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OPTIMIZATION OF TUBULAR TWO-DIMENSIONAL SHELLS

M. Daryadar

Department of Mechanics, Yerevan State University, Yerevan, Armenia, m.daryadar@ymail.com

Abstract- The shells, unlike slabs, are not smooth flexure. This circumstance has consequence of the shell's intense deformation state in total case, which is described by three functions independent from each other $U(\alpha, \beta)$, $V(\alpha, \beta)$ and $W(\alpha, \beta)$, which represent the transformation of points of shell's medium surface. Accordingly with α, β and γ key areas, that in case of slabs we deal only with one function. It is planned to introduce the results with the figure, and other methods.

Keywords: Shells, Optimization, Deformation, Function, Slabs.

I. INTRODUCTION

The shell's deformation state is described mathematically by the system of three differential equations, the form of which is very higher than one equation form in slabs.

1. The possible reduction of the shell's weight, when the main frequency of shell's own fluctuation is focused, in this paragraph we will observe two cases of the rotation of shells, closed and opened tubular shells [1, 2].
2. Three layered shells, the layers of which is made of homogeneous materials, the middle layer has h_0 constant thickness, but the outer layers, which were arranged symmetrically over the surface of the shell, have changeable $h_w(\alpha, \beta)$ thickness [3].

The optimal planning problems of shells for all listed types have the following general definition.

$$M \xrightarrow[h]{} \min \qquad (1)$$

$$\omega = \text{fix} \qquad (2)$$

where, M is the weight of shell, and the command parameter h is defined as function which describes the shell's thickness. We observe the cases when the command parameter h must comply the following geometric, physical, and isoperimetric limitations [4].

$$0 < h_1 \le h \le h_2 \quad \rightarrow \quad h_1, h_2 = \text{const} \qquad (3)$$

$$\iint_{\Omega} |grad h|^2 \, d\Omega \le C^2 \quad \rightarrow \quad C = \text{const} \qquad (4)$$

II. MATERIALS AND METHODS

Let's observe the tubular open shells in Figure 1 , for which main receptions of technical theories are held (note that the observation of total case will not cause significant differences in further recital, only additional members, which will made voluminous the observed relations and equations) [5].

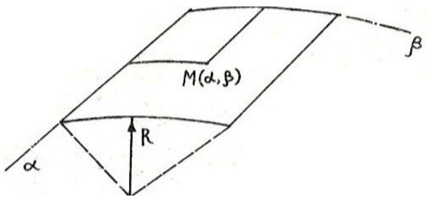

Figure 1. Tubular open shells

Free fluctuations with basic frequency of the shells ω_0 are described by Equations (5) to (7) as:

$$\frac{d}{d\alpha}\left[C_{11}\frac{\partial U}{\partial \alpha}+C_{12}\left(\frac{\partial V}{\partial \beta}+\frac{W}{R}\right)+C_{16}\left(\frac{\partial U}{\partial \beta}+\frac{\partial V}{\partial \alpha}\right)\right]+ +\frac{\partial}{\partial \beta}\left[C_{16}\frac{\partial U}{\partial \alpha}+C_{26}\left(\frac{\partial V}{\partial \beta}+\frac{W}{R}\right)+C_{66}\left(\frac{\partial U}{\partial \beta}+\frac{\partial V}{\partial \alpha}\right)\right]= =-\rho h\omega_0^2 U \qquad (5)$$

$$\frac{d}{d\alpha}\left[C_{16}\frac{\partial U}{\partial \alpha}+C_{26}\left(\frac{\partial V}{\partial \beta}+\frac{W}{R}\right)+C_{66}\left(\frac{\partial U}{\partial \beta}+\frac{\partial V}{\partial \alpha}\right)\right]+ +\frac{\partial}{\partial \beta}\left[C_{12}\frac{\partial U}{\partial \alpha}+C_{22}\left(\frac{\partial V}{\partial \beta}+\frac{W}{R}\right)+C_{26}\left(\frac{\partial U}{\partial \beta}+\frac{\partial V}{\partial \alpha}\right)\right]= =-\rho h\omega_0^2 V \qquad (6)$$

$$\frac{1}{R}\left[C_{12}\frac{\partial U}{\partial \alpha}+C_{22}\left(\frac{\partial V}{\partial \beta}+\frac{W}{R}\right)+C_{26}\left(\frac{\partial U}{\partial \beta}+\frac{\partial V}{\partial \alpha}\right)\right]+ +\frac{\partial^2}{\partial \alpha^2}\left[D_{11}\frac{\partial^2 W}{\partial \alpha^2}+D_{12}\frac{\partial^2 W}{\partial \beta^2}+2D_{16}\frac{\partial^2 W}{\partial \alpha \partial \beta}\right]+ +2\frac{\partial^2}{\partial \alpha \partial \beta}\left[D_{16}\frac{\partial^2 W}{\partial \alpha^2}+D_{26}\frac{\partial^2 W}{\partial \beta^2}+2D_{66}\frac{\partial^2 W}{\partial \alpha \partial \beta}\right]+ +\frac{\partial^2}{\partial \beta^2}\left[D_{12}\frac{\partial^2 W}{\partial \alpha^2}+D_{22}\frac{\partial^2 W}{\partial \beta^2}+2D_{26}\frac{\partial^2 W}{\partial \alpha \partial \beta}\right]= =-\rho h\omega_0^2 W \qquad (7)$$

where, C_{ij}, D_{ij} rigidity coefficients are given with the following equations [6, 7]:

$$C_{ij} = h(\alpha)B_{ij}, \; D_{ij} = \frac{h^3(\alpha)B_{ij}}{12}, \; B_{ij} = \text{const} \qquad (8)$$

