

## STATE SPACE MODEL AND SPEED CONTROL OF TWO-MASS RESONANT SYSTEM USING STATE FEEDBACK DESIGN

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**Abstract-** This paper presents a speed control strategy for the torsional vibration systems. The state feedback strategy with integral control based on the detailed small-signal model is applied to design the speed controller for the two-mass resonant system. The integral control will exhibit no steady state error in the response to the step input. Also the system responses are plotted using a step input for the system with controller and without controller.

**Keywords:** Two-Mass Resonant System, Speed Control, Feedback Design, Proportional-Integral-Derivative Controller.

### I. INTRODUCTION

A rotary system having its rotating components coupled by a long and deflective shaft usually experiences torsional resonance. Vibration suppression and disturbance rejection control in torsional system is an important problem in the future motion control. Mechanical system with physical structure rendering torsional resonance is commonly encountered in industry. Mechanical resonance is usually caused by a combination of high servo gains and a compliant coupling between the motor and load. A mechanical system composed of some masses connected with flexible shaft is called multi-mass resonant system. The dynamic performances of speed and position controlled multi-mass driving system can be deteriorated especially due to the elastic coupling, non-linear friction and backlash. The mechanical system in the industrial motor drive can be modeled by a multi-mass system. The simplest model of such resonant mechanical system is two-mass system [1, 2].

Control problem of a two-mass drive system is especially difficult when not all system variables are measurable. To overcome the problems, various control strategies have been proposed mainly for controlling two-inertia system [3, 4]. In [5] three kinds of typical pole assignments with identical radius/damping coefficient/real part are applied and compared, and the merits of each pole-assignment design are concluded. This method can slightly improve the damping ability of the drive, but the system dynamics decreases at the same time. In [6] an integral-proportional (IP) controller is designed for a higher-order plant by using the concept of plant model reduction and

the CRA method with the appropriate values of characteristic ratios in order to obtain the closed-loop step response without overshoot. A systematic comparative study of compensation schemes such as resonance ratio control and proportional-integral-derivative (PID) control for the coordinated motion control of two inertia mechanical systems presented in [7]. In [8], the systematic analysis of two-inertia stabilization system is proposed, and an optimal controller to achieve better reference tracking and disturbance rejection performance is introduced by pole assignment using ITAE criterion.

A simple digital filter which cuts the resonant frequency proposed in [9], where the controller is effective for command responses but it is not able to suppress the vibration prompted by disturbance actively. The speed control using two-degrees-of-freedom (TDOF) controller for vibration suppression and the disturbance rejection based  $H_\infty$  control theory is presented in [10]. In [11], low-order IP, modified IP and modified IPD controllers are design for the speed control of a two-mass system base on a normalized model and polynomial method.

In this paper, a speed control scheme for two-mass system is described. A state feedback controller with an integrator is designed. The mathematical model for analysis of the two-mass system uses the Laplace is described in section II. Section III describes the modeling of the two-mass resonant system with PID controller. Design of a state feedback controller based on transfer function of the plant model is described in section IV. The parameters of controller is given by using standard form in section V. Simulation results are illustrated in section VI to verify good performance obtained using the proposed controller. The paper is summarized in section VII.

### II. MATHEMATICAL MODEL OF TWO-MASS RESONANT SYSTEM

Figure 1 shows a schematic of a two-mass resonant system consisting of two lumped inertias  $J_M$  and  $J_L$ , representing the motor and load, respectively, coupled via a shaft of finite stiffness  $K_S$ , that is subject to torsional torque  $T_S$  and excited by a combination of electromagnetic torque  $T_M$  and load-torque perturbations  $T_L$ . Generally, the speed  $\omega_M$  and position  $\theta_M$  of the motor shaft differ from the respective variables  $\omega_L$  and  $\theta_L$ , on load side. The torsional torque equals the load torque only in the steady-state.

