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OPTIMIZATION IN SYMMETRIC ORTHOTROPIC SHELLS

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Abstract- The main problems of a modern mechanics, which are contemporary deformed, rigid bodies, are the problems of optimal design of structures. The topic of this work includes questions, which are related to the problems of multilayer structures, those are the problems of optimal design of plates and shells. It is planned to introduce final results based on the structure of Figure 1, and the proposed methods.

Keywords: Optimization, Deformation, Orthotropic, Multilayer, Shells.

I. INTRODUCTION

The shell's deformation state is described mathematically by the system of three differential equations, the form of which is very higher, than one equation form in slabs.

1. The possible reduction of the shell's weight, when the main frequency of shell's own fluctuation is focused, in this paragraph we will observe two cases of the rotation of shells, closed and opened tubular shells [1, 2].

2. Three layered shells, the layers of which is made of homogeneous materials, the middle layer has h_0 constant thickness, but the outer layers, which were arranged symmetrically over the surface of the shell, have changeable $h_w(\alpha,\beta)$ thickness. [3]

The optimal planning problems of shells for all listed types have the following general definition.

$$M \xrightarrow[h]{} \min \tag{1}$$

$$\omega = \text{fix} \tag{2}$$

where M is the weight of shell, and the command parameter h is defined as function which describes the shell's thickness [4].

$$(0 < h_1 \le h \le h_2)\ ,\ h_1, h_2 = \text{const} \tag{3}$$

$$\iint\limits_{\Omega} |gradh|^2\, d\Omega \le C^2\ ,\ C = \text{const} \tag{4}$$

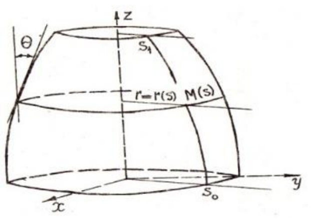

Figure 1. Multilayer structure problem

II. MATERIALS AND METHODS

$$\frac{d}{dS}\{rh[B_{11}(\frac{dU}{dS}+\frac{W}{R_1})+\frac{B_{12}}{r}\left(W\cos\theta+U\sin\theta\right)]\}+\frac{1}{R_1}\frac{d}{dS}\{\frac{rh^3}{12}.$$
$$.[B_{11}\frac{d}{dS}(-\frac{dW}{dS}+\frac{U}{R_1})+B_{12}\frac{\sin\theta}{r}(\frac{dW}{dS}-\frac{U}{R_!})]\}+h[B_{12}(\frac{dU}{dS}+$$
$$+\frac{W}{R_1})+B_{22}\frac{1}{r}\left(W\cos\theta-U\sin\theta\right)\sin\theta+\frac{\sin\theta}{R_1}\frac{h^3}{12}[B_{12}\frac{d}{dS}(-\frac{dW}{dS}+$$
$$+\frac{U}{R_1})+B_{22}\frac{\sin\theta}{r}(\frac{dW}{dS}-\frac{U}{R_1})]=-r\rho\omega^2 hU=-\omega^2\psi_3(t)U \tag{5}$$

$$\frac{d}{dS}\{\frac{d}{dS}[\frac{rh^3}{12}(B_{11}\frac{d}{dS}(-\frac{dW}{dS}+\frac{U}{R_1})+B_{12}\frac{\sin\theta}{r}.$$
$$.(\frac{dW}{dS}+\frac{U}{R_1}))]+\frac{h^3\sin\theta}{12}[B_{12}\frac{d}{dS}(-\frac{dW}{dS}+\frac{U}{R_1})+$$
$$+B_{22}\frac{\sin\theta}{r}(\frac{dW}{dS}-\frac{U}{R_!})]\}-rh\{\frac{1}{R_1}[B_{12}(\frac{dU}{dS}+\frac{W}{R_1})+$$
$$+\frac{B_{12}}{r}\left(W\cos\theta-U\sin\theta\right)]+\frac{1}{R_2}[B_{12}(\frac{dU}{dS}+\frac{W}{R_1})+$$
$$+B_{22}\frac{1}{r}(W\cos\theta-U\sin\theta)]\}=-r\rho\bar{\omega}^2 hW=-\omega^2\psi_3(D_3)W$$

$$\psi_3\left(D_3\right)\equiv r\rho h \tag{6}$$

$$D_3=h\left(S\right) \tag{7}$$

where S is the arc of the line of the shell surface between the point M and the point M_0 (the beginning of the focused settlement), $r=r\left(S\right)$ is the distance of the point M from OZ axis, $\theta=\theta\left(S\right)$ is the angle formed by angle of the point M with OZ axis, R_1, R_2 are the main radiuses of the curvature of a rotation surface. For certainty we would assume, that in the shell contour of $S=S_0$ and $S=S_1$ are taken place the following marginal conditions [5].

$$W\left(S\right)=\frac{dW}{dS}=U\left(S\right)=0\quad ,\quad S=S_0\quad ,\quad S=S_1 \tag{8}$$

The shells weight fixed with the accuracy of multiplier is given with the following functional way [6].

$$J=\int_{S_0}^{S} rhdS \tag{9}$$

III. RESULTS AND DISCUSSION

Let's give δh increase to the functional h, in that case (5) system of equations will be written of this enhancements and will receive as the following [9, 10].

