps, as well. At the first stage, the series is decomposed and at the second stage the original series is reconstructed and the reconstructed series is used for forecasting new data points. Following the methodology in [20], below a brief discussion of methodology of SSA technique is provided.

A. Stage 1: Decomposition

A.1. First Step: Embedding

In the first step of this stage, called the embedding step, the phase space of the time series is reconstructed. In other words, this step transfers the one-dimensional time series $Y_T = (y_1, ..., y_T)$ of length *T*, into a sequence of *L*-dimensional vectors $X_i = (y_{i-1}, ..., y_{i+L-2})^T$, $(i = 1, ..., K = T - L + 1)$. The single parameter of this embedding procedure is the window length *L*, $(1 < L < N)$. The K vectors X_i will form the columns of the $(L \times K)$ trajectory matrix X as:

$$X = \begin{bmatrix} y_0 & \cdots & y_{K-1} \\ \vdots & \ddots & \vdots \\ y_{L-1} & \cdots & y_{T-1} \end{bmatrix} \quad (1)$$

where, the trajectory matrix X is a Hankel matrix with equal elements on the diagonals $i+j=\text{const}$.

A.2. Second Step: Singular Value Decomposition (SVD)

The next step is the singular value decomposition (SVD) of the trajectory matrix X into a sum of rank-one bi-orthogonal elementary matrices. Denote the Eigen values of XX^T (where $(.)^T$ stands for matrix transpose) by $\lambda_i, i=1,...,L$ in descending order and let U_i and V_i denote the *i*th left and right orthonormal singular vectors of X, respectively (equivalent to the *i*th Eigen vector of XX^T and the *i*th Eigen vector of X^TX, respectively). Set:

$$d=\max(i, \text{ such that } \lambda_i>0)=\text{rank}(X) \quad (2)$$

Then the trajectory matrix X can be rewritten as:

$$X=X_1+...+X_d$$
$$X_i=s_iU_iV_i^T \quad (3)$$

where, s_i is the *i*th singular value of X (equivalent to the square root of the λ_i), and $X_i, i=1,...,d$ are matrices of rank one. In SSA literature, U_i is called *i*th factor empirical orthogonal function or simply EOF'; V_i is called *i*th principle component, the collections (s_i,U_i,V_i) is called the i th Eigen triple of the matrix X, and the set $\{s_i\}$ is called the spectrum of matrix X. It is shown in the literature that the above described decomposition is the optimal approximation of matrix X [19].

B. Stage 2: Reconstruction

B.1. First Step: Grouping

In this step, called the grouping step, it is made a partition of the indices set $j=1,...,d$ into M disjoint subsets $I_1,...,I_M$, corresponding to split the elementary matrices $X_i,\ i=1,...,d$ into M groups. Let $I=\{i_1,...,i_p\}$, then the resultant matrix X_I is defined as: $X_I=X_{i_1}+...+X_{i_p}$. By computing the resultant matrices X_I for $I=I_1,...,I_M$ and substituting them in Equation (3), it is resulted that:

$$X=X_{I_1}+...+X_{I_M} \quad (4)$$

where, the trajectory matrix X is represented as a sum of M resultant matrices. The choice of the sets $I_1,...,I_M$ is called the Eigen triple grouping. It should be noted that it is possible that some resultant matrices participate in original signal reconstruction or not, as it will be described in price forecasting later, in this paper.

B.2. Second Step: Diagonal Averaging

Diagonal averaging transfers each matrix $X_{I_n},\ n=1,\dots,M$ into a time series, which is an additive component of the initial series as in Equation (4). If z_{ij} stands for an element of a matrix *Z*, then the *k*th term of the resulting series is obtained by averaging z_{ij} over all *i*, *j* such that $i+j=k+2$. This procedure is called diagonal averaging, or Hankelization of the matrix *Z*. The result of Hankelization of a matrix *Z* is the Hankel matrix $\mathcal{H}Z$, which is the trajectory matrix corresponding to the series obtained as a result of the diagonal averaging. Now, consider Equation (4). Let X be a $(L\times K)$ matrix with elements $x_{ij},\ 1\le i\le L, 1\le j\le K$. Make $L^*=\min(L,\ K)$, $K^*=\max(L,K)$ and $T=L+K-1$. Lets define $x_{ij}^*=x_{ij}$, if $L<K$; and $x_{ij}^*=x_{ji}$, otherwise. Diagonal averaging transfers matrix X to a series $g_0,...,g_{T-1}$ following the formula:

$$g_k=\begin{cases}\dfrac{1}{k+1}\displaystyle\sum_{m=1}^{k+1}x_{m,k-m+2}^* & 0\le k<L^*-1\\ \dfrac{1}{L^*}\displaystyle\sum_{m=1}^{L^*}x_{m,k-m+2}^* & L^*-1\le k<K^*\\ \dfrac{1}{T-k}\displaystyle\sum_{m=k-k^*+2}^{N-k+1}x_{m,k-m+2}^* & K^*-1\le k<T\end{cases} \quad (5)$$

The expression in Equation (5) corresponds to averaging the elements along diagonals $i+j=k+2$. This diagonal averaging, applied to a resultant matrix X_{I_n}, produces a time series Y_n of length *T* and thus the initial series Y_T is decomposed into sum of M series. Calling the decomposed series as $\tilde{Y}$, it will be derived as:

$$\tilde{Y}=Y_1+...+Y_M \quad (6)$$

For proper choices of L and the sets $I_1,...,I_M$, it is possible to associate the components Y_n with the trend, oscillations or noise of the original time series Y_T. For a sufficiently vast class of series, its continuation (or forecast) can be successfully accomplished if a number of conditions are gathered. Namely, if the time series has a structure, then an algorithm identifying this structure is achieved, a method for the time series continuation (based on its structure) is available, and the structure of the time series is preserved for the time period that is to be continued [19]. Hence, bearing in mind the aforementioned conditions, a model will be built for forecast in Section 6.