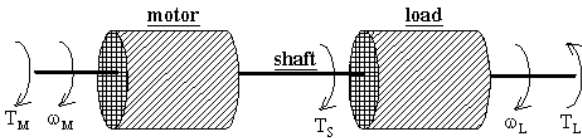


Figure 1. The plant model of the two-mass resonant system [12]

The state equation of the two-mass resonant system is as follows [13]:

$$\dot{\omega}_M = \frac{B_S}{J_M} \omega_L - \frac{B_M + B_S}{J_M} \omega_M - \frac{K_S}{J_M} (\theta_M - \theta_L) + \frac{1}{J_M} T_M \quad (1)$$

$$\dot{\omega}_L = \frac{B_S}{J_L} \omega_M - \frac{B_S + B_M}{J_L} \omega_L + \frac{K_S}{J_L} (\theta_M - \theta_L) - \frac{1}{J_L} T_L \quad (2)$$

$$\dot{\theta}_M = \omega_M \quad (3)$$

$$\dot{\theta}_L = \omega_L \quad (4)$$

State variables are  $\omega_M$ ,  $\omega_L$ ,  $\theta_M$  and  $\theta_L$ . Control input is the motor torque  $T_M$ . Output variable which can be measured is the motor speed  $\omega_M$ . Controlled variable is the load speed  $\omega_L$  and disturbance  $T_L$  is injected into the load. A block diagram of the compliantly coupled mechanism is shown in Figure 2. The transfer function load is denoted  $G_L(s)=1/(B_L+sJ_L)$  and the transfer function motor is denoted  $G_M(s)=1/(B_M+sJ_M)$ . During transients, speeds of motor and load differ, and torsional torque is given by:

$$T_S(s) = B_S [\omega_M(s) - \omega_L(s)] + K_S [\theta_M(s) - \theta_L(s)] \quad (2)$$

With the state equations the circuit can be easily modeled by using the functional blocks. The simplified block diagram of the two-mass system is shown in Figure 1.

Transfer functions of the motor, load and shaft are  $G_M(s)=1/(J_Ms+B_M)$ ,  $G_L(s)=1/(J_Ls+B_L)$  and  $G_S(s)=B_S+K_S/s$ , respectively. The load speed and motor speed are:

$$\omega_L(s) = \frac{G_L(s)G_M(s)G_S(s)}{1+G_M(s)G_S(s)+G_L(s)G_S(s)} T_M(s) - \frac{G_L(s)[1+G_M(s)G_S(s)]}{1+G_M(s)G_S(s)+G_L(s)G_S(s)} T_L(s) \quad (3)$$

$$\omega_M(s) = \frac{G_M(s)[1+G_S(s)G_L(s)]}{1+G_M(s)G_S(s)+G_L(s)G_S(s)} T_M(s) - \frac{G_L(s)G_M(s)G_S(s)}{1+G_M(s)G_S(s)+G_L(s)G_S(s)} T_L(s) \quad (4)$$

Let a two-input single-output process be represented by the block diagram shown in Figure 2. Since damping losses usually considered being relatively low, they are neglected without significantly affecting the accuracy of the forgoing analysis. Neglecting friction terms, transfer function from  $T_M$  to  $\omega_M$ , which plays an important role in the closed loop design, is given by:

$$H_{MM}(s) = \frac{1}{J_M} \frac{s^2 + \omega_A^2}{s(s^2 + \omega_R^2)} \quad (5)$$

where the anti-resonance frequency  $\omega_A$  is:

$$\omega_A = \sqrt{\frac{K_S}{J_L}} \quad (6)$$

Also, the transfer function from  $T_L$  to  $\omega_L$ , is given by:

$$H_{LL}(s) = \frac{1}{J_L} \frac{s^2 + \omega_{AL}^2}{s(s^2 + \omega_R^2)} \quad (7)$$

where the load anti-resonance frequency  $\omega_{AL}$  is:

$$\omega_{AL} = \sqrt{\frac{K_S}{J_M}} \quad (8)$$

Notice that  $\omega_A$  is always lower than  $\omega_R$ . The control bandwidth in closed-loop motion control system is limited by the  $\omega_A$ . An increase in the motor inertia constant decreases the  $\omega_R$  without affecting the  $\omega_A$ . Conversely, increasing the mechanical stiffness of the shaft ( $K_S$ ) increases both  $\omega_R$  and  $\omega_A$ , and also the mechanical bandwidth in closed loop system. Therefore, in the large systems, which have large inertias, generally produce low natural frequencies.