$$\begin{aligned}
&\frac{d}{dS}\{rh+\delta h[B_{11}(\frac{dU+\delta U}{dS}+\frac{W+\delta W}{R_1})+\frac{B_{12}}{r}(W+\delta W\cos\theta+\\
&U+\delta U\sin\theta)]\}+\frac{1}{R_1}\frac{d}{dS}\{\frac{rh+\delta h^3}{12}[B_{11}\frac{d}{dS}(-\frac{dW+\delta W}{dS}+\\
&\frac{U+\delta U}{R_1})+B_{12}\frac{\sin\theta}{r}(\frac{dW+\delta W}{dS}-\frac{U+\delta U}{R_!})]\}+h+\\
&+\delta h[B_{12}(\frac{dU+\delta U}{dS}+\frac{W+\delta W}{R_1})+B_{22}\frac{1}{r}(W+\delta W\cos\theta-U+\\
&+\delta U\sin\theta)]+\sin\theta+\frac{\sin\theta}{R_1}\frac{h+\delta h^3}{12}[B_{12}\frac{d}{dS}(-\frac{dW+\delta W}{dS}+\\
&+\frac{U+\delta U}{R_1})+B_{22}\frac{\sin\theta}{r}(\frac{dW+\delta W}{dS}-\frac{U+\delta U}{R_1})]+r\rho\omega^2 h+\\
&+\delta hU+\delta U=0
\end{aligned}\tag{10}$$

$$\begin{aligned}
&\frac{d}{dS}\{\frac{d}{dS}[\frac{rh+\delta h^3}{12}(B_{11}\frac{d}{dS}(-\frac{dW+\delta W}{dS}+\frac{U+\delta U}{R_1})+\\
&+B_{12}\frac{\sin\theta}{r}(\frac{dW+\delta W}{dS}+\frac{U+\delta U}{R_1}))]+\frac{h+\delta h^3\sin\theta}{12}.\\
&.[B_{12}\frac{d}{dS}(-\frac{dW+\delta W}{dS}+\frac{U+\delta U}{R_1})++B_{22}\frac{\sin\theta}{r}.\\
&.(\frac{dW+\delta W}{dS}-\frac{U+\delta U}{R_!})]\}-rh+\delta h\{\frac{1}{R_1}[B_{12}(\frac{dU+\delta U}{dS}+\\
&+\frac{W+\delta W|}{R_1})+\frac{B_{12}}{r}(W+\delta W\cos\theta-U+\delta U\sin\theta)]+\\
&+\frac{1}{R_2}[B_{12}(\frac{dU+\delta U}{dS}+\frac{W+\delta W}{R_1})+B_{22}\frac{1}{r}(W+\delta W\cos\theta-\\
&-U+\delta U\sin\theta)]\}+r\rho\bar{\omega}^2 h+\delta hW+\delta W=0
\end{aligned}\tag{11}$$

Taking into consideration (5) the equations and ignoring δh, δU, δW enhancement of the small quantities of the second order we will get [7, 8]:

$$\begin{aligned}
&\frac{d}{dS}\{r[B_{11}h\frac{d\delta U}{dS}+B_{11}\delta h\frac{dU}{dS}+B_{11}h\frac{\delta W}{R_1}+B_{11}\delta h\frac{W}{R_1}+\frac{B_{12}}{r}h.\\
&.\delta W\cos\theta+\frac{B_{12}}{r}\delta hW\cos\theta+\frac{B_{12}}{r}h\delta U\sin\theta+\frac{B_{12}}{r}\delta hU\sin\theta]\}+\\
&+\frac{1}{R_1}\frac{d}{dS}\{\frac{r}{12}[-B_{11}\frac{d}{dS}h^3\frac{d\delta W}{dS}-B_{11}\frac{d}{dS}3h^2\delta h\frac{dW}{dS}+\\
&+B_{11}\frac{d}{dS}h^3\frac{\delta U}{R_1}+B_{11}\frac{d}{dS}3h^2\delta h\frac{U}{R_1}+B_{12}\frac{\sin\theta}{r}\frac{d}{dS}h^3\frac{d\delta W}{dS}+\\
&+B_{12}\frac{\sin\theta}{r}\frac{d}{dS}3h^2\delta h\frac{dW}{dS}-B_{12}\frac{\sin\theta}{r}\frac{d}{dS}h^3\frac{\delta U}{R_1}-\\
&-B_{12}\frac{\sin\theta}{r}\frac{d}{dS}3h^2\delta h\frac{U}{R_!}]\}+[B_{12}h\frac{d\delta U}{dS}+B_{12}\delta h\frac{dU}{dS}+\\
&+B_{12}h\frac{\delta W}{R_1}+B_{12}\delta h\frac{W}{R_1}+B_{22}\frac{1}{r}h\delta W\cos\theta+B_{22}\frac{1}{r}\delta hW\cos\theta-\\
&-B_{22}\frac{1}{r}h\delta U\sin\theta-B_{22}\frac{1}{r}\delta hU\sin\theta]\sin\theta+\frac{\sin\theta}{R_1}\frac{1}{12}.\\
&.[-B_{12}\frac{d}{dS}h^3\frac{d\delta W}{ds}-B_{12}\frac{d}{dS}3h^2\delta h\frac{dW}{dS}+B_{12}\frac{d}{dS}h^3\frac{\delta U}{R_1}+\\
&+B_{12}\frac{d}{dS}3h^2\delta h\frac{U}{R_1}+B_{22}\frac{\sin\theta}{r}h^3\frac{d\delta W}{dS}+\\
&+B_{22}\frac{\sin\theta}{r}3h^2\delta h\frac{dW}{dS}-B_{22}\frac{\sin\theta}{r}h^3\frac{\delta U}{R_1}-\\
&-B_{22}\frac{\sin\theta}{r}3h^2\delta h\frac{U}{R_1}]+r\rho\omega^2 h\delta U+r\rho\omega^2\delta hU=0
\end{aligned}\tag{12}$$