IV. SINGULAR SPECTRAL ANALYSIS OF PRICE TIME SERIES

A. The Experimental WAP Data

The experimental data is the WAP time series of Iran electricity market in US$ for 844 days from 24 September 2005 up to 15 Dec. 2007 [23]. The mentioned data versus time have been shown in Figure 1(a). To get a better view, this figure has been zoomed out for a 2500 hours period in Figure 1(b). From the first graph in these two figures, a quasi-periodic behavior (with period 24 hours) is observed for daily variations in WAP, where its variance is different in various time intervals.

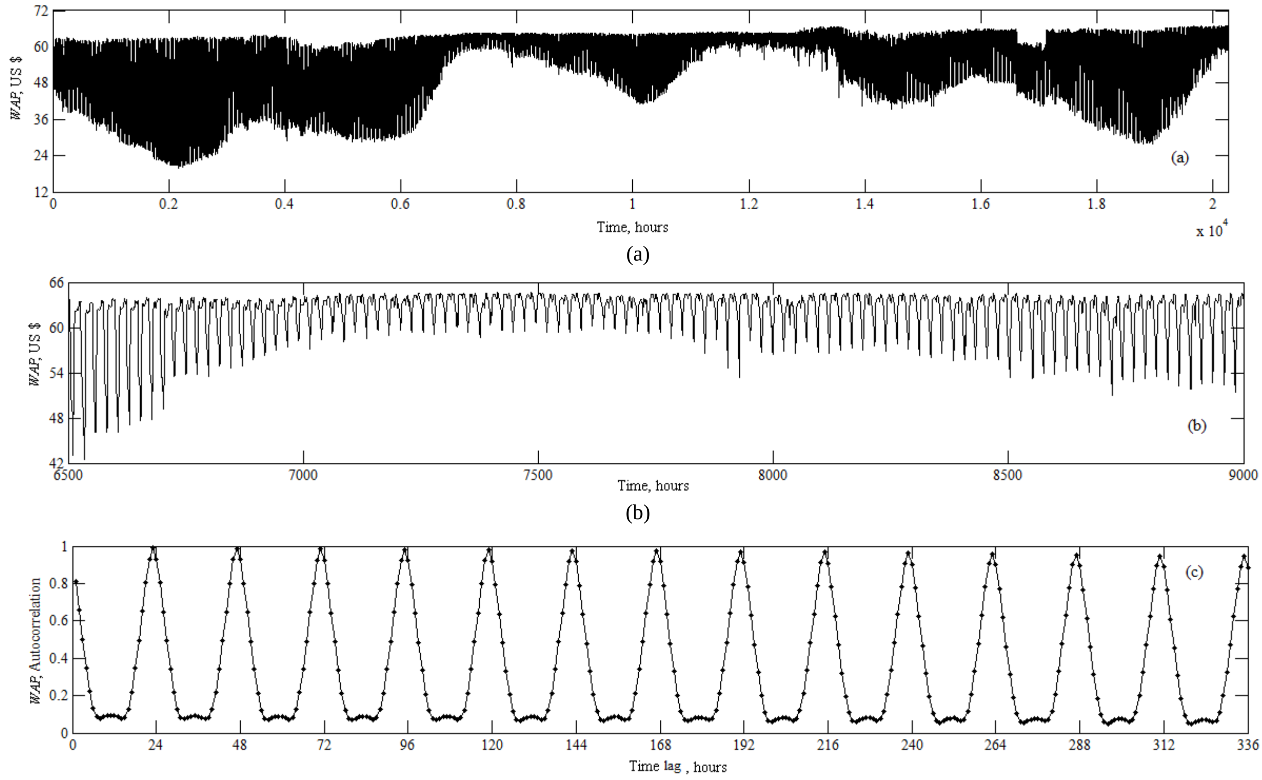

Figure 1. (a) The experimental weighted average price (*WAP*) from Iran electricity market for 844 days, (b) some parts of Figure 1(a) magnified, (c) the autocorrelation of *WAP* time series