### III. PID CONTROLLER

PID control with its three terms functionality covering treatment to both transient and steady-states response, offers the simplest and yet most efficient solution for many real control problems [14]. One of the most widely used control laws in two-mass systems is the PID type controller. A block diagram of the speed control system using conventional PID controller [ $G_V(s)=K_P+K_I/s+sK_D$ ] with motor speed feedback is shown in Figure 3. When a PID controller is used for two-mass system, the closed-loop transfer function from the speed command  $\omega_C$  to the motor speed is given by:

$$H_C(s) = \frac{\omega_M(s)}{\omega_C(s)} = \frac{(s^2 + \omega_A^2)(K_D s^2 + K_P s + K_I)}{\Delta_S(s)} \quad (9)$$

where  $\Delta_S(s)$  is closed-loop characteristic equation and given by:

$$\Delta_S(s) = (K_D + J_M) s^4 + K_P s^3 + (J_M \omega_R^2 + K_I + K_D \omega_A^2) s^2 + K_P \omega_A^2 s + K_I \omega_A^2 \quad (10)$$

### IV. STATE FEEDBACK CONTROL

The tasks of speed controller for two-mass resonant system are suppression of shaft torsional vibration, faster tracking the load speed of the speed reference without overshoot, rejection the effect of the load disturbance torque and robust controller [15].

In the design of a controller for the system, all states of system are not measurable. It is often impossible to measure the  $\omega_L$ ,  $T_S$  and  $T_L$  in real system. In this section, we discussed the state feedback control.

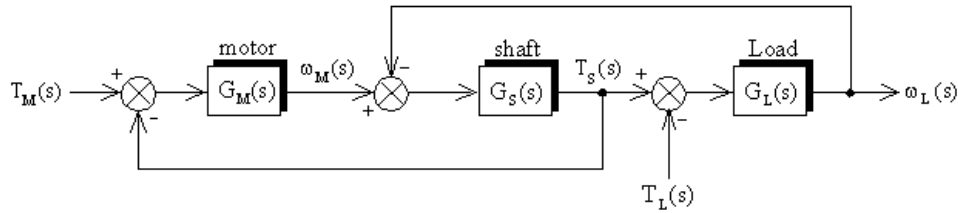


Figure 2. Block diagram of two-mass resonant system [16]

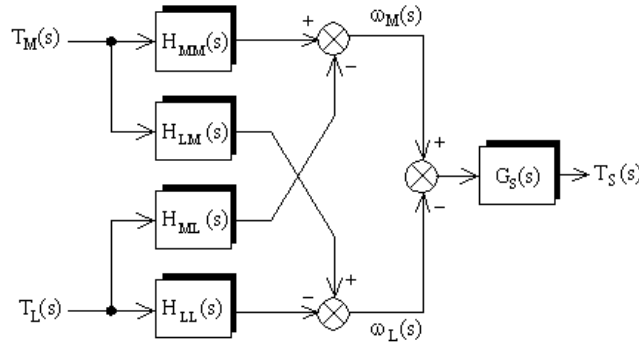


Figure 3. Two-input and single-output of the system [17]

In state feedback, every state variable is multiplied by a gain and fed back into the input terminal. The state feedback control is basically a proportional control and a steady state error may exist due to model uncertainty. The integral control action together with state feedback scheme is used to properly stabilize the system. Figure 4 shows a block diagram of the speed control system using a state feedback controller with an integrator, where  $E_V$  is output of the integrator. The system is to insert an integrator in the feed forward path between the error comparator and the model. The control signal is:

$$T_M = -K_M \omega_M - K_L \omega_L - K_H T_S + E_V \tag{11}$$

The system dynamic in close-loop can be described by  $\dot{\hat{X}} = \hat{A}\hat{X} + \hat{B}\hat{U}$ , where  $\hat{A} = A - BK$ ,  $\hat{B} = BK_E \omega_c$  and  $\hat{X} = [\omega_M \ T_S \ \omega_L \ E_V]^T$ . The  $\omega_c$  is reference input speed and  $K$  is gain vector:

$$K = [K_M \ K_H \ K_L]^T \tag{12}$$

If the desired eigenvalues of matrix  $\hat{A}$  are specified, then the state feedback gains vector and the integral gain constant  $K_E$  can be determined by the pole placement technique.