At the same time must be satisfied $\alpha = 0$, $\alpha = a$, $\beta = 0$, $\beta = b$, one of the following marginal conditions given in contour [10].

$$T_1=0,\ S_{12}+\frac{H_{12}}{R}=0,\ N_1+\frac{\partial H_{12}}{\partial\beta}=0,\ M_1=0 \quad (9)$$

$$U=0,\ V=0,\ W=0,\ M_1=0 \quad (10)$$

$$V=0,\ W=0,\ T_1=0,\ M_1=0 \quad (11)$$

$$N_1+\frac{\partial H_{12}}{\partial\beta}=0,\ U=0,\ V=0,\ M_1=0 \quad (12)$$

$$U=0,\ V=0,\ W=0,\ \tilde{\theta}=-\frac{\partial W}{\partial\alpha}=0 \quad (13)$$

where, $\tilde{\theta}$ is the normal turn of the middle surface of shell to line tangent, and the shells weight fixed with the accuracy of multiplier is given with the following functional way [8]:

$$J=\int_0^a\int_0^b h(\alpha,\beta)\,d\alpha.d\beta \quad (14)$$

III. RESULTS AND DISCUSSIONS

Let's give δh increase to functional h, in that case (5) system of equations will be written of this enhancements and will receive the following look [9, 10].

$$\begin{aligned}&\frac{\partial}{d\alpha}\Big(B_{11}(h+\delta h)\frac{\partial(U+\delta U)}{\partial\alpha}+B_{12}(h+\delta h)\frac{(W+\delta W)}{R}+\\&+B_{12}(h+\delta h)\frac{\partial(V+\delta V)}{\partial\beta}+B_{16}(h+\delta h)\frac{\partial(U+\delta U)}{\partial\beta}+\\&+B_{16}(h+\delta h)\frac{\partial(V+\delta V)}{\partial\alpha}\Big)+\frac{\partial}{\partial\beta}\Big[B_{16}(h+\delta h)\frac{\partial(U+\delta U)}{\partial\alpha}+\\&+B_{26}(h+\delta h)\frac{(W+\delta W)}{R}+B_{26}(h+\delta h)\frac{\partial(V+\delta V)}{\partial\beta}+\\&+B_{66}(h+\delta h)\frac{\partial(U+\delta U)}{\partial\beta}+B_{66}(h+\delta h)\frac{\partial(V+\delta V)}{\partial\alpha}\Big]+\\&+\rho\omega_0^2(h+\delta h)(U+\delta U)=0\end{aligned} \quad (15)$$

$$\begin{aligned}&\frac{\partial}{d\alpha}\Big(B_{16}(h+\delta h)\frac{\partial(U+\delta U)}{\partial\alpha}+B_{26}(h+\delta h)\frac{(W+\delta W)}{R}+\\&+B_{26}(h+\delta h)\frac{\partial(V+\delta V)}{\partial\beta}+B_{66}(h+\delta h)\frac{\partial(U+\delta U)}{\partial\beta}+\\&+B_{66}(h+\delta h)\frac{\partial(V+\delta V)}{\partial\alpha}\Big)+\frac{\partial}{\partial\beta}\Big[B_{12}(h+\delta h)\frac{\partial(U+\delta U)}{\partial\alpha}+\\&+B_{22}(h+\delta h)\frac{(W+\delta W)}{R}+B_{22}(h+\delta h)\frac{\partial(V+\delta V)}{\partial\beta}+\\&+B_{26}(h+\delta h)\frac{\partial(U+\delta U)}{\partial\beta}+B_{26}(h+\delta h)\frac{\partial(V+\delta V)}{\partial\alpha}\Big]+\\&+\rho\omega_0^2(h+\delta h)(V+\delta V)=0\end{aligned} \quad (16)$$

$$\begin{aligned}&\frac{1}{R}\Big(B_{12}(h+\delta h)\frac{\partial(U+\delta U)}{\partial\alpha}+B_{22}(h+\delta h)\frac{(W+\delta W)}{R}+\\&+B_{22}(h+\delta h)\frac{\partial(V+\delta V)}{\partial\beta}+B_{12}(h+\delta h)\frac{\partial(U+\delta U)}{\partial\beta}+\\&B_{26}(h+\delta h)\frac{\partial(V+\delta V)}{\partial\alpha}\Big)+\\&+\frac{\partial^2}{\partial\alpha^2}\Big[\frac{B_{11}(h+\delta h)^3}{12}\frac{\partial^2(W+\delta W)}{\partial\alpha^2}+\\&+\frac{B_{12}(h+\delta h)^3}{12}\frac{\partial^2(W+\delta W)}{\partial\beta^2}+\\&+2\frac{B_{16}(h+\delta h)^3}{12}\frac{\partial^2(W+\delta W)}{\partial\alpha.\partial\beta}\Big]+\\&+2\frac{\partial^2}{\partial\alpha\partial\beta}\Big[\frac{B_{16}(h+\delta h)^3}{12}\frac{\partial^2(W+\delta W)}{\partial\alpha^2}+\\&+\frac{B_{26}(h+\delta h)^3}{12}\frac{\partial^2(W+\delta W)}{\partial\beta^2}+\\&+2\frac{B_{66}(h+\delta h)^3}{12}\frac{\partial^2(W+\delta W)}{\partial\alpha.\partial\beta}\Big]+\\&+\frac{\partial^2}{\partial\beta^2}\Big[B_{12}\frac{(h+\delta h)^3}{12}\frac{\partial^2(W+\delta W)}{\partial\alpha^2}+\\&+\frac{B_{22}(h+\delta h)^3}{12}\frac{\partial^2(W+\delta W)}{\partial\beta}+\\&+2\frac{B_{26}(h+\delta h)^3}{12}\frac{\partial^2(W+\delta W)}{\partial\alpha.\partial\beta}\Big]+\\&+\rho\omega_0^2(h+\delta h)(W+\delta W)=0\end{aligned} \quad (17)$$