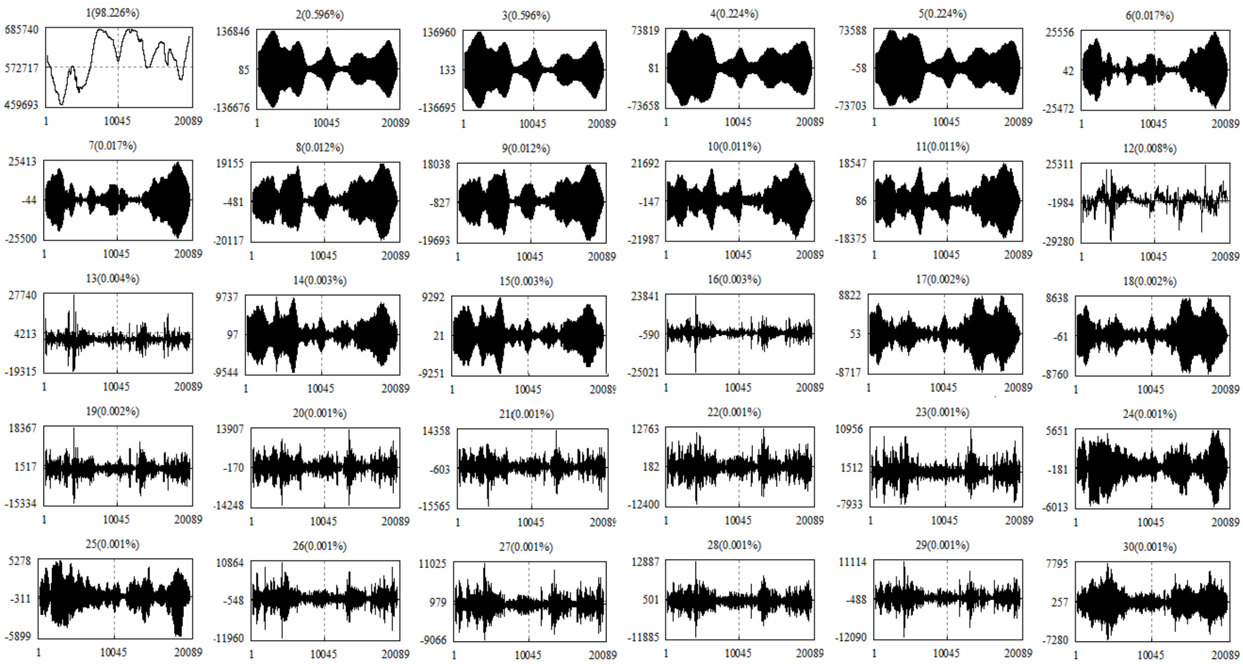

Figure 2. Principal components related to the first 30 Eigen triples

This fact has been shown in [18] as the un-stationary and seasonality of *WAP* time series. The autocorrelation of *WAP* is also shown in Figure 1(c). From Figure 1(c), it is seen that *WAP* time series is highly correlated with its lagged values of 24k to 24k+5 where k is a non-negative integer. This correlation slowly decays as k increases and may be taken as a hallmark of predictability.

B. Decomposition of WAP

As mentioned earlier, the window length L is the only parameter in the decomposition stage. Selection of the proper window length depends on the problem in hand and on preliminarily information about the time series. Based on theoretical literature, L should be large enough but not greater than T/2. Furthermore, if we know that the time series may have a periodic component with an integer period (for example, if this component is a seasonal component), then to get better reparability of this periodic component it is advisable to take the window length proportional to that period [19]. Using these recommendations, L = 168 hour is assumed, here, which corresponds to weekly variations of WAP time series. So, this window length and the SVD of the trajectory matrix result in 168 Eigen triples, ordered by their contribution (share) in the decomposition.

Note that the rows and columns of the trajectory matrix X are subseries of the original time series. Therefore, the left eigenvectors U_i and principal components V_i (right Eigen vectors) also have a temporal structure and hence can also be regarded as time series' as well. Figure 2 illustrates the principal components related to the first 30 Eigen triples of WAP time series. From this figure, it is observed that the leftmost graph in the first row of the graph, which is roughly considered as the weekly trend, has a share of 98.266% of the WAP time series. For $20 \le i \le 35$, the share of Eigen triples (comprising the seasonality of the series) remain at about 1e-3% and for $36 \le i$ the share of each Eigen triple reduces to 1e-4% or less (not shown, here). In order for reconstruction of time series some additional information may be employed, which are presented in continue.

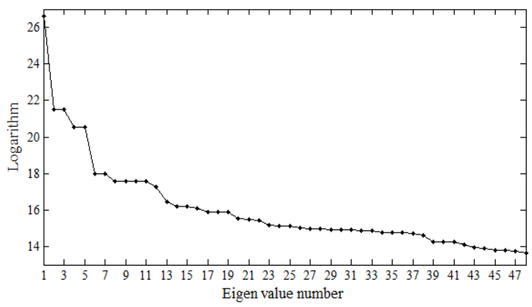

Figure 3. Logarithms of the 48 Eigen values

C. Supplementary Information

In SSA analysis, there is some extra information, which proves to be very helpful in the identification of the useful Eigen triples of the SVD of the trajectory matrix of the original series. Supplementary information helps us to make the proper groups to extract the trend, harmonic components and noise and so, increases the ability to build the proper model. Therefore, supplementary information can be considered as a bridge between the decomposition and reconstruction step:

Decomposition → Supplementary information → Reconstruction

Auxiliary information about the initial series always makes the situation clearer and helps in choosing the parameters of the models. Not only can this information help us to select the proper group, but it is also useful for forecasting based on the SSA technique [19]. Some methods for complementary analysis of *WAP* series based on SSA results are also employed as follows:

- Singular Values: Usually every harmonic component with a different frequency produces two Eigen triples with close singular values (except for frequency 0.5 which provides one Eigen triples with saw-tooth singular vector). It will be clearer if T, L and K are sufficiently large. Another useful insight is provided by checking breaks in the Eigen value spectra. As a rule, a pure noise series produces a slowly decreasing sequence of singular values. Therefore, explicit plateau in the Eigen value spectra prompt the ordinal numbers of the paired Eigen triples [19]. Figure 3 depicts the plot of the logarithms of the 48 singular values for the *WAP* series. From this figure, the Eigen pairs of (2, 3), (4, 5), (6, 7), (8, 9), (10, 11), (14, 15), (17, 18), and (24, 25) produce the main harmonics of *WAP* series. The pairs (32, 33), (34, 35), and (36, 37) produce harmonics, too. But, considering the results of the SVD, these harmonics may be ignored for WAP reconstruction.

