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SYNCHRONIZATION OF CHAOTIC FRACTIONAL ORDER ENERGY RESOURCES OF DEMAND SUPPLY SYSTEMS VIA ACTIVE HYBRID CONTROL

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Abstract- In this paper, synchronization of energy resources demand-supply system in China is considered. The original system behavior can be modeled via a fractional order chaotic system. A new active hybrid method based on feedback linearization control method has been proposed for synchronization of the energy resources demand and supply systems. Besides, the stability of the synchronized system been investigated, analytically and the necessary and sufficient condition for asymptotic stability of the error dynamics has been achieved. Next, numerical simulations are performed to verify the effectiveness of the proposed synchronization scheme. The results show the out-performance of the proposed method with respect to the methods in the literature. Low controller effort, short settling time, insignificant steady state error and simple controller design are advantages of using this method.

Keywords: Chaos, Fractional Order System, Chaotic Behavior, Energy Resources Demand Supply System, Hybrid Control, Fractional Calculus.

I. INTRODUCTION

Fractional calculus is a 300 years old mathematical topic. Although it has a long history, for many years it was not used in physics and engineering. However, during the last decades, fractional calculus starts to attract increasing attention of physicists and engineers from an application point of view [1-3]. It was found that many systems in interdisciplinary fields can be elegantly described with the help of fractional derivatives. Many systems are known to display fractional-order dynamics, such as viscoelastic systems [4], dielectric polarization [5], electrode-electrolyte polarization [6], electromagnetic waves [7], quantitative finance [8], and quantum evolution of complex systems [9].

In this era, synchronization of identical and non-identical fractional order chaotic systems has attracted a vast range of researches and many effective methods have been developed for synchronization of different fractional order chaotic systems [10-15]. One of the new real systems whose dynamic behavior can be well described by fractional order chaotic systems is the real energy resources demand-supply system. That is, the main problem of such systems is to synchronize the demand and supply energy systems to achieve good functionality of the entire energy resource demand-supply system. In the literature, different methods have been employed for synchronization of the energy resources demand-supply systems such as adaptive synchronization [16-18], robust chaos synchronization [19], and linear feedback synchronization [20, 21]. Some other control methods such as state feedback and PID [22, 23] can be used for synchronizing, as well.

The fractional chaotic model of energy resources demand-supply system for two regions in china has been first introduced in [24], where projective synchronization method has been used for synchronization of the system in [25]. In this paper, a new active hybrid synchronization method for this system is proposed. The synchronization method is an active method which achieves synchronization based on the state feedback control method.

This rest of this paper is organized as follows. In section II, basic definitions in fractional calculus, notations and numerical algorithms are given. In the section III, the stability and chaos prediction in fractional order systems are surveyed. In the section IV, the synchronization scheme is described. The proposed synchronization method for the chaotic fractional-order energy resource demand-supply systems are introduced in section V. Section VI provided some simulation to show the effectiveness of the proposed method. Finally, the paper is concluded in section VII.

II. BASIC DEFINITION AND PRELIMINARIES OF FRACTIONAL ORDER CALCULUS

Fractional-order integration and differentiation are the generalization of the integer order ones. Efforts to extend the specific definitions of the traditional integer-order to the more general arbitrary order context led to different definitions for fractional order differentiation. There are three commonly used definitions of the fractional-order differential operator. Grunwald-Letnikov, Riemann-Liouville, and Caputo definitions. The Grunwald-Letnikov (GL) definition is given by [26]:

$$D_t^q f(t) = \lim_{h\to 0} h^{-q} \sum_{i=0}^{\left|\frac{t-\alpha}{h}\right|} \binom{-q}{i} f(t-iq) \quad (1)$$

The Riemann-Liouville (RL) definition is described by:

$$D_t^q f(t) = \frac{1}{\Gamma(n-q)} \frac{d^n}{dt^n} \int_0^t \frac{f(\tau)}{(t-\tau)^{q-1-n}} d\tau \quad (2)$$

where, n is the first integer which is not less than q, i.e. $n-1 \le q < n$ and $\Gamma(.)$ is the well-known Euler's gamma function:

$$\Gamma(s) = \int_0^\infty t^{s-1} e^{-t} dt \quad (3)$$

The Caputo definition is written as:

$$D_t^q = \begin{cases} \dfrac{1}{\Gamma(n-q)} \displaystyle\int_0^t (t-\tau)^{n-q-1} \frac{d^n f(\tau)}{d\tau^n} d\tau \ , \ n-1 < q < n \\ \dfrac{d^n u(x,t)}{dt^n} \qquad q = n \in N \end{cases} \quad (4)$$

where, n is the first integer which is not less than q, i.e. $n-1 \le q < n$. The Laplace transform of the Riemann-Liouville fractional derivative is:

$$L\{D^q f(t)\} = S^q L\{f(t)\} - \sum_{k=0}^{n-1} s^k \left[\frac{d^{q-1-k} f(t)}{dt^{q-1-k}}\right]\Bigg|_{t=0} \quad (5)$$

where, L means Laplace transform, and s is a complex variable. It shows that the non-integer order derivative of the function at $t = 0$ is required in the Laplace transform of the Riemann-Liouville fractional derivative. However, this problem does not exist in the Laplace transform of the Caputo definition, which is given by:

$$L\{D^q f(t)\} = S^q L\{f(t)\} - \sum_{k=0}^{n-1} s^{q-1-k} f^{(k)}(0) \quad (6)$$

However, only the integer order derivatives of the function appear in the Laplace transform of the Caputo derivative. Upon considering the initial conditions to zero, this formula reduces to:

$$L\{D^q f(t)\} = S^q L\{f(t)\} \quad (7)$$

On the other hand, the Grunwald-Letnikov definition and the Riemann-Liouville definition are equivalent for a wide class of functions. However, the initial conditions for the fractional differential equation (FDE) with the Caputo derivative are in the same form as for integer-order derivatives which have well understood physical meaning. So the Caputo fractional derivative is more popular than the Riemann-Liouville definition of fractional derivative, when modeling real-world phenomena with FDE. Hence, we choose the Caputo derivative in this paper, as it is common in the literature [26].

III. STABILITY ANALYSIS FOR FRACTIONAL ORDER SYSTEMS

Fractional order differential equations are at least as stable as their integer orders counterparts, because systems with memory are typically more stable than their memory less alternatives [27]. So, the autonomous dynamic $D^q x = Ax$, $x(0) = x_0$ is asymptotically stable if the following condition is met [28]:

$$|\arg(\mathrm{eig}(A))| > \frac{q\pi}{2} \quad (8)$$

where, $0 < q < 1$ and $eig(A)$ represents the eigenvalues of matrix A. Here, each component of states decays toward 0, like t^{-q}. Furthermore, the system is stable if $|\arg(\mathrm{eig}(A))| > \frac{q\pi}{2}$ and those critical eigenvalues which satisfy $|\arg(\mathrm{eig}(A))| = \frac{q\pi}{2}$ have geometric multiplicity of 1. The stability region for $0 < q < 1$ is shown in Figure 1. Now, consider the following autonomous commensurate order of fractional system:

$$D^q x = f(x) \quad (9)$$

where, $0 < q < 1$ and $x \in R^n$. The equilibrium points of system Equation (9) are found by solving the equation:

$$f(x) = 0 \quad (10)$$

These points are locally and asymptotically stable if all eigenvalues of the Jacobin matrix $A = \frac{\partial f}{\partial x}$, which are evaluated at the equilibrium points-satisfy the following condition [27, 28]:

$$|\arg(\mathrm{eig}(A))| < \frac{q\pi}{2} \quad (11)$$

The minimum value of q which results in chaotic behavior of system, calculated by (12) [29]:

$$q_{\min} = \max\{\frac{2}{\pi} |\arg(\mathrm{eig}(A))|\} \quad (12)$$

The main advantage of fractional expression of system in stability analysis is; all parameters of system (including the region of stability) could be affected by q. This means more compactness in the system representation will be achieved rather than classic representation of systems.

In other words, in comparison with integer order expression with the same resolution, fractional order expression would provide better conditions both in stability analysis and design procedure.

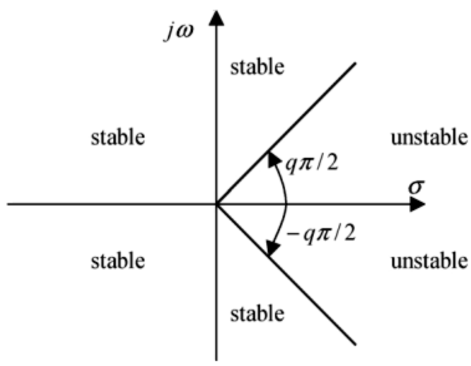

Figure1. Stability region of the FOLTI system with fractional order, $0 < q \le 1$

Theorem 1 [29]: The following n-dimensional linear fractional-order autonomous system:

$$D^q x = Ax,\ x(0) = x_0 \tag{13}$$

$A \in R^{n\times n}$, $x(0) = (x_{10}, x_{20}, ..., x_{n0})^T$, $q = (q_1, q_2, ..., q_n)^T$ and $0 < q_i < 1$ $(i = 1, 2, ..., n)$, is asymptotically stable if A is an upper or lower triangular Matrix and all eigenvalues of A are negative real numbers.