Taking into consideration Equations (5) to (7) and ignoring δh, δU, δW, δV enhancement of the small quantities of the second order we will get [11, 12].

$$\begin{aligned}&\frac{\partial}{d\alpha}\Big(B_{11}h\frac{\partial\delta U}{\partial\alpha}+B_{11}\delta h\frac{\partial U}{\partial\alpha}+B_{12}\frac{h\delta W}{R}+B_{12}\frac{\delta hW}{R}+\\&+B_{12}h+\frac{\partial\delta V}{\partial\beta}+B_{12}\delta h\frac{\partial V}{\partial\beta}+B_{16}h\frac{\partial\delta U}{\partial\beta}+B_{16}\delta h\frac{\partial U}{\partial\beta}+\\&+B_{16}h\frac{\partial\delta V}{\partial\alpha}+B_{16}\delta h\frac{\partial V}{\partial\alpha}\Big)+\frac{\partial}{\partial\beta}\Big[B_{16}h\frac{\partial\delta U}{\partial\alpha}+\\&+B_{16}\delta h\frac{\partial U}{\partial\alpha}+B_{26}\frac{h\delta W}{R}+B_{26}\frac{\delta hW}{R}+B_{26}h\frac{\partial\delta V}{\partial\beta}+\\&+B_{26}\delta h\frac{\partial V}{\partial\beta}+B_{66}h\frac{\partial\delta U}{\partial\beta}+B_{66}\delta h\frac{\partial U}{\partial\beta}+B_{66}h\frac{\partial\delta V}{\partial\alpha}+\\&+B_{66}\delta h\frac{\partial V}{\partial\alpha}\Big]+\rho\omega_0^2h\delta U+\rho\omega_0^2\delta hU=0\end{aligned} \quad (18)$$

$$\begin{aligned}&\frac{\partial}{d\alpha}\Big(B_{16}h\frac{\partial\delta U}{\partial\alpha}+B_{16}\delta h\frac{\partial U}{\partial\alpha}+B_{26}\frac{h\delta W}{R}+\\&+B_{26}\frac{\delta hW}{R}+B_{26}h\frac{\partial\delta V}{\partial\beta}+B_{26}\delta h\frac{\partial V}{\partial\beta}+\\&+B_{66}h\frac{\partial\delta U}{\partial\beta}+B_{66}\delta h\frac{\partial U}{\partial\beta}+B_{66}h\frac{\partial\delta V}{\partial\alpha}+\\&+B_{66}\delta h\frac{\partial V}{\partial\alpha}\Big)+\frac{\partial}{\partial\beta}\Big[B_{12}h\frac{\partial\delta U}{\partial\alpha}+\\&+B_{12}\delta h\frac{\partial U}{\partial\alpha}+B_{22}\frac{h\delta W}{R}+B_{22}\frac{\delta hW}{R}+\\&+B_{22}h\frac{\partial\delta V}{\partial\beta}+B_{22}\delta h\frac{\partial V}{\partial\beta}+B_{26}h\frac{\partial\delta U}{\partial\beta}+\\&+B_{26}\delta h\frac{\partial U}{\partial\beta}+B_{26}h\frac{\partial\delta V}{\partial\alpha}+B_{26}\delta h\frac{\partial V}{\partial\alpha}\Big]+\\&+\rho\omega_0^2h\delta V+\rho\omega_0^2\delta hV=0\end{aligned} \quad (19)$$

$$\frac{1}{R}\left(B_{12}\, h\frac{\partial \delta U}{\partial \alpha}+B_{12}\delta h\frac{\partial U}{\partial \alpha}+B_{22}\,\frac{h\delta W}{R}+\right.$$
$$+B_{22}\,\frac{\delta hW}{R}+B_{22}h\frac{\partial \delta V}{\partial \beta}+B_{22}\delta h\frac{\partial V}{\partial \beta}+$$
$$+B_{12}h\frac{\partial \delta U}{\partial \beta}+B_{12}\delta h\frac{\partial U}{\partial \beta}+B_{26}h\frac{\partial \delta V}{\partial \alpha}+$$
$$\left.+B_{26}\delta h\frac{\partial V}{\partial \alpha}\ \right)+\frac{\partial^2}{\partial \alpha^2}\Big[\ \frac{B_{11}h^2\delta h}{4}\frac{\partial^2 W}{\partial \alpha^2}+$$
$$+\frac{B_{12}h^2\delta h}{4}\frac{\partial^2 W}{\partial \beta^2}+\frac{B_{16}h^2\delta h}{2}\frac{\partial^2 W}{\partial \alpha \partial \beta}\ \Big]+ \tag{20}$$
$$+2\frac{\partial^2}{\partial \alpha \partial \beta}\Big[\ \frac{B_{16}h^2\delta h}{4}\frac{\partial^2 W}{\partial \alpha^2}+\frac{B_{26}h^2\delta h}{4}\frac{\partial^2 W}{\partial \beta^2}+$$
$$+\frac{B_{66}h^2\delta h}{2}\frac{\partial^2 W}{\partial \alpha \partial \beta}\ \Big]+\frac{\partial^2}{\partial \beta^2}\Big[\ \frac{B_{12}h^2\delta h}{4}\frac{\partial^2 W}{\partial \alpha^2}+$$
$$+\frac{B_{22}h^2\delta h}{4}\frac{\partial^2 W}{\partial \beta}+\frac{B_{26}h^2\delta h}{2}\frac{\partial^2 W}{\partial \alpha \partial \beta}\ \Big]+$$
$$+\rho\omega_0^2 h\delta W+\rho\omega_0^2\delta hW=0$$