$$\begin{aligned}
&\frac{d}{dS}\{\frac{d}{dS}[\frac{r}{12}(-B_{11}h^3\frac{d\delta W}{dS}-B_{11}3h^2\delta h\frac{dW}{dS}+B_{11}h^3\frac{\delta U}{R_1}+\\
&+B_{11}3h^2\delta h\frac{U}{R_1}+B_{12}\frac{\sin\theta}{r}h^2\frac{d\delta W}{dS}+B_{12}\frac{\sin\theta}{r}3h^2\delta h\frac{dW}{ds}+\\
&+B_{12}\frac{\sin\theta}{r}h^3\frac{\delta U}{R_1}+B_{12}\frac{\sin\theta}{r}3h^2\delta h\frac{U}{R_1})]+\frac{\sin\theta}{12}[-B_{12}\frac{d}{dS}.\\
&.h^3\frac{d\delta W}{dS}-B_{12}\frac{d}{dS}3h^2\delta h\frac{dW}{dS}+B_{12}\frac{d}{dS}h^3\frac{\delta U}{R_1}+B_{12}\frac{d}{dS}3h^2.\\
&.\delta h\frac{U}{R_1}+B_{22}\frac{\sin\theta}{r}h^3\frac{d\delta W}{dS}+B_{22}\frac{\sin\theta}{r}3h^2\delta h\frac{dW}{dS}-\\
&-B_{22}\frac{\sin\theta}{r}h^3\frac{\delta U}{R_1}-B_{22}\frac{\sin\theta}{r}3h^2\delta h\frac{U}{R_!}]\}-r\{\frac{1}{R_1}[B_{12}h\frac{d\delta U}{dS}+\\
&+B_{12}\delta h\frac{dU}{dS}+B_{12}h\frac{\delta W}{R_1}+B_{12}\delta h\frac{W}{R_1}+B_{12}\frac{1}{r}h\delta W\cos\theta+\\
&+B_{12}\frac{1}{r}\delta hW\cos\theta-B_{12}\frac{1}{r}h\delta U\sin\theta-B_{12}\frac{1}{r}\delta hU\sin\theta]+\\
&+\frac{1}{R_2}[B_{12}h\frac{d\delta U}{ds}+B_{12}\delta h\frac{dU}{dS}+B_{12}h\frac{\delta W}{R_1}+B_{12}\delta h\frac{W}{R_1}+\\
&+B_{22}\frac{1}{r}h\delta W\cos\theta+B_{22}\frac{1}{r}\delta hW\cos\theta-B_{22}\frac{1}{r}h\delta U\sin\theta-\\
&-B_{22}\frac{1}{r}\delta hU\sin\theta]\}+r\rho\bar{\omega}^2 h\delta W+r\rho\bar{\omega}^2\delta hW=0
\end{aligned}\tag{13}$$

The Functional purpose will also receive increases

$$\delta J=\int_{S_0}^{S} r\delta h dS \tag{14}$$

Let's multiply the first and the second left parts of the systems Equation (13) accordingly with the U_1, W_1 functions. The amount of obtained expressions integrate to S_0 - S, than to add Equation (14) [9, 10].