IV. SYSTEM MODEL

In this section the model of chaotic fractional-order energy resource demand-supply system is proposed. The following model with fractional-order derivative was proposed by Sun et al. [24] to describe the real energy resources demand-supply system for two regions in China. That is:

$$\begin{cases} \dfrac{d^{q_1} x}{dt^{q_1}} = a_1 x(1-\dfrac{x}{M}) - a_2(y+z) \\ \dfrac{d^{q_2} y}{dt^{q_2}} = -b_1 y - b_2 z + b_3 x(N - x + z) \\ \dfrac{d^{q_3} z}{dt^{q_3}} = c_1 z(c_2 x - c_3) \end{cases} \tag{14}$$

where, x is the energy resources demand in region A, y is the energy resources supply from region B to region A, z, is the energy resources import in region $A, a_i, b_i, c_i, M, N > 0$ are positive constants, and $N < M$ and $0 < q_1, q_2, q_3 < 1$.

V. SYNCHRONIZATION SCHEME

Assume that the master (drive) and slave (response) systems are described as:

Master system:

$$\begin{cases} \dfrac{d^{q_1} x_m}{dt^{q_1}} = a_1 x_m(1-\dfrac{x_m}{M}) - a_2(y_m+z_m) \\ \dfrac{d^{q_2} y_m}{dt^{q_2}} = -b_1 y_m - b_2 z_m + b_3 x_m(N - x_m + z_m) \\ \dfrac{d^{q_3} z_m}{dt^{q_3}} = c_1 z_m(c_2 x_m - c_3) \end{cases} \tag{15}$$

Slave system:

$$\begin{cases} \dfrac{d^{q_1} x_s}{dt^{q_1}} = a_1 x_s(1-\dfrac{x_s}{M}) - a_2(y_s+z_s) + u_1 \\ \dfrac{d^{q_2} y_s}{dt^{q_2}} = -b_1 y_s - b_2 z_s + b_3 x_s(N - x_s + z_s) + u_2 \\ \dfrac{d^{q_3} z_s}{dt^{q_3}} = c_1 z_s(c_2 x_s - c_3) + u_3 \end{cases} \tag{16}$$

Error is defined as:

$$\begin{cases} e_1 = x_s - x_m \\ e_2 = y_s - y_m \\ e_3 = z_s - z_m \end{cases} \tag{17}$$

Control law is defined as follows:

$$u_i = u_{fi} + u_{li} \qquad i = 1, 2, 3 \tag{18}$$

where, u_{fi} is obtained by feedback linearization method and u_{li} is obtained by state feedback. Error system is now defined as follows:

$$\begin{cases} \dfrac{d^{q_1} e_1}{dt^{q_1}} = a_1 e_1 - a_2(e_2+e_3) + u_{f1} + u_{l1} - \dfrac{a_1}{M}(x_s^2 - x_m^2) \\ \dfrac{d^{q_2} e_2}{dt^{q_2}} = -b_1 e_2 - b_2 e_3 + b_3 N e_1 - (x_s^2 - x_m^2) + \\ \qquad +(x_s z_s - x_m z_m) + u_{f2} + u_{l2} \\ \dfrac{d^{q_3} e_3}{dt^{q_3}} = -c_1 c_3 e_3 + c_1 c_2(x_s z_s - x_m z_m) + u_{f3} + u_{l3} \end{cases} \tag{19}$$

By feedback linearization method, u_{fi} is obtained as:

$$\begin{cases} u_{f1} = \dfrac{a_1}{M}(x_s^2 - x_m^2) \\ u_{f2} = (x_s^2 - x_m^2) - (x_s z_s - x_m z_m) \\ u_{f3} = -c_1 c_2(x_s z_s - x_m z_m) \end{cases} \tag{20}$$