- Pair-Wise Scatter Plots: In practice, the singular values of the two Eigen triples of a harmonic series are often very close to each other, and this fact simplifies the visual identification of the harmonic components. An analysis of the pair-wise scatter plots of the singular vectors allows one to visually identify those Eigen triples that correspond to the harmonic components of the series, provided these components are separable from the residual component.

Consider a pure harmonic with a frequency w, certain phase, amplitude and ideal situation where $P = 1/w$ is a divisor of the window length L and K. Since P is an integer, it is a period of the harmonic. In the ideal situation, the left Eigen vectors and principal components have the form of sine and cosine sequences with the same P and the same phase. Thus, the identification of the components that are generated by a harmonic is reduced to the determination of these pairs. The pure sine and cosine with equal frequencies, amplitudes, and phases create the scatter plot with the points lying on a circle. If $P = 1/w$ is an integer, then this points are the vertices of the regular P-vertex polygon. For the rational frequency $w = m/n < 0.5$ with relatively prime integer m and n, the points are the vertices of the scatter plots of the regular n-vertex polygon [19-20].

Figure 4 depicts scatter plots of the paired eigenvectors in the *WAP* series, corresponding to the harmonics with periods 24, 12, 8, 4.8, 6, 4, 2.4 and 3.6. They are ordered by their contribution (share) in the SVD step. Therefore, Eigen triples producing this harmonics are those which should be selected for grouping. In this era, the major question is that which one of the Eigen pairs is essential to be grouped for model construction? Periodogram analysis answers this question.

- Periodogram Analysis: The periodogram analysis of the original series and Eigen vectors may help us a lot in making the proper grouping; it tells one which frequency must be considered. One must then look for the Eigen triples whose frequencies coincide with the frequencies of the original series. If the periodograms of the Eigen vector have sharp spark(s) around some frequencies, then the corresponding Eigen triples must be regarded as those related to the signal component [19].

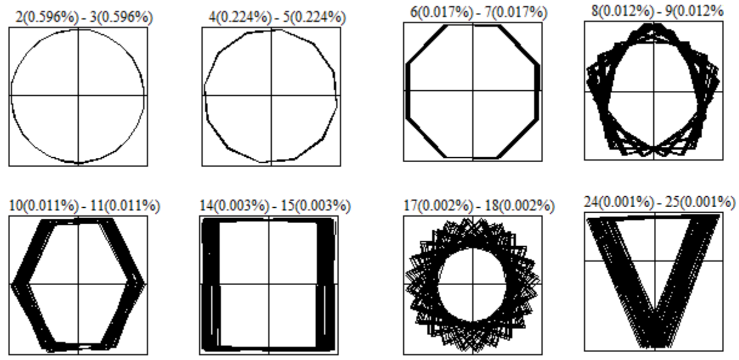

Figure 4. Scatter plots of the paired harmonic eigenvectors

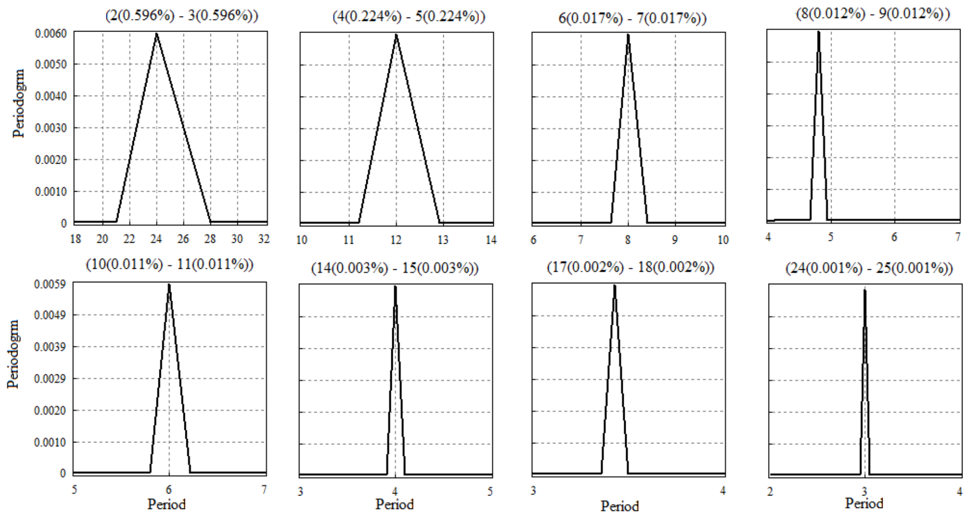

Figure 5. Periodograms of the paired Eigen triples (2, 3), (4, 5), (6, 7), (8, 9), (10, 11), (14, 15), (17, 18), and (24, 25) for *WAP* series