$$[[\frac{d}{dS}\{r[B_{11}h\frac{d\delta U}{dS}+B_{11}\delta h\frac{dU}{dS}+B_{11}h\frac{\delta W}{R_1}+B_{11}\delta h\frac{W}{R_1}+$$
$$+\frac{B_{12}}{r}h\delta W\cos\theta+\frac{B_{12}}{r}\delta hW\cos\theta+\frac{B_{12}}{r}h\delta U\sin\theta+$$
$$+\frac{B_{12}}{r}\delta hU\sin\theta]\}+\frac{1}{R_1}\frac{d}{dS}\{\frac{r}{12}[-B_{11}\frac{d}{dS}h^3\frac{d\delta W}{dS}-$$
$$-B_{11}\frac{d}{dS}3h^2\delta h\frac{dW}{dS}+B_{11}\frac{d}{dS}h^3\frac{\delta U}{R_1}+B_{11}\frac{d}{dS}3h^2\delta h\frac{U}{R_1}+$$
$$+B_{12}\frac{\sin\theta}{r}\frac{d}{dS}h^3\frac{d\delta W}{dS}+B_{12}\frac{\sin\theta}{r}\frac{d}{dS}3h^2\delta h\frac{dW}{dS}-$$
$$-B_{12}\frac{\sin\theta}{r}\frac{d}{dS}h^3\frac{\delta U}{R_1}-B_{12}\frac{\sin\theta}{r}\frac{d}{dS}3h^2\delta h\frac{U}{R_!}]\}+$$
$$+[B_{12}h\frac{d\delta U}{dS}+B_{12}\delta h\frac{dU}{dS}+B_{12}h\frac{\delta W}{R_1}+B_{12}\delta h\frac{W}{R_1}+$$
$$+B_{22}\frac{1}{r}h\delta W\cos\theta+B_{22}\frac{1}{r}\delta hW\cos\theta-B_{22}\frac{1}{r}h\delta U\sin\theta-$$
$$-B_{22}\frac{1}{r}\delta hU\sin\theta]\sin\theta+\frac{\sin\theta}{R_1}\frac{1}{12}[-B_{12}\frac{d}{dS}h^3\frac{d\delta W}{ds}-$$
$$-B_{12}\frac{d}{dS}3h^2\delta h\frac{dW}{dS}+B_{12}\frac{d}{dS}h^3\frac{\delta U}{R_1}+B_{12}\frac{d}{dS}3h^2\delta h\frac{U}{R_1}+$$
$$+B_{22}\frac{\sin\theta}{r}h^3\frac{d\delta W}{dS}+B_{22}\sin 3h^2\delta h\frac{dW}{dS}-B_{22}\frac{\sin\theta}{r}h^3\frac{\delta U}{R_1}-$$
$$B_{22}\frac{\sin\theta}{r}3h^2\delta h\frac{U}{R_1}]+r\rho\omega^2 h\delta U+r\rho\omega^2\delta hU]U_1+ \quad (15)$$
$$+[\frac{d}{dS}\{\frac{d}{dS}[\frac{r}{12}(-B_{11}\frac{d}{dS}h^3\frac{d\delta W}{dS}-B_{11}\frac{d}{dS}3h^2\delta h\frac{dW}{dS}+$$
$$+B_{11}\frac{d}{dS}h^3\frac{\delta U}{R_1}+B_{11}\frac{d}{dS}3h^2\delta h\frac{U}{R_1}+B_{12}\frac{\sin\theta}{r}h^2\frac{d\delta W}{dS}+$$
$$+B_{12}\frac{\sin\theta}{r}3h^2\delta h\frac{dW}{ds}+B_{12}\frac{\sin\theta}{r}h^3\frac{\delta U}{R_1}+B_{12}\frac{\sin\theta}{r}3h^2\delta h\frac{U}{R_1})]+$$
$$+\frac{\sin\theta}{12}[-B_{12}\frac{d}{dS}h^3\frac{d\delta W}{dS}-B_{12}\frac{d}{dS}3h^2\delta h\frac{dW}{dS}+B_{12}\frac{d}{dS}h^3\frac{\delta U}{R_1}+$$
$$+B_{12}\frac{d}{dS}3h^2\delta h\frac{U}{R_1}+B_{22}\frac{\sin\theta}{r}h^3\frac{d\delta W}{dS}+B_{22}\frac{\sin\theta}{r}3h^2\delta h\frac{dW}{dS}-$$
$$-B_{22}\frac{\sin\theta}{r}h^3\frac{\delta U}{R_1}-B_{22}\frac{\sin\theta}{r}3h^2\delta h\frac{U}{R_!}]\}-r\{\frac{1}{R_1}[B_{12}h\frac{d\delta U}{dS}+$$
$$+B_{12}\delta h\frac{dU}{dS}+B_{12}h\frac{\delta W}{R_1}+B_{12}\delta h\frac{W}{R_1}+B_{12}\frac{1}{r}h\delta W\cos\theta+$$
$$+B_{12}\frac{1}{r}\delta hW\cos\theta-B_{12}\frac{1}{r}h\delta U\sin\theta-B_{12}\frac{1}{r}\delta hU\sin\theta]+$$
$$+\frac{1}{R_2}[B_{12}h\frac{d\delta U}{ds}+B_{12}\delta h\frac{dU}{dS}+B_{12}h\frac{\delta W}{R_1}+B_{12}\delta h\frac{W}{R_1}+$$
$$+B_{22}\frac{1}{r}h\delta W\cos\theta+B_{22}\frac{1}{r}\delta hW\cos\theta-B_{22}\frac{1}{r}h\delta U\sin\theta-$$
$$-B_{22}\frac{1}{r}\delta hU\sin\theta]\}+r\rho\overline{\omega}^2 h\delta W+r\rho\overline{\omega}^2\delta hW]W_1+\delta h]dS$$

In the integrate excretion we get let's do grouping by δh, δU, δW enhancements. So (15) this integrate will have the following outlook [11, 12].