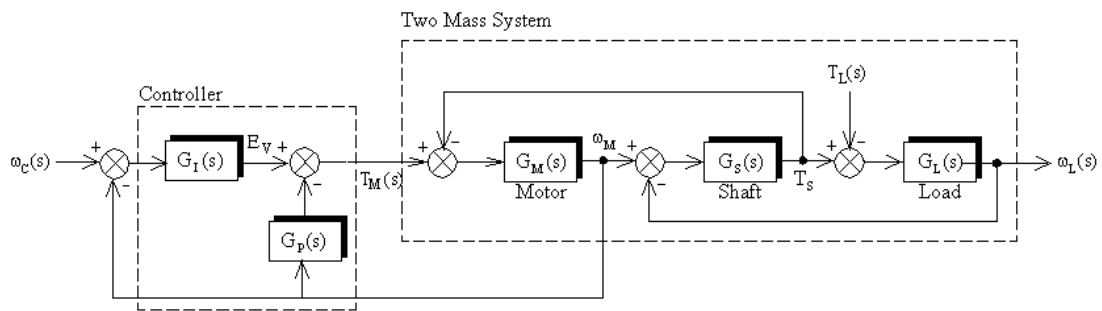


Figure 4. Speed control system using PID controller with motor speed feedback

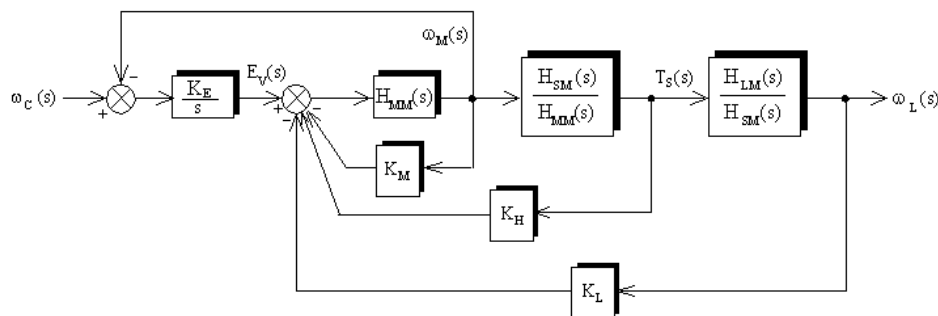


Figure 5. Speed control system using a state feedback controller with an integrator [18]

V. DESIGN CONTROLLER

The desired eigenvalues are depends on the performance criteria, such as settling time, rise time and overshoot, used in the design. The gains controller is evaluated using the integral of time multiplied by the absolute error (ITAE) criterion for a step reference input. The ITAE performance index provides the best selectivity by minimizing overshoot and settling time for a given undershoot [19, 20]. As for the ITAE criterion, the standard form coefficients for a step input is given by:

$$s^4 + 2.1\omega_n s^3 + 3.4\omega_n^2 s^2 + 2.7\omega_n^3 s + \omega_n^4 \tag{13}$$

where  $\omega_n$  represents the 3dB bandwidth. The  $\omega_n$  and gains of the controller obtained from ITAE criterion for PID controller are given by:

$$\omega_n = 0.8819 \omega_A \tag{14}$$

$$K_D = 0.9620 J_L - J_M \tag{15}$$

$$K_I = 0.5819 J_L \omega_n^2 \tag{16}$$

$$K_P = 1.7816 J_L \omega_n \tag{17}$$

The gains of the controller obtained from ITAE criterion for state feedback controller are given by:

$$K_E = \frac{J_M J_L}{K_S} \omega_n^4 \tag{18}$$

$$K_L = \frac{2.7 J_M J_L \omega_n^3}{K_S} - 2.1 J_M \omega_n \tag{19}$$

$$K_H = \frac{J_M}{K_S} (3.4 \omega_n^2 - \frac{J_L}{K_S} \omega_n^4 - \frac{K_S}{J_L}) - 1 \tag{20}$$

$$K_M = 2.1 J_M \omega_n \tag{21}$$

In the case  $\omega_n$  is not free and choice that the coefficient  $K_H$  and  $K_L$  are positive.

VI. SIMULATION RESULTS

The simulation results of the speed control of the two-mass using the proposed controller will be shown in this section in order to demonstrate the efficiency of the controller. The system parameters used in this paper are listed on Table 1.

The uncontrolled system has one critical oscillatory mode with eigenvalue,  $\lambda = -10.26 \pm j137.14$ , and with low damping ratio,  $\eta = 0.0746$ . The frequency response of motor speed to motor torque [ $H_{MM}(s)$ ] and load speed to load torque [ $H_{LL}(s)$ ] in open loop system are show in Figure 5. The step responses of shaft torque relative to motor torque and load torque for open loop system are show in Figure 6. In all the figures, the response relative to motor torque is shown with dotted line with legend  $T_M$  and the response relative to load torque is shown with line with legend  $T_L$ .

To assess the effectiveness of the proposed controller different 3dB bandwidth are considered. The gains and system eigenvalues with the proposed controller are given in Table 2. In the case  $\omega_n$  must choice between 79.3122 and 219.6992. The simulation results for  $\omega_n = 134.06$  show in Figures 8-10.