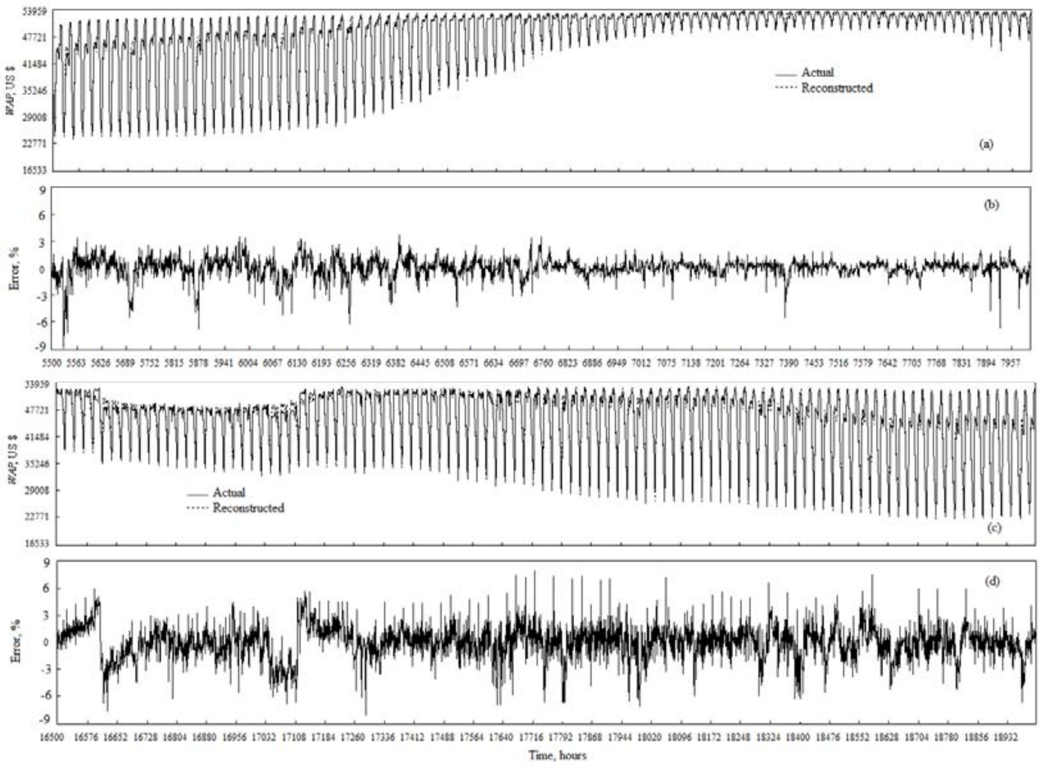

Figure 6. The reconstructed *WAP* in comparison with the actual data and the corresponding relative errors in %

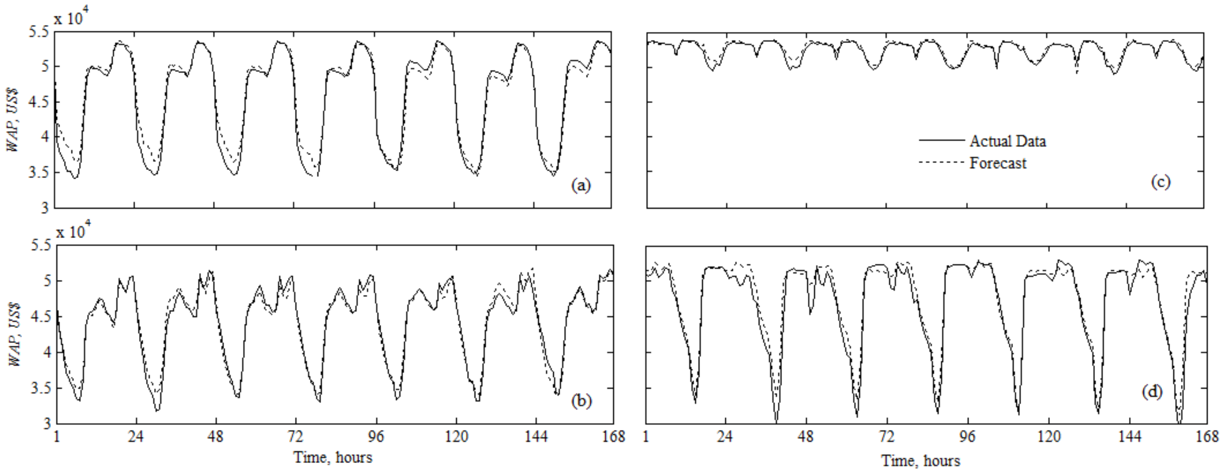

Figure 7. Comparison of forecasted *WAP* with its actual value for Iran electricity marker:
(a) the February week, (b) The May week, (c) The August week, and (d) The November Week

Figure 5 depicts the periodogram of the paired Eigen triples (2, 3), (4, 5), (6, 7), (8, 9), (10, 11), (14, 15), (17, 18), and (24, 25) for *WAP* series. The information arising from Figure 5 confirms that the above mentioned Eigen triples correspond to the periods 24, 12, 8, 4.8, 6, 4, 2.5 and 3.5 which must be regarded as selected Eigen triples in the grouping step in conjunction with another Eigen triple (the first Eigen triple) that we need to reconstruct the series.

Based on the above arguments, in this section it is tried to reconstruct the *WAP* series using the selected pairs. For this purpose, each Eigen pairs has been considered in one group. Figure 6 shows the reconstructed of *WAP* in comparison with the actual time series and the corresponding relative errors for two different time intervals with completely different behavior. As it is seen in this figure, the reconstruction has been done with a satisfactory error in both cases.