By merging Equations (20) and (19), error system is defined as:

$$\begin{cases} \dfrac{d^{q_1} e_1}{dt^{q_1}} = a_1 e_1 - a_2(e_2+e_3) + u_{l1} \\ \dfrac{d^{q_2} e_2}{dt^{q_2}} = -b_1 e_2 - b_2 e_3 + b_3 N e_1 + u_{l2} \\ \dfrac{d^{q_3} e_3}{dt^{q_3}} = -c_1 c_3 e_3 + u_{l3} \end{cases} \tag{21}$$

Now, u_{li} is described by (22):

$$\begin{cases} u_{l1} = a_{11}e_1 + a_{12}e_2 + a_{13}e_3 \\ u_{l2} = a_{21}e_1 + a_{22}e_2 + a_{23}e_3 \\ u_{l3} = a_{31}e_1 + a_{32}e_2 + a_{33}e_3 \end{cases} \tag{22}$$

where,

$$u_l = Ke, \qquad K = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \tag{23}$$

From, Equation (22), the matrix A is obtained as:

$$A = \begin{bmatrix} a_{11}+a_1 & a_{12}-a_2 & a_{13}-a_2 \\ a_{21}+b_3 N & a_{22}-b_1 & a_{23}-b_2 \\ a_{31} & a_{32} & a_{33}-c_1 c_3 \end{bmatrix} \tag{24}$$

where, $a_{i,j}$, $i, j = 1, 2, 3$ are obtained satisfying conditions of Theorem 1.

VI. SIMULATION RESULTS

In this part, the real energy resources demand-supply system for two regions in China is simulated. Two cases have been considered for simulation. In the first case, the operation conditions have been assumed as in [25].

In [25] projective synchronization method has been employed for synchronization of the energy resources demand-supply system. Simulation results are compared by the results in [25]. In the second case, another operating condition has been considered where the observed chaos is more severe than the first case. As it will be seen in both cases the proposed method has shown very good performance for synchronization of the systems.

A. Case 1

Consider the parameters of the system in Equation (15) as:

$$\begin{cases} (q_1, q_2, q_3) = (0.98, 0.85, 0.92) \\ (a_1, a_2) = (0.1, 0.3) \\ (b_1, b_2, b_3) = (0.01, 0.02, 0.2) \\ (c_1, c_2, c_3) = (0.5, 0.8, 0.1) \\ (M, N) = (2,1) \\ (x_0, y_0, z_0) = (0.2, 0.1, 0.8) \end{cases}$$

For this system, its chaotic state trajectories as well as its attractor have been shown in Figures 2 and 3. In order to perform the synchronization, we consider $(x_{0m}, y_{0m}, z_{0m}) = (0.2, 0.1, 0.8)$ and $(x_{0s}, y_{0s}, z_{0s}) = (0.6, -0.3, 1.2)$ as considered in [25].

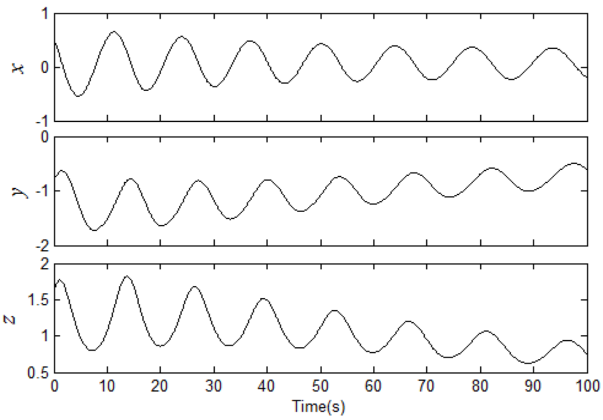

Figure 2. State trajectories of the master chaotic system

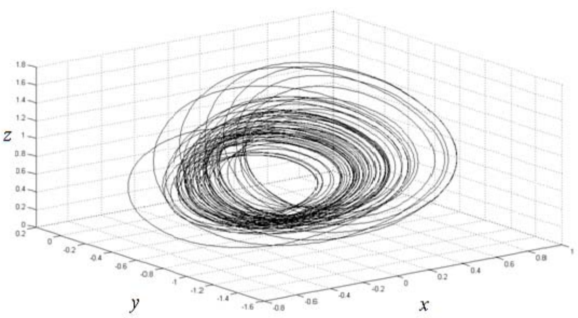

Figure 3. Chaotic attractor of the master chaotic system

Applying the input control signals as in Equations (20) and (22), and from Theorem 1, the matrix K is obtained as:

$$K = \begin{bmatrix} -1 & 0 & 0 \\ -0.2 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