$$\int_{S_0}^{S}[\{r[B_{11}\delta U\frac{d}{dS}h\frac{dU_1}{dS}-B_{11}\delta h\frac{dU_1}{dS}\frac{dU}{dS}-\frac{B_{11}}{R_1}h\frac{dU_1}{dS}\delta W-$$
$$-\frac{B_{11}}{R_1}\delta h\frac{dU_1}{dS}W+\frac{B_{12}}{r}h\frac{dU_1}{dS}\delta W\cos\theta-\frac{B_{12}}{r}\delta h\frac{dU_1}{dS}W\cos\theta-$$
$$-\frac{B_{12}}{r}h\frac{dU_1}{dS}\delta U\sin\theta-\frac{B_{12}}{r}\delta h\frac{dU_1}{dS}U\sin\theta]\}+\frac{1}{R_1}\{\frac{r}{12}.$$
$$.[-B_{11}\delta W\frac{d^2}{dS^2}h^3\frac{dU_1}{dS}+3B_{11}h^2\delta h\frac{d^2W}{dS^2}\frac{dU_1}{dS}+\frac{B_{11}}{R_1}\frac{d}{dS}h^3\frac{dU_1}{dS}.$$
$$.\delta U+\frac{3B_{11}}{R_1}\frac{d}{dS}h^2\delta h\frac{dU_1}{dS}U+B_{12}\frac{\sin\theta}{r}\frac{d}{dS}h^3\frac{dU_1}{dS}\delta W-$$
$$-B_{12}\frac{\sin\theta}{r}\frac{d}{dS}3h^2\delta h\frac{dU_1}{dS}W-\frac{B_{12}}{R_1}\frac{\sin\theta}{r}\frac{d}{dS}h^3\frac{dU_1}{dS}\delta U-$$
$$-\frac{B_{12}}{R_1}\frac{\sin\theta}{r}\frac{d}{dS}3h^2\delta h\frac{dU_1}{dS}U]\}+[B_{12}\delta U\frac{dhU_1}{dS}-B_{12}\delta h\frac{dU}{dS}U_1-$$
$$-\frac{B_{12}}{R_1}h\delta WU_1-\frac{B_{12}}{R_1}\delta hWU_1-B_{22}\frac{1}{r}h\delta WU_1\cos\theta+$$
$$+B_{22}\frac{1}{r}\delta hWU_1\cos\theta+B_{22}\frac{1}{r}h\delta UU_1\sin\theta+B_{22}\frac{1}{r}\delta hUU_1\sin\theta].$$
$$.\sin\theta+\frac{\sin\theta}{R_1}\frac{1}{12}[-B_{12}\delta W\frac{d^2}{dS^2}h^3\frac{dU_1}{ds}++3B_{12}h^2\delta h\frac{d^2W}{dS^2}.$$
$$.\frac{dU_1}{dS}-\frac{B_{12}}{R_1}\frac{d}{dS}h^3\frac{dU_1}{dS}\delta U-\frac{3B_{12}}{R_1}\frac{d}{dS}h^2\delta h\frac{dU_1}{dS}U+B_{22}\frac{\sin\theta}{r}.$$
$$.h^3\frac{dU_1}{dS}\delta W-B_{22}\frac{\sin\theta}{r}3h^2\delta h\frac{dU_1}{dS}W+\frac{B_{22}}{R_1}\frac{\sin\theta}{r}h^3\delta UU_1+$$
$$+\frac{B_{22}}{R_1}\frac{\sin\theta}{r}3h^2\delta hUU_1]+r\rho\omega^2 h\delta UU_1+r\rho\omega^2\delta hUU_1+$$
$$+[\frac{r}{12}(B_{11}\delta W\frac{d^3}{dS^3}h^3\frac{dW_1}{dS}-B_{11}\frac{d^3}{dS^3}3h^2\delta h\frac{dW_1}{dS}\frac{dW}{dS}-$$
$$-\frac{B_{11}}{R_1}\frac{d^3}{dS^3}h^3\frac{dW_1}{dS}\delta U-\frac{B_{11}}{R_1}\frac{d^3}{dS^3}3h^2\delta h\frac{dW_1}{dS}U+ \quad (16)$$
$$+B_{12}\delta W\frac{\sin\theta}{r}\frac{d^2}{dS^2}h^2\frac{dW_1}{dS}-B_{12}\frac{\sin\theta}{r}\frac{d^2}{dS^2}3h^2\delta h\frac{dW_1}{dS}\frac{dW}{dS}-$$
$$-\frac{B_{12}}{R_1}\frac{\sin\theta}{r}\frac{d^2}{dS^2}h^3\delta UW_1-B_{12}\frac{\sin\theta}{r}\frac{d^2}{dS^2}3h^2\delta hUW_1)]+$$
$$+\frac{\sin\theta}{12}[-B_{12}\delta W\frac{d^3}{dS^3}h^3\frac{dW_1}{dS}+B_{12}\frac{d^2}{dS^2}3h^2\delta h\frac{dW_1}{dS}\frac{dW}{dS}+$$
$$+\frac{B_{12}}{R_1}\frac{d^2}{dS^2}h^3\frac{dW_1}{dS}\delta U+\frac{B_{12}}{R_1}\frac{d^2}{dS^2}3h^2\delta h\frac{dW_1}{dS}U+$$
$$+B_{22}\frac{\sin\theta}{r}\frac{d^2}{dS^2}h^3\frac{dW_1}{dS}\delta W-B_{22}\frac{\sin\theta}{r}\frac{d}{dS}3h^2\delta h\frac{dW_1}{dS}\frac{dW}{dS}-$$
$$-\frac{B_{22}}{R_1}\frac{\sin\theta}{r}\frac{d^2}{dS^2}h^3W_1\delta U-\frac{B_{22}}{R_1}\frac{\sin\theta}{r}\frac{d^2}{dS^2}3h^2\delta hW_1U]\}-$$
$$-r\{\frac{1}{R_1}[B_{12}\frac{d}{dS}h\frac{dW_1}{dS}\delta U-B_{12}\delta h\frac{dW_1}{dS}\frac{dU}{dS}-\frac{B_{12}}{R_1}h\frac{dW_1}{dS}\delta W-$$
$$-\frac{B_{12}}{R_1}\delta h\frac{dW_1}{dS}W-B_{12}\frac{1}{r}h\frac{dW_1}{dS}\delta W\cos\theta+B_{12}\frac{1}{r}\delta h\frac{dW_1}{dS}.$$