V. MODIFIED SSA FORECASTING METHOD

A. Forecasting with SSA

In SSA forecasting, the model is expressed by a Linear Recurrent Formula (LRF). This LRF applied to the last $L-1$ terms of the initial time series gives a continuation of it. The same idea can be applied to a component of the time series. Once, the data is decomposed as described in Section 3.1, the coefficients of the LRF can be collected in vector $R=(a_{L-1},...,a_1)^T$, given by [16-17]:

$$R=\frac{1}{1-v^2}\sum_{i=1}^{r}\pi_i U_i^v \qquad (6)$$
$$v^2=\pi_i^2+...+\pi_r^2$$

where, r $(r<L)$ is a design parameter and corresponds to the number of Eigen triples which are intended to participate in forecasting, π_i is the last component of vector U_i, and $U_i^v\in\Re^{L-1}$ is the vector consisting of the first $L-1$ components of U_i. Forecasting with SSA method can then be derived as [19]:

$$\tilde{y}_{i+d}=\sum_{k=1}^{d}a_k y_{i+d-k}, \quad 1\le i\le T-d \qquad (8)$$

where, $\tilde{y}_{i+d}$ stands for the forecast of y_{i+d}, T is the time series length, and d is derived from Equation (2).

B. The Proposed Method

As stated earlier, in this paper the basic SSA has been modified to be employed for price forecasting. The employed modifications are is directed in order to meet two goals: 1) to use the real data for short term price forecasting to avoid cumulative error; and 2) to estimate the trend of data from the history of the data to account for trend variations in the price time series. To meet the first purpose, once again consider the forecasting Equation (8). That is, in this equation, as time goes on (i.e. for $1<i$), the last forecasted terms should be used in order for forecasting new terms. This phenomenon, however, results in unavoidable cumulative error which is increasing as a function of time. To improve the forecasting (for omission of the cumulative error), the forecasting formula has been modified to:

$$\begin{aligned}
&\tilde{y}_{d+1}=a_1y_d+a_2y_{d-1}+...+a_dy_1\\
&\tilde{y}_{d+2}=a_1\tilde{y}_{d+1}+a_2y_d+...+a_dy_2\cong a_1y_{d+1-L}+a_2y_d+...+a_dy_2\\
&\tilde{y}_{d+3}=a_1\tilde{y}_{d+2}+a_2\tilde{y}_{d+1}+...+a_dy_3\cong a_1y_{d+2-L}+a_2y_{d+1-L}+...+a_dy_3\\
&\vdots\\
&\tilde{y}_{d+L}=a_1\tilde{y}_{d+L-1}+a_2\tilde{y}_{d+L-2}+...+a_d\tilde{y}_L\cong a_1y_{d-1}+a_2y_{d-2}+...+a_dy_L
\end{aligned} \qquad (9)$$

where, in Equation (9), the forecasted values of $\{\tilde{y}_{d+1},\tilde{y}_{d+2},\tilde{y}_{d+3},\cdots,\tilde{y}_{d+L-1}\}$ has been replaced with the real values of $\{y_{d+1-L},y_{d+2-L},y_{d+3-L},\cdots,y_{d-1}\}$.

The other modification is done based on the fact that the trend of *WAP* trend experiences relatively slow variations versus time which is due to its non-stationarity and seasonality. Therefore, in order for accounting for this phenomenon, in the modified SSA forecasting model, the time interval for data analysis and short term forecasting model construction has been confined to the last 14 days before the desired forecasting date, i.e. $T=14\times 24=336$ hours. In this way, the non-stationary behavior of *WAP* has been accounted for and the model is updated regularly to track the market behavior.

VI. FORECASTING RESULTS

The proposed method has been applied for short term price forecasting for the analyzed *WAP* time series of Iran electricity market. The forecasting has been done in two time scales, i.e. one day ahead, and one week ahead. For these forecasting periods, the parameter L has been considered as 24 and 168 hours, respectively. The parameter d is considered as $d = L$ and the Eigen pairs derived in Section 4 (i.e. Eigen pairs of (2, 3), (4, 5), (6, 7), (8, 9), (10, 11), (14, 15), (17, 18), and (24, 25)) are used for time series reconstruction.

The performance of the proposed method for price forecasting has been investigated in Figure 7 as well as Table 1. In Figure 7, the modified SSA forecasting result in comparison with the underlying real data for the *WAP* time series has been illustrated. In this examination, four weeks of year 2007 including February 12 to 18, May 14 to 20, August 13 to 19, and November 12 to 18, representative of four seasons of the year have been considered, respectively. This is a usual way for presentation of typical results of a method for the whole year [14, 24]. As seen, the model outputs follow the real data very well. This is, however, is representative of the true selection of Eigen triples.