From the mentioned matrix K, the eigenvalues of the error dynamics are calculated as:

$\lambda_{1,2,3} = -1.01, -0.9, -1.05$

As it is seen, all of these eigenvalues are stable. The master and slave state trajectories, the related error signals and the control efforts are shown in Figures 4 to 6, respectively. As it is observed that the synchronization has been done very well, with small settling time and reasonable control effort. Besides, in order to show the out-performance of the proposed method, the master and slave state trajectories of the system synchronized by the projective synchronization method of [25] are shown in Figure 7.

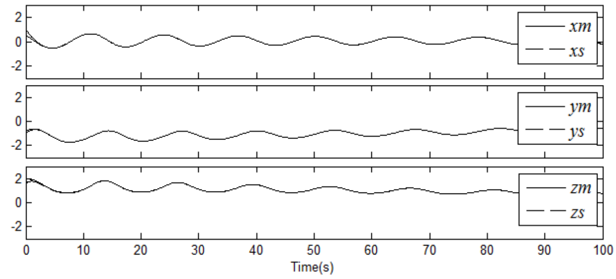

Figure 4. Master and slave systems state trajectories synchronized by the proposed method for Case 1

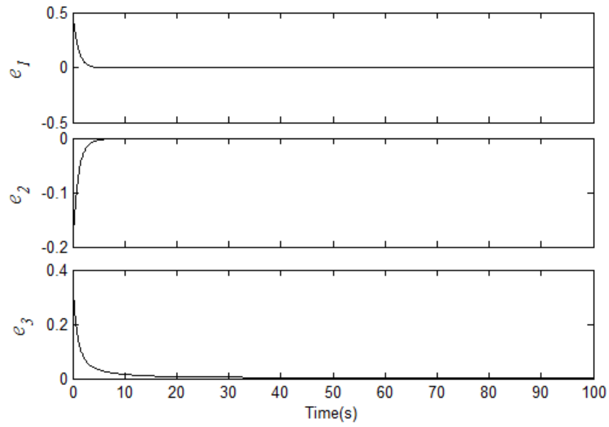

Figure 5. Error trajectories of the synchronized master and slave systems for Case 1

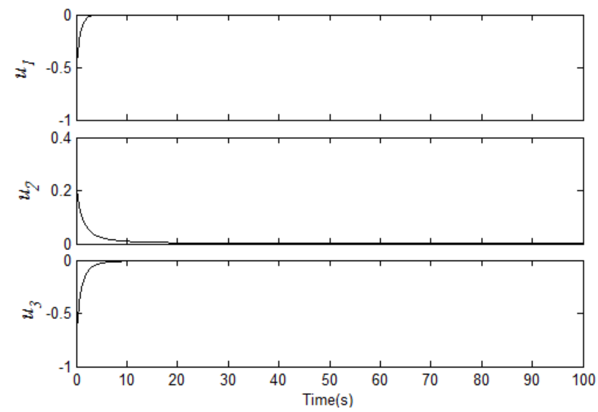

Figure 6. Control signals for the synchronized system by the proposed method for Case 1

As seen in Figure 7, although the states of the slave system somehow follow the overall behavior of those the master system, but the synchronization error never converges to zero. That is, even the steady state synchronization error does not remain constant, but, it is time varying and noticeably large. This makes the errorless predictable to compensate for it. On the other hand, the error in case of the proposed active hybrid method tends to zero in a settling time of less than 30 seconds. Therefore, the synchronization goal has been met very well via the proposed method. Overally speaking, the proposed method has very good performance in both speed of response and steady state tracking capability in comparison with that of [25].

B. Case 2

In this case, new conditions are applied and the synchronization has been achieved via the proposed method. Let us consider $(q_1, q_2, q_3) = (0.98, 0.85, 0.82)$, $(x_{0m}, y_{0m}, z_{0m}) = (0.2, 0.1, 0.8)$ and $(x_{0s}, y_{0s}, z_{0s}) = (0.6, -0.3, 1.2)$. Applying the input control signals as in Equations (20) and (22), and from Theorem.1, the matrix K is obtained as:

$$K = \begin{bmatrix} -1 & 0 & 0 \\ -0.5 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

From the mentioned matrix K, the eigenvalues of the error system are stated at below points:

$\lambda_{1,2,3} = -1.05, -0.65, -1.26$

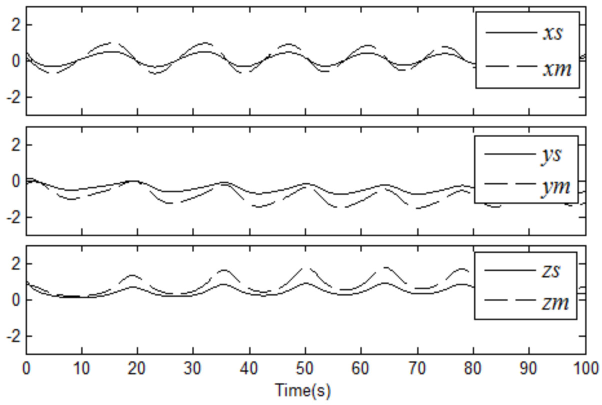

Figure 7. Master and slave systems state trajectories synchronized by the method in [25] for Case 2

It is clear that all of these eigenvalues are stable. In this case, in order to show the role of controller clearer, the control signals have been applied at t = 5 and thereafter. The master and slave state trajectories, the related error signals and the control efforts are shown in Figures 8 to 10, respectively. As seen, the state variables of the two systems have become synchronized in a very short time. Besides, the control efforts are reasonably small. The error signals converge to zero and it means that the synchronization performs, thoroughly.

VII. CONCLUSIONS

In this paper, a new active hybrid control method is proposed for synchronization of the chaotic fractional-order energy resources demand-supply systems in China. The method is based on feedback linearization method. The stability analysis is performed for the closed loop synchronized system and the necessary and sufficient condition for asymptotic stability of the error dynamics has been achieved. Besides, based on the provided analyses, the controller parameters can be tuned for the desired transient response requirements. From the presented results, it is demonstrated that the presented method has good features such as perfect synchronization, fast response, no tracking error and low cost (low control signal amplitude).

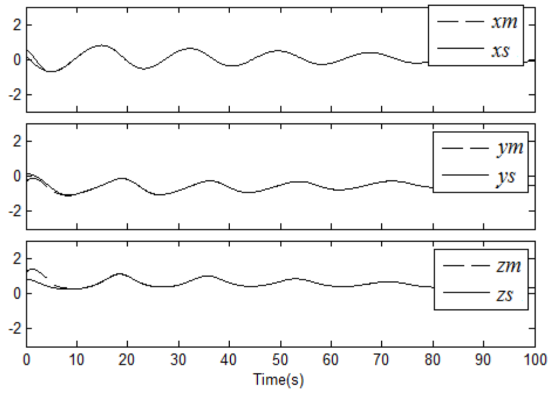

Figure 8. Master and slave systems state trajectories synchronized by the proposed method for Case 2

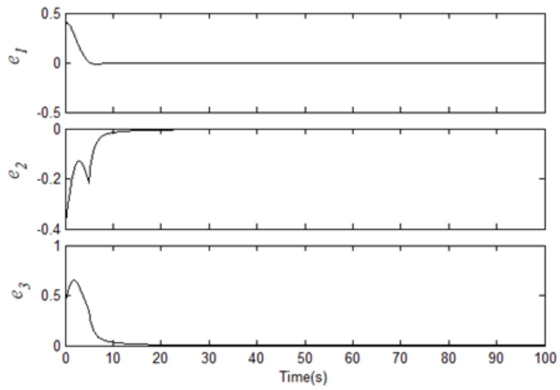

Figure 9. Error trajectories of the synchronized master and slave systems for Case 2

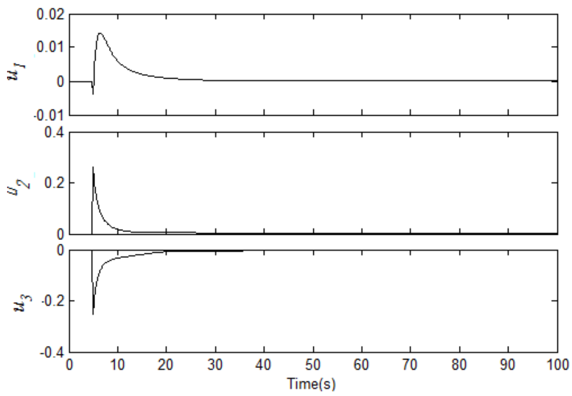

Figure 10. Control signals for the synchronized system by the proposed method for Case 2

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