$$.W\cos\theta + B_{12}\frac{1}{r}h\frac{dW_1}{dS}\delta U\sin\theta + B_{12}\frac{1}{r}\delta h\frac{dW_1}{dS}U\sin\theta] +$$
$$+\frac{1}{R_2}[B_{12}\frac{d}{dS}h\frac{dW_1}{ds}\delta U - B_{12}\delta h\frac{dW_1}{dS}\frac{dU}{dS} - \frac{B_{12}}{R_1}h\frac{\delta W_1}{dS}\delta W -$$
$$-\frac{B_{12}}{R_1}\delta h\frac{dW_1}{dS}W + B_{22}\frac{1}{r}h\frac{dW_1}{dS}\delta W\cos\theta - B_{22}\frac{1}{r}\delta h\frac{dW_1}{dS}.$$
$$.W\cos\theta + B_{22}\frac{1}{r}h\frac{dW_1}{dS}\delta U\sin\theta + +B_{22}\frac{1}{r}\delta h\frac{dW_1}{dS}U\sin\theta]\} +$$
$$+r\rho\overline{\omega}^2 h\delta WW_1 + r\rho\overline{\omega}^2\delta hWW_1]dS$$

In (16) integrate expressions let's do grouping by δh, δU, δW, δV enhancement, demand that δh, δU, δW, δV enhancement parameters be equal to 0. It is possible, as U_1, V_1, W_1 functions don't subject to any limitation and now we demand them to satisfy the equations mentioned above.

$$\{r[B_{11}\frac{d}{dS}h\frac{dU_1}{dS} - \frac{B_{12}}{r}h\frac{dU_1}{dS}\sin\theta]\} + \frac{1}{R_1}\{\frac{r}{12}[+\frac{B_{11}}{R_1}\frac{d}{dS}h^3\frac{dU_1}{dS} -$$
$$-\frac{B_{12}}{R_1}\frac{\sin\theta}{r}\frac{d}{dS}h^3\frac{dU_1}{dS}]\} + [B_{12}\frac{dhU_1}{dS} + B_{22}\frac{1}{r}hU_1\sin\theta]\sin\theta +$$
$$+\frac{\sin\theta}{R_1}\frac{1}{12}[-\frac{B_{12}}{R_1}\frac{d}{dS}h^3\frac{dU_1}{dS} + \frac{B_{22}}{R_1}\frac{\sin\theta}{r}h^3U_{11}] + r\rho\omega^2 hU_1 + \quad (17)$$
$$[\frac{r}{12}(-\frac{B_{11}}{R_1}\frac{d^3}{dS^3}h^3\frac{dW_1}{dS} - \frac{B_{12}}{R_1}\frac{\sin\theta}{r}\frac{d^2}{dS^2}h^3W_1)] + \frac{\sin\theta}{12}.$$
$$.[\frac{B_{12}}{R_1}\frac{d^2}{dS^2}h^3\frac{dW_1}{dS} - \frac{B_{22}}{R_1}\frac{\sin\theta}{r}\frac{d^2}{dS^2}h^3W_1]\} - r\{\frac{1}{R_1}[B_{12}\frac{d}{dS}h\frac{dW_1}{dS} +$$
$$+B_{12}\frac{1}{r}h\frac{dW_1}{dS}\sin\theta] + \frac{1}{R_2}[B_{12}\frac{d}{dS}h\frac{dW_1}{ds} + B_{22}\frac{1}{r}h\frac{dW_1}{dS}\sin\theta]\} = 0$$

$$\{r[-\frac{B_{11}}{R_1}h\frac{dU_1}{dS} + \frac{B_{12}}{r}h\frac{dU_1}{dS}\cos\theta]\} + \frac{1}{R_1}\{\frac{r}{12}[-B_{11}\frac{d^2}{dS^2}.$$
$$.h^3\frac{dU_1}{dS} + B_{12}\frac{\sin\theta}{r}\frac{d}{dS}h^3\frac{dU_1}{dS}]\} + [-\frac{B_{12}}{R_1}hU_1 - B_{22}\frac{1}{r}hU_1.$$
$$.\cos\theta]\sin\theta + \frac{\sin\theta}{R_1}\frac{1}{12}[-B_{12}\frac{d^2}{dS^2}h^3\frac{dU_1}{ds} + B_{22}\frac{\sin\theta}{r}h^3\frac{dU_1}{dS}] +$$
$$+[\frac{r}{12}(B_{11}\frac{d^3}{dS^3}h^3\frac{dW_1}{dS} + B_{12}\frac{\sin\theta}{r}\frac{d^2}{dS^2}h^2\frac{dW_1}{dS})] + \frac{\sin\theta}{12}. \quad (18)$$
$$.[-B_{12}\frac{d^3}{dS^3}h^3\frac{dW_1}{dS} + B_{22}\frac{\sin\theta}{r}\frac{d^2}{dS^2}h^3\frac{dW_1}{dS}]\} - r\{\frac{1}{R_1}[-\frac{B_{12}}{R_1}.$$
$$.h\frac{dW_1}{dS} - B_{12}\frac{1}{r}h\frac{dW_1}{dS}\cos\theta] + \frac{1}{R_2}[-\frac{B_{12}}{R_1}h\frac{\delta W_1}{dS} + B_{22}\frac{1}{r}h\frac{dW_1}{dS}.$$
$$.\cos\theta]\} + r\rho\overline{\omega}^2 hW_1 = 0$$