Table 1. Nominal parameters of a two-mass plant [21]

Component	Quantity	Rating value
$K_S$	shaft stiffness	138 Nm/rad
$B_S$	shaft damping coefficient	$1 \times 10^{-1}$ Nm/rad/s
$J_M$	motor inertia	$4.8 \times 10^{-2}$ kg.m <sup>2</sup>
$B_M$	motor viscosity coefficient	$1.3 \times 10^{-3}$ Nm/rad/s
$J_L$	load inertia	$8.6 \times 10^{-3}$ kg.m <sup>2</sup>
$B_L$	load viscosity coefficient	$6.9 \times 10^{-2}$ Nm/rad/s
$\omega_R$	resonance frequency	137.56 rad/s
$\omega_A$	anti-resonance frequency	126.68 rad/s
$\omega_{AL}$	load anti-resonance frequency	53.62 rad/s

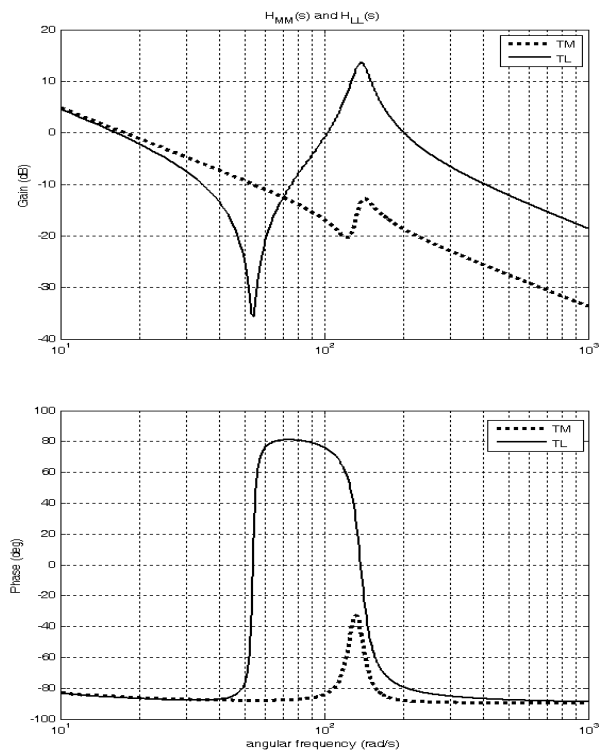


Figure 6. The frequency response of motor speed to motor torque and load speed to motor torque in open loop system

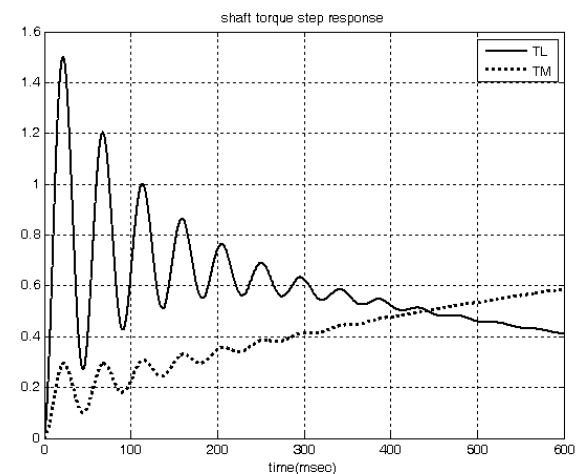


Figure 7. Step response of the shaft torque in open-loop system

The frequency response of motor speed to command speed in close-loop system is show in Figure 7. The step response of motor speed and shaft torque relative to command speed for close-loop system are show in Figures 8 and 9, respectively. It can see the response is stable.

Table 2. System eigenvalues with controller

$\omega_n$	Gains and eigenvalues
11.17	$K_E=0.05, K_L=-1.12, K_M=1.13, K_H=-6.43$ $\lambda_{1,2}=-4.74 \pm j14.11, \lambda_{3,4}=-6.99 \pm j4.63$
33.52	$K_E=3.77, K_L=-3.07, K_M=3.38, K_H=-5.28$ $\lambda_{1,2}=-14.21 \pm j42.33, \lambda_{3,4}=-20.98 \pm j13.88$
96.07	$K_E=254.87, K_L=-2.52, K_M=9.68, K_H=2.19$ $\lambda_{1,2}=-40.73 \pm j121.34, \lambda_{3,4}=-60.15 \pm j39.79$
111.72	$K_E=465.9, K_L=0, K_M=11.26, K_H=4.80$ $\lambda_{1,2}=-47.37 \pm j141.10, \lambda_{3,4}=-69.94 \pm j46.27$
134.06	$K_E=966.18, K_L=5.95, K_M=13.51, K_H=7.67$ $\lambda_{1,2}=-56.84 \pm j69.32, \lambda_{3,4}=-83.92 \pm j55.52$