From Equation (14) functional, purpose will also receive increases,

$$\delta \bar{J}=\int_0^a\int_0^b \delta h.d\alpha .d\beta \tag{21}$$

Considering Equation (20), let's multiply first, second and the third left parts of the systems equations accordingly with the U_1, V_1, W_1 functions. The amount of obtained expressions integrates to $0-\ell$, than to add Equation (21) expression.

$$\Big[\Big[\ \Big(B_{11}\, h\frac{\partial \delta U}{\partial \alpha}+B_{11}\delta h\frac{\partial U}{\partial \alpha}+B_{12}\,\frac{h\delta W}{R}+$$
$$+B_{12}\,\frac{\delta hW}{R}+B_{12}h\frac{\partial \delta V}{\partial \beta}+B_{12}\delta h\frac{\partial V}{\partial \beta}+B_{16}h\frac{\partial \delta U}{\partial \beta}+$$
$$+B_{16}\delta h\frac{\partial U}{\partial \beta}+B_{16}h\frac{\partial \delta V}{\partial \alpha}+B_{16}\delta h\frac{\partial V}{\partial \alpha}\ \Big)+$$
$$+\frac{\partial}{\partial \beta}\Big[B_{16}\, h\frac{\partial \delta U}{\partial \alpha}+B_{16}\delta h\frac{\partial U}{\partial \alpha}+B_{26}\,\frac{h\delta W}{R}+$$
$$+B_{26}\,\frac{\delta hW}{R}+B_{26}h\frac{\partial \delta V}{\partial \beta}+B_{26}\delta h\frac{\partial V}{\partial \beta}+B_{66}h\frac{\partial \delta U}{\partial \beta}+$$
$$+B_{66}\delta h\frac{\partial U}{\partial \beta}+B_{66}h\frac{\partial \delta V}{\partial \alpha}+B_{66}\delta h\frac{\partial V}{\partial \alpha}\ \Big]+\rho\omega_0^2 h\delta U+$$
$$+\rho\omega_0^2\delta hU\ \Big]U_1+\Big[\ \frac{\partial}{d\alpha}\Big(B_{16}\, h\frac{\partial \delta U}{\partial \alpha}+B_{16}\delta h\frac{\partial U}{\partial \alpha}+$$
$$+B_{26}\,\frac{h\delta W}{R}+B_{26}\,\frac{\delta hW}{R}+B_{26}h\frac{\partial \delta V}{\partial \beta}+B_{26}\delta h\frac{\partial V}{\partial \beta}+$$
$$+B_{66}h\frac{\partial \delta U}{\partial \beta}+B_{66}\delta h\frac{\partial U}{\partial \beta}+B_{66}h\frac{\partial \delta V}{\partial \alpha}+B_{66}\delta h\frac{\partial V}{\partial \alpha}\ \Big)+$$
$$+\frac{\partial}{\partial \beta}\Big[B_{12}\, h\frac{\partial \delta U}{\partial \alpha}+B_{12}\delta h\frac{\partial U}{\partial \alpha}+B_{22}\,\frac{h\delta W}{R}+B_{22}\,\frac{\delta hW}{R}+$$
$$+B_{22}h\frac{\partial \delta V}{\partial \beta}+B_{22}\delta h\frac{\partial V}{\partial \beta}+B_{26}h\frac{\partial \delta U}{\partial \beta}+B_{26}\delta h\frac{\partial U}{\partial \beta}+$$
$$+B_{26}h\frac{\partial \delta V}{\partial \alpha}+B_{26}\delta h\frac{\partial V}{\partial \alpha}\ \Big]+\rho\omega_0^2 h\delta V+\rho\omega_0^2\delta hV\ \Big]V_1+$$
$$+\Big[\ \frac{1}{R}\Big(B_{12}\, h\frac{\partial \delta U}{\partial \alpha}+B_{12}\delta h\frac{\partial U}{\partial \alpha}+B_{22}\,\frac{h\delta W}{R}+B_{22}\,\frac{\delta hW}{R}+$$
$$+B_{22}h\frac{\partial \delta V}{\partial \beta}+B_{22}\delta h\frac{\partial V}{\partial \beta}+B_{12}h\frac{\partial \delta U}{\partial \beta}+B_{12}\delta h\frac{\partial U}{\partial \beta}+$$
$$+B_{26}h\frac{\partial \delta V}{\partial \alpha}+B_{26}\delta h\frac{\partial V}{\partial \alpha}\ \Big)+\frac{\partial^2}{\partial \alpha^2}\Big[\ \frac{B_{11}h^2\delta h}{4}\frac{\partial^2 W}{\partial \alpha^2}+$$
$$+\frac{B_{12}h^2\delta h}{4}\frac{\partial^2 W}{\partial \beta^2}+\frac{B_{16}h^2\delta h}{2}\frac{\partial^2 W}{\partial \alpha \partial \beta}\ \Big]+ \tag{22}$$
$$+2\frac{\partial^2}{\partial \alpha \partial \beta}\Big[\ \frac{B_{16}h^2\delta h}{4}\frac{\partial^2 W}{\partial \alpha^2}+\frac{B_{26}h^2\delta h}{4}\frac{\partial^2 W}{\partial \beta^2}+$$
$$+\frac{B_{66}h^2\delta h}{2}\frac{\partial^2 W}{\partial \alpha \partial \beta}\ \Big]+\frac{\partial^2}{\partial \beta^2}\Big[\ \frac{B_{12}h^2\delta h}{4}\frac{\partial^2 W}{\partial \alpha^2}+$$
$$+\frac{B_{22}h^2\delta h}{4}\frac{\partial^2 W}{\partial \beta}+\frac{B_{26}h^2\delta h}{2}\frac{\partial^2 W}{\partial \alpha \partial \beta}\ \Big]+\rho\omega_0^2 h\delta W+$$
$$+\rho\omega_0^2\delta hW\ \Big]W_1+\delta h\ \Big]d\alpha d\beta$$