In this case δh factor must be 0, as $\delta\overline{J} = 0$ at any δh case.

$$\{r[-B_{11}\frac{dU_1}{dS}\frac{dU}{dS} - \frac{B_{11}}{R_1}\frac{dU_1}{dS}W - \frac{B_{12}}{r}\frac{dU_1}{dS}W\cos\theta - \frac{B_{12}}{r}\frac{dU_1}{dS}U.$$
$$\sin\theta]\} + \frac{1}{R_1}\{\frac{r}{12}[+3B_{11}h^2\frac{d^2W}{dS^2}\frac{dU_1}{dS} + \frac{3B_{11}}{R_1}\frac{d}{dS}h^2\frac{dU_1}{dS}U -$$
$$-B_{12}\frac{\sin\theta}{r}\frac{d}{dS}3h^2\frac{dU_1}{dS}W - \frac{B_{12}}{R_1}\frac{\sin\theta}{r}\frac{d}{dS}3h\frac{dU_1}{dS}U]\} + [-B_{12}\frac{dU}{dS}.$$
$$.U_1 - \frac{B_{12}}{R_1}WU_1 + B_{22}\frac{1}{r}WU_1\cos\theta + B_{22}\frac{1}{r}UU_1\sin\theta]\sin\theta + \frac{\sin\theta}{R_1}\frac{1}{12}.$$
$$.[3B_{12}h^2\frac{d^2W}{dS^2}\frac{dU_1}{dS} - \frac{3B_{12}}{R_1}\frac{d}{dS}h^2\frac{dU_1}{dS}U - B_{22}\frac{Sin\theta}{r}3h^2\frac{dU_1}{dS}W +$$
$$+\frac{B_{22}}{R_1}\frac{\sin\theta}{r}3h^2UU_1] + r\rho\omega^2UU_1 + [\frac{r}{12}(-B_{11}\frac{d^3}{dS^3}3h^2\frac{dW_1}{dS}\frac{dW}{dS} -$$
$$-\frac{B_{11}}{R_1}\frac{d^3}{dS^3}3h^2\frac{dW_1}{dS}U - B_{12}\frac{\sin\theta}{r}\frac{d^2}{dS^2}3h^2\frac{dW_1}{dS}\frac{dW}{dS} - \quad (19)$$
$$-B_{12}\frac{\sin\theta}{r}\frac{d^2}{dS^2}3h^2UW_1)] + \frac{\sin\theta}{12}[B_{12}\frac{d^2}{dS^2}3h^2\frac{dW_1}{dS}\frac{dW}{dS} +$$
$$+\frac{B_{12}}{R_1}\frac{d^2}{dS^2}3h^2\frac{dW_1}{dS}U - B_{22}\frac{\sin\theta}{r}\frac{d}{dS}3h^2\frac{dW_1}{dS}\frac{dW}{dS} - \frac{B_{22}}{R_1}\frac{\sin\theta}{r}.$$
$$.\frac{d^2}{dS^2}3h^2W_1U]\} - r\{\frac{1}{R_1}[-B_{12}\frac{dW_1}{dS}\frac{dU}{dS} - \frac{B_{12}}{R_1}\frac{dW_1}{dS}W + B_{12}\frac{1}{r}\frac{dW_1}{dS}.$$
$$.W\cos\theta + B_{12}\frac{1}{r}\frac{dW_1}{dS}U\sin\theta] + \frac{1}{R_2}[-B_{12}\frac{dW_1}{dS}\frac{dU}{dS} - \frac{B_{12}}{R_1}\frac{dW_1}{dS}W -$$
$$-B_{22}\frac{1}{r}\frac{dW_1}{dS}W\cos\theta + B_{22}\frac{1}{r}\frac{dW_1}{dS}U\sin\theta]\} + r\rho\overline{\omega}^2WW_1 = 0$$