Table 1. DME, DPE, WME and WPE of four weeks for the forecasted *WAP* time series

Test Week	*DME* (%)	*DPE* (%)	*WME* (%)	*WPE* (%)
February	1.33	5.90	2.39	7.9
May	0.55	2.18	0.66	2.74
August	1.29	4.30	1.80	6.80
November	1.80	5.58	2.10	9.10
Average	1.24	4.49	1.73	6.63

In Table 1, the daily mean error (*DME*), daily peak error (*DPE*), weekly mean error (*WME*), and weekly peak error (*WPE*) of the proposed method are presented. *DME*, *DPE*, *WME*, and *WPE* are well-known statistical indices for evaluation of prediction methods, defined as follows [24]:

$$DME = \frac{1}{24}\sum_{i=1}^{168}\frac{|L_{iACT} - L_{iFOR}|}{L_{iACT}} \qquad DPE = \max_{1\le i\le 24}\left(\frac{|L_{iACT} - L_{iFOR}|}{L_{iACT}}\right) \tag{10}$$

$$WME = \frac{1}{168}\sum_{i=1}^{168}\frac{|L_{iACT} - L_{iFOR}|}{L_{iACT}} \qquad WPE = \max_{1\le i\le 168}\left(\frac{|L_{iACT} - L_{iFOR}|}{L_{iACT}}\right) \tag{11}$$

where L_{iACT} and L_{iFOR} are the actual and forecasted *WAP* of hour i, respectively. In Equation (10), the mean and max operators are executed for 24 hours or one day, while in Equation (11) the time interval reduces to 168 hours or one week. In each cell of the Table 1, the number has been indicated in %. As seen from the values in this table, the prediction of *WAP* has been very well done via the modified SSA method. That is, both the mean and peak of error remain satisfactorily small and the performance is justified.

VII. CONCLUSIONS

In this paper, the problem of analysis and forecasting of *WAP* time series of Iran electricity market as a pay-as-bid payment market has been considered. *WAP* is the topmost descriptive index of Iran electricity market which is publically used for forecasting the market behavior and generating bidding curve to achieve maximized profit. For dealing with this index, in this paper, singular spectral analysis has been employed as the tool. At first, the *WAP* time series has been analyzed with SSA to achieve the effective and predictable components of SSA. Then a modified SSA technique has been proposed to forecast the future values of the time series. The proposed modified SSA method has been examined for the experimental data of Iran electricity market. The obtained results show that the method has a good ability in characterizing and prediction of the desired WAP time series.

REFERENCES

[1] N. Taghizadegan, N. M. Tabatabaei, A. Demiroren, N.S. Boushehri, "LFC Dynamic Improvement Using Fuzzy Logic Based Control of TCPS in Deregulated Power System", International Journal on Technical and Physical Problems of Engineering (IJTPE), Issue 9, Vol. 3, No. 4, pp. 86-90, December 2011.

[2] H. Shayeghi, H.A. Shayanfar, A. Ghasemi, "Application of ABC Algorithm for Action Based Dispatch in the Restructured Power Systems", International Journal on Technical and Physical Problems of Engineering (IJTPE), Issue 10, Vol. 4, No. 1, pp. 114-119, March 2012.

[3] N. Hasanabadi, K. Afshar, N. Bigdeli, "A Coordination Mechanism for Maintenance Scheduling of Generating Units in the Restrutured Power Systems Based on Game Theory Considering Pay-as-Bid and MCP Payment Mechanism", International Journal on Technical and Physical Problems of Engineering (IJTPE), Issue 11, Vol. 4, No. 2, pp. 42-50, June 2012.

[4] F.J. Nogales, J. Contreras, A.J. Conejo, R. Espinola, "Forecasting Next-Day Electricity Prices by Time Series Model", IEEE Transactions on Power Systems, Vol. 17, No. 2, pp. 342-348, May 2002.

[5] N. Bigdeli, K. Afshar, N. Amjady, "Market Data Analysis and Short Term Price Forecasting in the Iran Electricity Market with Pay-as-Bid Payment Mechanism", Electeric Power Systems Research (EPSR) Vol. 79, No. 6, pp. 888-898, June 2009.

[6] N. Amjady, "Day-Ahead Price Forecasting of Electricity Markets by a New Fuzzy Neural Network", IEEE Transactions on Power Systems, Vol. 21, No. 2, pp. 887-894, May 2006.

[7] Y.S. Son, R. Baldick, K. Lee, S. Siddiqi, "Short-Term Electricity Market Auction Game Analysis: Uniform and Pay-as-Bid Pricing", IEEE Transactions on Power Systems Vol. 19, No. 4, pp. 1990-1998, May 2004.

[8] D.J. Pedregal, J.R. Trapero, "Electricity Prices Forecasting by Automatic Dynamic Harmonic Regression Models", Energy Conversions and Management, Vol. 48, No. 5, pp. 1710-1719, May 2007.

[9] J.P.S. Catalao, S.J.P.S. Mariano, V.M.F. Mendes, L. A.F.M. Ferreira, "Short-Term Electricity Prices Forecasting in a Competitive Market: A Neural Network Approach",