In the integrate excretion we get let's do grouping by δh, δU, δW, δV enhancements. Therefore, Equation (12) this integrates will have the following outlook.

$$\int_0^l\Big[-B_{11}\delta h\frac{\partial U_1}{\partial \alpha}\frac{\partial U}{\partial \alpha}+B_{11}\delta U\frac{\partial}{\partial \alpha}\left(h\frac{\partial U_1}{\partial \alpha}\right)-$$
$$-B_{12}\delta h\frac{\partial U_1}{\partial \beta}\frac{\partial V}{\partial \alpha}+B_{12}\delta V\frac{\partial}{\partial \alpha}\left(h\frac{\partial U_1}{\partial \beta}\right)-$$
$$-\frac{B_{12}}{R}\delta h\frac{\partial U_1}{\partial \alpha}W-\frac{B_{12}}{R}h\frac{\partial U_1}{\partial \alpha}\delta W-B_{16}\delta h\frac{\partial U_1}{\partial \alpha}\frac{\partial V}{\partial \alpha}+$$
$$+B_{16}\delta V\frac{\partial}{\partial \alpha}\left(h\frac{\partial U_1}{\partial \alpha}\right)-B_{16}\delta h\frac{\partial U_1}{\partial \beta}\frac{\partial U}{\partial \alpha}-$$
$$-B_{16}\delta U\frac{\partial}{\partial \alpha}h\frac{\partial U_1}{\partial \beta}+\rho\omega_0^2\left(\delta hU+h\delta U\right)U_1-$$
$$-B_{16}\delta h\frac{\partial V_1}{\partial \alpha}\frac{\partial V}{\partial \alpha}+B_{16}\delta U\frac{\partial}{\partial \alpha}\left(h\frac{\partial V_1}{\partial \alpha}\right)-\frac{B_{26}}{R}\delta Wh\frac{\partial V_1}{\partial \alpha}-$$
$$-B_{16}\delta h\frac{\partial V_1}{\partial \beta}\frac{\partial V}{\partial \alpha}+B_{16}\delta V\frac{\partial}{\partial \alpha}h\frac{\partial V_1}{\partial \alpha}-\frac{B_{26}}{R}\delta h\frac{\partial V_1}{\partial \alpha}W-$$
$$-B_{66}\delta h\frac{\partial V_1}{\partial \alpha}\frac{\partial V}{\partial \alpha}+B_{66}\delta V\frac{\partial}{\partial \alpha}\left(h\frac{\partial V_1}{\partial \alpha}\right)-B_{66}\delta h\frac{\partial V_1}{\partial \beta}\frac{\partial U}{\partial \alpha}+$$
$$+\rho\omega_0^2\left(\delta hV+h\delta V\right)V_1-\frac{B_{12}}{R}\delta h\frac{\partial U}{\partial \alpha}W_1-\frac{B_{12}}{R}\delta U\frac{\partial hW_1}{\partial \alpha}-$$
$$-\frac{B_{22}}{R}\delta h\frac{\partial V}{\partial \beta}W_1+\frac{B_{22}}{R}\delta V\frac{\partial hW_1}{\partial \beta}-\frac{B_{22}}{R^2}\left(\delta hW\right)W_1-$$
$$-\frac{B_{22}}{R^2}\left(h\delta W\right)W_1-\frac{B_{26}}{R}W_1\left(\delta h\frac{\partial V}{\partial \alpha}\right)+\frac{B_{26}}{R}\delta V\frac{\partial\left(hW_1\right)}{\partial \alpha}-$$
$$-\frac{B_{26}}{R^2}W_1\delta h\frac{\partial U}{\partial \beta}+\frac{B_{26}}{R^2}\delta U\frac{\partial hW_1}{\partial \beta}-$$
$$-\frac{B_{11}}{12}\delta W\frac{\partial^2}{\partial \alpha^2}\left(h^3\frac{\partial^2 W_1}{\partial \alpha^2}\right)+\frac{B_{11}}{4}h^2\delta h\frac{\partial^2 W}{\partial \alpha}\frac{\partial^2 W_1}{\partial \alpha^2}-$$