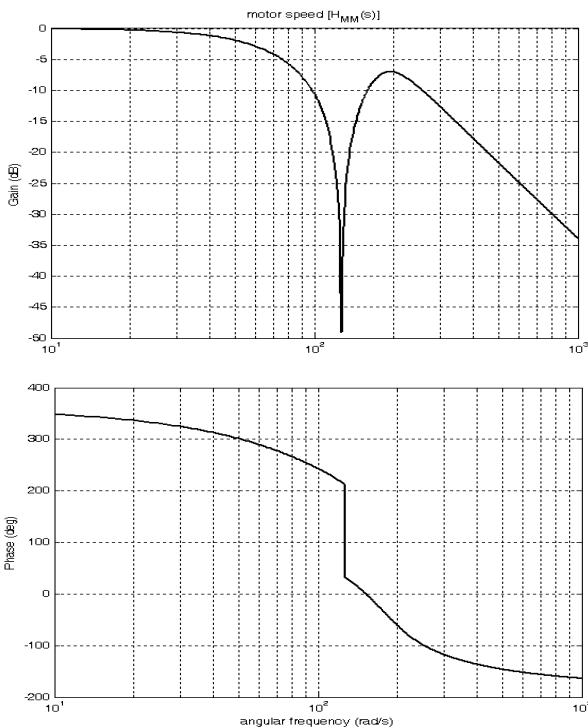


Figure 8. The frequency response of motor speed to command speed in close loop system

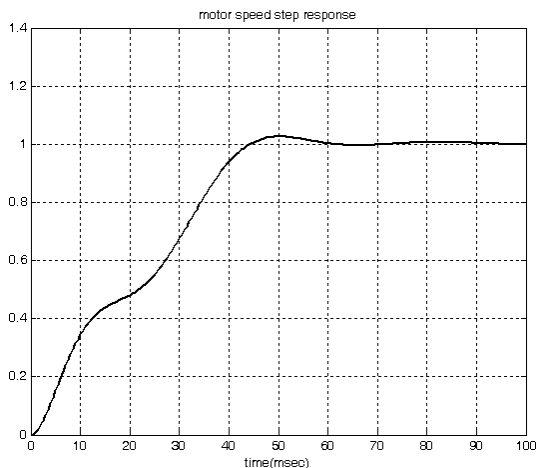


Figure 9. Step response of the motor speed in close-loop system

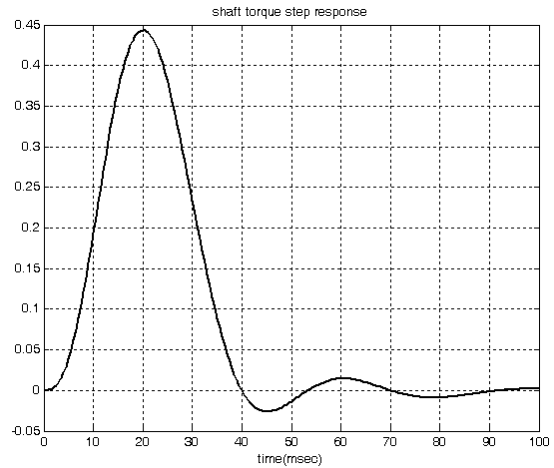


Figure 10. Step response of the shaft torque in close-loop system

### VI. CONCLUSIONS

Vibration suppression and attainment of robustness in motion control systems is an important problem in industry applications. In this paper a space-state mathematical analysis and design methods for a two-inertia system is develop. To eliminate the steady state error, an integral feedback is added to the motor speed feedback. The simulation results in controlling the speed of two-mass system by the proposed controller are investigated to verify the effectiveness of the proposed method.

### NOMENCLATURES

- $\omega_{AL}$ : load anti-resonance frequency
- $\omega_R$ : anti-resonance frequency
- $B_L$ : load viscosity coefficient
- $J_L$ : load inertia
- $B_M$ : motor viscosity coefficient
- $B_S$ : shaft damping coefficient
- $K_S$ : shaft stiffness
- $J_M$ : motor inertia
- $H_{MM}(s)$ : transfer function of motor speed to motor torque
- $H_{LL}(s)$ : transfer function of load speed to load torque

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