Based on (5), (7) and (18) systems similarity we have:

$$U_1 = -UC \quad , \quad W_1 = WC \quad (20)$$

If Equation (20) put on Equation (19) we will get:

$$\{r[B_{11}\frac{d(UC)}{dS}\frac{dU}{dS} + \frac{B_{11}}{R_1}\frac{d(UC)}{dS}W + \frac{B_{12}}{r}\frac{d(UC)}{dS}W\cos\theta +$$
$$+\frac{B_{12}}{r}\frac{d(UC)}{dS}U\sin\theta]\} + \frac{1}{R_1}\{\frac{r}{12}[-3B_{11}h^2\frac{d^2W}{dS^2}\frac{d(UC)}{dS} -$$
$$-\frac{3B_{11}}{R_1}\frac{d}{dS}h^2\frac{d(UC)}{dS}U + B_{12}\frac{\sin\theta}{r}\frac{d}{dS}3h^2\frac{d(UC)}{dS}W +$$
$$+\frac{B_{12}}{R_1}\frac{\sin\theta}{r}\frac{d}{dS}3h\frac{d(UC)}{dS}U]\} + [B_{12}\frac{dU}{dS}(UC) + \frac{B_{12}}{R_1}W(UC) -$$
$$-B_{22}\frac{1}{r}W(UC)\cos\theta - B_{22}\frac{1}{r}U(UC)\sin]\sin\theta + +\frac{\sin\theta}{R_1}\frac{1}{12}.$$
$$.[-3B_{12}h^2\frac{d^2W}{dS^2}\frac{d(UC)}{dS} + \frac{3B_{12}}{R_1}\frac{d}{dS}h^2\frac{d(UC)}{dS}U + B_{22}\frac{\sin\theta}{r}.$$
$$.3h^2\frac{d(UC)}{dS}W - \frac{B_{22}}{R_1}\frac{\sin\theta}{r}3h^2U(UC)] - r\rho\omega^2U(UC) + \quad (21)$$
$$+[\frac{r}{12}(-B_{11}\frac{d^3}{dS^3}3h^2\frac{d(WC)}{dS}\frac{dW}{dS} - \frac{B_{11}}{R_1}\frac{d^3}{dS^3}3h^2\frac{d(WC)}{dS}U -$$

$$-B_{12}\frac{\sin\theta}{r}\frac{d^2}{dS^2}3h^2\frac{d(WC)}{dS}\frac{dW}{dS}-B_{12}\frac{\sin\theta}{r}\frac{d^2}{dS^2}3h^2U(WC))]+$$
$$+\frac{\sin\theta}{12}[B_{12}\frac{d^2}{dS^2}3h^2\frac{d(WC)}{dS}\frac{dW}{dS}+\frac{B_{12}}{R_1}\frac{d^2}{dS^2}3h^2\frac{d(WC)}{dS}U-$$
$$-B_{22}\frac{\sin\theta}{r}\frac{d}{dS}3h^2\frac{d(WC)}{dS}\frac{dW}{dS}-\frac{B_{22}}{R_1}\frac{\sin\theta}{r}\frac{d^2}{dS^2}3h^2(WC)U]\}-$$
$$-r\{\frac{1}{R_1}[-B_{12}\frac{d(WC)}{dS}\frac{dU}{dS}-\frac{B_{12}}{R_1}\frac{d(WC)}{dS}W+B_{12}\frac{1}{r}\frac{d(WC)}{dS}W.$$
$$.\cos\theta+B_{12}\frac{1}{r}\frac{d(WC)}{dS}U\sin\theta]+\frac{1}{R_2}[-B_{12}\frac{d(WC)}{dS}\frac{dU}{dS}-$$
$$-\frac{B_{12}}{R_1}\frac{d(WC)}{dS}W-B_{22}\frac{1}{r}\frac{d(WC)}{dS}W\cos\theta+B_{22}\frac{1}{r}\frac{d(WC)}{dS}U.$$
$$.\sin\theta]\}+r\rho\bar{\omega}^2W(WC)=0$$

IV. CONCLUSIONS

The Equation (21) with the Equation (5) to (7) system made close system $U(\alpha)$, $V(\alpha)$, $W(\alpha)$, $h(\alpha)$ for unknown functions. Solving this equations system, taking into consideration the appropriate (8) edge conditions, we will get the function of transformation of U, V, W and the function of describing the thickness of h.

$$\frac{h^2}{4}\{B_{11}[\frac{d}{dS}(\frac{dW}{dS}+\frac{U}{R_1})]^2+2B_{12}(\frac{dW}{dS}-\frac{U}{R_1})\frac{d}{dS}(\frac{dW}{dS}-\frac{U}{R_1})\frac{\sin\theta}{r}+$$
$$+B_{22}(\frac{dW}{dS}-\frac{U}{R_1})^2\frac{\sin^2\theta}{r^2}\}+B_{11}(\frac{dU}{dS}-\frac{U}{R_1})^2+2B_{12}(\frac{dU}{dS}+\frac{W}{R_1}).\quad(22)$$
$$.(\frac{W}{R_2}-\frac{U\sin\theta}{r})+B_{22}(\frac{W}{R_2}-\frac{U\sin\theta}{r})^2=\rho\omega^2U^2+\rho\omega^2W^2$$

Suggested technology has some advantages:
- It has an optimization system;
- It is cheaper;
- Getting a high qualitative production.

NOMENCLATURES

∂: Partial differentiation operator
u, *v*, *w*: Displacements in the *x*, *y*, and *z* directions
M: Weight of shell
E: Young's modulus
Σ: Summation
D: Modulus of rigidity for isotropic material
h: Shell thickness
$h_w(\alpha, \beta)$: Changeable thickness
dx: Differential distance in the x direction
dy: Differential distance in the y direction
dz: Differential distance in the z direction
D_{ij}, (i,j=1,2,6): Flexural rigidity matrix
ω: Free fluctuation of shell
v: Modulus of rigidity for isotropic material

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BIOGRAPHY

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