$$-\frac{B_{12}}{12}\delta W\frac{\partial^2}{\partial\alpha^2}h^3\frac{\partial^2 W_1}{\partial\beta^2}+\frac{B_{12}}{4}h^2\delta h\frac{\partial^2 W}{\partial\alpha}\frac{\partial^2 W_1}{\partial\beta^2}-$$
$$-\frac{B_{16}}{6}\delta W\frac{\partial^2}{\partial\alpha^2}\left(h^3\frac{\partial W_1}{\partial\alpha\partial\beta}\right)+\frac{B_{16}}{4}h^2\delta h\frac{\partial^2 W}{\partial\alpha}\frac{\partial^2 W_1}{\partial\alpha\partial\beta}-$$
$$-\frac{B_{16}}{6}\delta W\frac{\partial^2}{\partial\alpha\partial\beta}h^3\frac{\partial^2 W_1}{\partial\alpha^2}+\frac{B_{16}}{2}h^2\delta h\frac{\partial^2 W}{\partial\beta}\frac{\partial^2 W_1}{\partial\alpha^2}-$$
$$-\frac{B_{26}}{6}\delta W\frac{\partial^2}{\partial\alpha\partial\beta}h^3\frac{\partial^2 W_1}{\partial\beta^2}+\frac{B_{26}}{2}h^2\delta h\frac{\partial^2 W}{\partial\beta}\frac{\partial^2 W_1}{\partial\beta^2}-$$
$$-\frac{B_{66}}{3}\delta W\frac{\partial^2}{\partial\alpha\partial\beta}h^3\frac{\partial^2 W_1}{\partial\alpha\partial\beta}+B_{66}h^2\delta h\frac{\partial^2 W}{\partial\beta}\frac{\partial^2 W_1}{\partial\alpha\partial\beta}- \quad (23)$$
$$-\frac{B_{12}}{12}\delta W\frac{\partial^2}{\partial\alpha^2}h^3\frac{\partial^2 W_1}{\partial\alpha^2}+\frac{B_{12}}{4}h^2\delta h\frac{\partial^2 W}{\partial\beta^2}\frac{\partial^2 W_1}{\partial\alpha}-$$
$$-\frac{B_{22}}{12}\delta W\frac{\partial^2}{\partial\beta^2}h^3\frac{\partial^2 W_1}{\partial\beta^2}+\frac{B_{22}}{4}h^2\delta h\frac{\partial^2 W}{(\partial\beta^2)\partial\alpha}\frac{\partial^2 W_1}{\partial\beta^2}-$$
$$-\frac{B_{26}}{6}\delta W\frac{\partial^2}{\partial\beta^2}h^3\frac{\partial^2 W_1}{\partial\alpha\partial\beta}+\frac{B_{26}}{2}h^2\delta h\frac{\partial^2 W}{(\partial\beta^2)\partial\alpha}\frac{\partial^2 W_1}{\partial\alpha\partial\beta}+$$
$$+\rho\omega_0^2(\delta hW+h\delta W)W_1\ \left.\right]d\alpha d\beta$$

In Equation (23) integrate expressions let's do grouping by δh, δU, δW, δV enhancement, demand that δh, δU, δW, δV enhancement parameters be equal to zero. It's possible, as U_1, V_1, W_1 functions don't subject to any limitation and now we demand them to satisfy the equations mentioned above.

$$B_{11}\frac{\partial}{\partial\alpha}\left(h\frac{\partial U_1}{\partial\alpha}\right)-B_{16}\frac{\partial}{\partial\alpha}h\frac{\partial U_1}{\partial\beta}+\rho\omega_0^2hU_1+$$
$$+B_{16}\frac{\partial}{\partial\alpha}\left(h\frac{\partial V_1}{\partial\alpha}\right)-\frac{B_{12}}{R}\frac{\partial(hW_1)}{\partial\alpha}+\frac{B_{26}}{R^2}\frac{\partial hW_1}{\partial\beta}=0 \quad (24)$$

$$-\frac{B_{12}}{R}h\frac{\partial U_1}{\partial\alpha}-\frac{B_{26}}{R}h\frac{\partial V_1}{\partial\alpha}+\frac{B_{16}}{R}h^3\frac{\partial^2 V_1}{\partial\alpha}-$$
$$-\frac{B_{11}}{12}\frac{\partial^2}{\partial\alpha^2}\left(h^3\frac{\partial^2 W_1}{\partial\alpha^2}\right)-\frac{B_{16}}{6}\frac{\partial^2}{\partial\alpha^2}h^3\frac{\partial W_1}{\partial\alpha\partial\beta}-$$
$$-\frac{B_{12}}{12}\frac{\partial^2}{\partial\alpha^2}\left(h^3\frac{\partial^2 W_1}{\partial\beta^2}\right)-\frac{B_{16}}{6}\frac{\partial_2}{\partial\alpha\partial\beta}h^3\frac{\partial^2 W_1}{\partial\alpha^2}-$$
$$-\frac{B_{26}}{6}\frac{\partial^2}{\partial\alpha\partial\beta}h^3\frac{\partial^2 W_1}{\partial\beta^2}-\frac{B_{66}}{3}\frac{\partial^2}{\partial\alpha\partial\beta}h^3\frac{\partial^2 W_1}{\partial\alpha\partial\beta}- \quad (25)$$
$$-\frac{B_{12}}{12}\frac{\partial^2}{\partial\alpha^2}h^3\frac{\partial^2 W_1}{\partial\alpha^2}-\frac{B_{22}}{12}\frac{\partial^2}{\partial\beta^2}h^3\frac{\partial^2 W_1}{\partial\beta^2}-$$
$$-\frac{B_{26}}{6}\frac{\partial^2}{\partial\beta^2}h^3\frac{\partial^2 W_1}{\partial\alpha\partial\beta}-\frac{B_{22}}{R^2}hW_1+\rho\omega^2hW_1=0$$

$$B_{12}\frac{\partial}{\partial\alpha}\left(h\frac{\partial U_1}{\partial\beta}\right)+B_{16}\frac{\partial}{\partial\alpha}\left(h\frac{\partial U_1}{\partial\alpha}\right)+$$
$$+B_{16}\frac{\partial}{\partial\alpha}\left(h\frac{\partial V_1}{\partial\alpha}\right)+B_{66}\frac{\partial}{\partial\alpha}h\frac{\partial V_1}{\partial\alpha}+ \quad (26)$$
$$+\frac{B_{22}}{R}\frac{\partial hW_1}{\partial\beta}+\frac{B_{26}}{R}\frac{\partial hW_1}{\partial\alpha}+\rho\omega_0^2hV_1=0$$