Electric Power Systems Research (EPSR), Vol. 77, No. 10, pp. 1297-1304, Aug. 2007.
[10] D.W. Bunn, "Forecasting Loads and Prices in Competitive Power Markets", Proceedings of the IEEE, Vol. 88, No. 2, pp. 163-169, Feb. 2000.
[11] C.P. Rodriguez, G.J. Anders, "Energy Price Forecasting in the Ontario Competitive Power System Market", IEEE Transactions on Power Systems, Vol. 19, No. 3, pp. 366-374, Feb. 2004.
[12] B.R. Szkuta, L.A. Sanabria, T.S. Dillon, "Electricity Price Short-Term Forecasting Using Artificial Neural Networks", IEEE Transactions on Power Systems, Vol. 14, No. 3, pp. 851-857, Aug. 1999.
[13] H.Y. Yamin, M Shahidehpour, Z. Li, "Adaptive Short-Term Electricity Price Forecasting Using Artificial Neural Networks in the Restructured Power Markets", Electrical Power and Energy Systems, Vol. 26, No. 8, pp. 571-581, Oct. 2004.
[14] P. Doulai, W. Cahill, "Short-Term Price Forecasting in Electric Energy Market", International Power Engineering Conference (IPEC2001), pp. 749-754, Vol. 7, No. 2, Feb. 2001.
[15] L. Zhang, L. B Luh, "An Integrated Neural Network Method for Market Clearing Price Prediction and Confidence Interval Estimation", 4th World Congress on Intelligent Control and Automation, Vol. 3, pp. 2045-2050, Nov. 2002.
[16] R.C. Garcia, J. Contreras, M. Akkeren, J.B.C. Garcia, "A GARCH Forecasting Model to Predict Day-ahead Electricity Prices", IEEE Transactions on Power Systems, Vol. 20, No. 2, pp. 867-874, May 2005.
[17] A.J. Conejo, M.A. Plazas, R. Espinola, A.B. Molina, "Day-Ahead Electricity Price Forecasting Using the Wavelet Transform and ARIMA Models", IEEE Transactions on Power Systems, Vol. 20, No. 2, pp. 1035- 1042, May 2005.
[18] M. Shahidehpour, H. Yamin, Z. Li, "Market Operations in Electric Power Systems: Forecasting, Scheduling, and Risk Management", John Wiley & Sons Inc., 2002.
[19] N. Amjady, M. Hemmati, "Energy Price Forecasting: Problems and Proposals for Such Predictions", IEEE Power and Energy Magazine, Vol. 4, No. 2, pp. 20-29, March-April 2006.
[20] N. Bigdeli, K. Afshar, "Analysis of Properties of Iran Electricity Market Indices using Recurrence Plots", Physica A: Statistical and Mechanical Applications, Vol. 388, No. 8, pp. 1577-1592, March 2009.
[21] N. Bigdeli, K. Afshar, "Chaotic Behavior of Price in the Power Market with Pay-as-Bid Payment Mechanism", Chaos, Solitons and Fractals, Vol. 42, No. 4, pp. 2560-2569, Nov. 2009.
[22] H. Hassani, "Singular Spectrum Analysis: Methodology and Comparison", Journal of Data Sciences, Vol. 5, No. 2, pp. 239-257, May 2007.
[23] N. Golyandina, V. Nekrutkin, A. Zhigljavsky, "Analysis of Time Series Structure: SSA and Related Techniques", Chapman & Hall/CRC, 2001.
[24] D.S. Broomhead, G.P. King, "Extracting Qualitative Dynamics from Experimental Data", Physica D: Nonlinear Phenomenon, Vol. 20, No. 1, pp. 217-236, Aug. 1986.
[25] M.S. Ghaziadeh, M.K. Sheilkh-El-Eslami, H. Seifi, "Electricity Restructuring: Compromise between Competition and Stability", IEEE Power and Energy Magazine, Vol. 5, No. 2, pp. 16-20, May 2007.
[26] http://www.igmc.ir/pmt/ .
[27] N. Amjady, "Short-Term Bus Load Forecasting of Power Systems by a New Hybrid Method", IEEE Transactions on Power Systems, Vol. 22, No. 1, pp. 333-341, Feb. 2007.

BIOGRAPHIES

Karim Afshar was born in 1978 in Iran, and completed his Ph.D. degree in Electrical Engineering majoring in Power Systems at Sharif University of Technology, Tehran, Iran in 2007. From then, he is with Electrical Engineering Department of Imam Khomeini International University, Qazvin, Iran and currently is an Assistant Professor. His recent research interest includes: reliability issues, economic operation and forecasting as well as bidding strategy in power markets.

Nooshin Bigdeli was born in 1978 in Iran, and completed her Ph.D. degree in Electrical Engineering majoring in Control at Sharif University of Technology, Tehran, Iran in 2007. From then, she is with Electrical Engineering Department of Imam Khomeini International University, Qazvin, Iran and currently is an Assistant Professor and also is the head of department from June 2010. Her recent research interest includes: data mining, chaotic systems, analysis and forecasting as well as bidding strategy in economic markets.

Hossein Shayeghi received the B.S. and M.S.E. degrees in Electrical and Control Engineering in 1996 and 1998, respectively. He received his Ph.D. degree in Electrical Engineering from Iran University of Science and Technology, Tehran, Iran in 2006. Currently, he is an Associate Professor in Technical Engineering Department of University of Mohaghegh Ardabili, Ardabil, Iran. His research interests are in the application of robust control, artificial intelligence and heuristic optimization methods to power system control design, operation and planning and power System restructuring. He has authored and co-authored of five books in Electrical Engineering area all in Farsi, two book chapters in international publishers and more than 205 papers in international journals and conference proceedings. Also, he collaborates with several international journals as reviewer boards and works as editorial committee of six international journals. He has served on several other committees and panels in government, industry, and technical conferences. He was selected as distinguished researcher of the University of Mohaghegh Ardabili several times and in 2007, 2010 and 2011 he was elected as distinguished researcher in engineering field in Ardabil province of Iran. Also, he is a member of Iranian Association of Electrical and Electronic Engineers (IAEEE) and IEEE. Currently, he is head of Ardabil Technology Incubation Center (ATIC) at University of Mohaghegh Ardabili since 2008.