In this case, δh must be zero, as $\delta\bar{J}=0$ at any δh case.

$$-B_{11}\frac{\partial U_1}{\partial\alpha}\frac{\partial U}{\partial\alpha}-B_{12}\frac{\partial U_1}{\partial\beta}\frac{\partial V}{\partial\alpha}-\frac{B_{12}}{R}\frac{\partial V_1}{\partial\alpha}W-$$
$$-B_{16}\frac{\partial U_1}{\partial\alpha}\frac{\partial V}{\partial\alpha}-B_{16}\frac{\partial U_1}{\partial\beta}\frac{\partial U}{\partial\alpha}-B_{16}\frac{\partial V_1}{\partial\alpha}\frac{\partial V}{\partial\alpha}-$$
$$-B_{16}\frac{\partial V_1}{\partial\beta}\frac{\partial V}{\partial\alpha}-\frac{B_{26}}{R}\frac{\partial V_1}{\partial\alpha}W-B_{66}\frac{\partial V_1}{\partial\alpha}\frac{\partial V}{\partial\alpha}-$$
$$-B_{66}\frac{\partial V_1}{\partial\beta}\frac{\partial U}{\partial\alpha}-\frac{B_{12}}{R}\frac{\partial U}{\partial\alpha}W_1-\frac{B_{22}}{R}\frac{\partial V}{\partial\beta}W_1-$$
$$-\frac{B_{22}}{R^2}WW_1-\frac{B_{26}}{R}W_1\frac{\partial V}{\partial\alpha}-\frac{B_{26}}{R^2}W_1\frac{\partial U}{\partial\beta}+ \quad (27)$$
$$+\frac{B_{11}}{4}h^2\frac{\partial^2 W}{\partial\alpha}\frac{\partial^2 W_1}{\partial\alpha^2}+\frac{B_{12}}{4}h^2\frac{\partial^2 W}{\partial\alpha}\frac{\partial^2 W_1}{\partial\beta^2}+$$
$$+\frac{B_{16}}{4}h^2\frac{\partial^2 W}{\partial\alpha}\frac{\partial^2 W_1}{\partial\alpha\partial\beta}+\frac{B_{16}}{2}h^2\frac{\partial^2 W}{\partial\beta}\frac{\partial^2 W_1}{\partial\alpha^2}+$$
$$+\frac{B_{26}}{2}h^2\frac{\partial^2 W}{\partial\beta}\frac{\partial^2 W_1}{\partial\beta^2}+B_{66}h^2\frac{\partial^2 W}{\partial\beta}\frac{\partial^2 W_1}{\partial\alpha\partial\beta}+$$
$$+\frac{B_{12}}{4}h^2\frac{\partial^2 W}{\partial\beta^2}\frac{\partial^2 W_1}{\partial\alpha}+\frac{B_{22}}{4}h^2\frac{\partial^2 W}{\partial\beta^2\partial\alpha}\frac{\partial^2 W_1}{\partial\beta^2}+$$
$$+\frac{B_{26}}{2}h^2\frac{\partial^2 W}{\partial\beta^2\partial\alpha}\frac{\partial^2 W_1}{\partial\alpha\partial\beta}+\rho\omega_0^2UU_1+\rho\omega_0^2VV_1+\rho\omega_0^2WW_1=0$$

Based on Equations (5) and (14) systems similarity we can say that:

$$U_1=-UC\ ,\ V_1=-VC\ ,\ W_1=WC \quad (28)$$

If Equation (28) put in Equation (27), we will get:

$$-B_{11}\frac{\partial(-UC)}{\partial\alpha}\frac{\partial U}{\partial\alpha}-B_{12}\frac{\partial(-UC)}{\partial\beta}\frac{\partial V}{\partial\alpha}-\frac{B_{12}}{R}\frac{\partial(-VC)}{\partial\alpha}W-$$
$$-B_{16}\frac{\partial(-UC)}{\partial\alpha}\frac{\partial V}{\partial\alpha}-B_{16}\frac{\partial(-UC)}{\partial\beta}\frac{\partial U}{\partial\alpha}-B_{16}\frac{\partial(-VC)}{\partial\alpha}\frac{\partial V}{\partial\alpha}-$$
$$-B_{16}\frac{\partial(-VC)}{\partial\beta}\frac{\partial V}{\partial\alpha}-\frac{B_{26}}{R}\frac{\partial(-VC)}{\partial\alpha}W-B_{66}\frac{\partial(-VC)}{\partial\alpha}\frac{\partial V}{\partial\alpha}-$$
$$-B_{66}\frac{\partial(-VC)}{\partial\beta}\frac{\partial U}{\partial\alpha}-\frac{B_{12}}{R}\frac{\partial U}{\partial\alpha}(WC)-\frac{B_{22}}{R}\frac{\partial V}{\partial\beta}(WC)-$$
$$-\frac{B_{22}}{R^2}(WWC)-\frac{B_{26}}{R}(WC)\frac{\partial V}{\partial\alpha}-\frac{B_{26}}{R^2}(WC)\frac{\partial U}{\partial\beta}+$$
$$+\frac{B_{11}}{4}h^2\frac{\partial^2 W}{\partial\alpha}\frac{\partial^2(WC)}{\partial\alpha^2}+\frac{B_{12}}{4}h^2\frac{\partial^2 W}{\partial\alpha}\frac{\partial^2(WC)}{\partial\beta^2}+ \quad (29)$$
$$+\frac{B_{16}}{4}h^2\frac{\partial^2 W}{\partial\alpha}\frac{\partial^2(WC)}{\partial\alpha\partial\beta}+\frac{B_{16}}{2}h^2\frac{\partial^2 W}{\partial\beta}\frac{\partial^2(WC)}{\partial\alpha^2}+$$
$$+\frac{B_{26}}{2}h^2\frac{\partial^2 W}{\partial\beta}\frac{\partial^2(WC)}{\partial\beta^2}+B_{66}h^2\frac{\partial^2 W}{\partial\beta}\frac{\partial^2(WC)}{\partial\alpha\partial\beta}+$$
$$+\frac{B_{12}}{4}h^2\frac{\partial^2 W}{\partial\beta^2}\frac{\partial^2(WC)}{\partial\alpha}+\frac{B_{22}}{4}h^2\frac{\partial^2 W}{\partial\beta^2\partial\alpha}\frac{\partial^2(WC)}{\partial\beta^2}+$$
$$+\frac{B_{26}}{2}h^2\frac{\partial^2 W}{\partial\beta^2\partial\alpha}\frac{\partial^2(WC)}{\partial\alpha\partial\beta}+\rho\omega_0^2U(-UC)+$$
$$+\rho\omega_0^2V(-VC)+\rho\omega_0^2W(WC)=0$$

IV. CONCLUSIONS

The Equation (17) with in Equation (5) system made close system $U(\alpha)$, $V(\alpha)$, $W(\alpha)$, $h(\alpha)$ for unknown functions. Solving this equations system, taking into consideration the appropriate in Equations (9) to (13) edge conditions, we will get the function of transformation of U, V, W and the function of describing the thickness of h.

$$\begin{aligned}
&B_{11}\left(\frac{\partial U}{\partial \alpha}\right)^2 + B_{12}\frac{\partial U}{\partial \beta}\frac{\partial V}{\partial \alpha} + \frac{B_{12}}{R}\frac{\partial V}{\partial \alpha}W + B_{16}\frac{\partial U}{\partial \alpha}\frac{\partial V}{\partial \alpha} + \\
&+B_{16}\frac{\partial U}{\partial \beta}\frac{\partial U}{\partial \alpha} + B_{16}\left(\frac{\partial V}{\partial \alpha}\right)^2 + B_{16}\frac{\partial V}{\partial \beta}\frac{\partial V}{\partial \alpha} + \frac{B_{26}}{R}\frac{\partial V}{\partial \alpha}W + \\
&+B_{66}\left(\frac{\partial V}{\partial \alpha}\right)^2 + B_{66}\frac{\partial V}{\partial \beta}\frac{\partial U}{\partial \alpha} - \frac{B_{12}}{R}\frac{\partial U}{\partial \alpha}W - \frac{B_{22}}{R}\frac{\partial V}{\partial \beta}W - \\
&-\frac{B_{22}}{R^2}W^2 - \frac{B_{26}}{R}W\frac{\partial V}{\partial \alpha} - \frac{B_{26}}{R^2}W\frac{\partial U}{\partial \beta} + \frac{B_{11}}{4}h^2\frac{\left(\partial^2 W\right)^2}{\partial \alpha^3} + \\
&+\frac{B_{12}}{4}h^2\frac{\partial^2 W}{\partial \alpha}\frac{\partial^2 W}{\partial \beta^2} + \frac{B_{16}}{4}h^2\frac{\partial^2 W}{\partial \alpha}\frac{\partial^2 W}{\partial \alpha \partial \beta} + \qquad (30) \\
&+\frac{B_{16}}{2}h^2\frac{\partial^2 W}{\partial \beta}\frac{\partial^2 W}{\partial \alpha^2} + \frac{B_{26}}{2}h^2\frac{\left(\partial^2 W\right)^2}{\partial \beta^3} + \\
&+B_{66}h^2\frac{\partial^2 W}{\partial \beta}\frac{\partial^2 W}{\partial \alpha \partial \beta} + + \frac{B_{12}}{4}h^2\frac{\partial^2 W}{\partial \beta^2}\frac{\partial^2 W}{\partial \alpha} + \\
&+\frac{B_{22}}{4}h^2\frac{\partial^2 W}{\partial \beta^2 \partial \alpha}\frac{\partial^2 W}{\partial \beta^2} + + \frac{B_{26}}{2}h^2\frac{\partial^2 W}{\partial \beta^2 \partial \alpha}\frac{\partial^2 W}{\partial \alpha \partial \beta} + \\
&+\rho\omega_0^2 U^2 + \rho\omega_0^2 V^2 + \rho\omega_0^2 W^2 = 0
\end{aligned}$$

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BIOGRAPHY

Mohammad Daryadar was born in Tehran, Iran in 1985. He received the M.Sc. degree from Department of Mechanics, Yerevan State University, Yerevan, Armenia and currently he is a Ph.D. student in the same university.