

LEARNING FOOTHOLDS IN ROUGH TERRAIN QUADRUPED LOCOMOTION

N. Sharghivand S.N. Razavi M.R. Feizi Derakhshi

Computer Engineering Department, Faculty of Electrical and Computer Engineering, University of Tabriz, Tabriz, Iran
n.sharghivand@gmail.com, n.razavi@tabrizu.ac.ir, mfeizi@tabrizu.ac.ir

Abstract- In this paper, a new foothold selection approach is proposed for quadruped locomotion over perceived rough terrain, in which a variety of learning and optimization techniques has been applied to achieve robust, fast locomotion. Here, the feature set gained by template learning from expert demonstrations, with combination of other linear features, form the input to a reinforcement learning algorithm that learns optimal foothold choices using an approximation of the value function. We evaluate the performance of our method by testing it on a simulated quadruped robot in Matlab over perceived uneven terrains. Results show that even with low number of expert demonstrations, the overall performance is quiet fair in terms of collisions and fall overs.

Keywords: Quadruped Locomotion, Rough Terrain, Reinforcement Learning, Template Learning, Foothold Selection.

I. INTRODUCTION

The problem of legged locomotion over rough terrain has always been an appealing branch of research in robotics. The limitations existing in wheeled locomotion have restricted its applications to few cases and only about half of the earth's land mass is accessible by wheeled vehicles [1]. Indeed, the main motivation behind this research area is the potential of legged animals to traverse much wider variety of terrains, including very rough terrains. However, despite the advances achieved in the field, current legged robots still have a long way to go to be as good as their biological counterparts, while most of the work done in this area focuses on legged locomotion over flat terrains or terrains with small irregularities and obstacles compared to the size of the robot, ignoring the potential of such robots to traverse much more challenging terrains.

One of the main challenges in robust rough terrain legged locomotion, is to find a suitable foothold for the swing leg. Because, a bad choice of foot position may end up with a collision, fall over, or leading to a bad state, making further progress impossible.

In this work, we present a foothold selection approach for quadruped locomotion over perceived rough terrain. Since it is nearly impossible to hand-code a system complex and robust enough to function well on various types of terrains, using machine learning techniques seems unavoidable. So, here, reinforcement learning is applied to select suitable foot positions, which uses an approximation of the value function. This approximation is learned from data provided by expert demonstrations. Given the target foot position, a set of joint angles, which lead the swing leg to its desired place, is found using inverse kinematics.

Reinforcement learning has been applied to the problem of rough terrain quadruped locomotion previously [2]. Here, the same idea has been used to select optimal foothold choices. However, in addition to some of the previously used features, the idea of terrain templates [3] has been applied to learn an approximation to the value function. The rest of this paper is laid as follows. In section II, a brief overview of the work on robotic legged locomotion is given. In section III, we discuss a high-level overview of our software architecture, followed by its details. Finally, in section IV we explain the simulated quadruped, the experimental setup and the obtained results from logs of runs executed by robot on various types of terrain, and finish in section V with conclusions and ideas for future work.

II. RELATED WORKS

One of the first works in the legged locomotion area was a human-operated robot developed by GE [4]. However, the first computer-controlled quadruped robot was built by McGhee [5], followed by his studies on the stability properties of quadruped creeping gaits [6].

Quadruped locomotion was pushed forward by the seminal work of Raibert on balancing robots [1], [7]. This line of research resulted in the BigDog robot, which is highly robust to pushes and can traverse a wide variety of terrains [8].

Another branch of research in rough terrain locomotion has focused on biologically inspired legged robots with biomimetic hardware designs suitable for rough terrains, such as, Rhex [9], RiSE [10], Stickybot [11], LAURON [12], and HITCR-II [13, 14].

Also, some other researchers have focused on biologically inspired controllers, usually in the form of Central Pattern Generators (CPGs), such as the work by Ajallooeian et al. [15]. CPGs are neural circuits found in both invertebrate and vertebrate animals that can produce rhythmic patterns of neural activity without receiving rhythmic inputs [16].

Recently, a new line of research was spawned by DARPA in the context of its learning locomotion project with the LittleDog robot [17, 18]. This project aimed at producing algorithms for fast and robust legged locomotion over rough terrain facing perceived environments and led into several papers with remarkable results [3], [19-21], which have used various techniques such as machine learning.

Machine learning has a wide range of usage in robotics, including mobile robots [22, 23]. This technique provides robots with the ability to cope with new situations successfully.

Here we present an approach based on different learning and optimization techniques for the problem of quadruped foothold selection over rough terrains.

III. LEARNING FOOTHOLDS

Our goal is to provide a sequence of suitable footholds for fast, robust quadruped locomotion over rough terrain. The overall architecture of the proposed approach is presented in Figure 1. Here, a high level description of this architecture is given, followed by a complete description of its components in the following sections.

First, a set of expert demonstrations are collected for optimal foot placement choices. In the next step, a linear binary classifier is run to learn a small set of the templates extracted from these demonstrations. The learned templates along with other linear features form the feature vector used for value function approximation. Using this approximated value function, a suitable foothold position is selected in each state. Finally, inverse kinematics is used to place the swing leg in its chosen foot location.

A. Problem Formulation

The basic model of reinforcement learning consists of:

- S : a set of states
- A : a set of actions
- $R : S \times A \rightarrow \mathbb{R}$ is the reward function
- $F : S \times A \rightarrow S$ is the state transition function
- $\gamma \in [0,1)$ is called the discount factor

A policy is a function mapping from perceived states to the actions. The value function for a policy π is defined as:

$$V^\pi(s_0) = \sum_{i=0}^{\infty} \gamma^i R(s_i, a_i) \quad (1)$$

where $a_i = \pi(s_i)$, $s_{i+1} = F(s_i, a_i)$. The optimal value function is defined as:

$$V^*(s) = \max_{\pi} V^\pi(s) \quad (2)$$

Also, the Bellman optimality equation is defined as:

$$V^*(s) = \max_a R(s, a) + \gamma V^*(s') \quad (3)$$

where s' is the successor state for taking action a in state s . Hence, the optimal policy π^* can be computed as follows:

$$\pi^*(s) = \arg \max_a R(s, a) + \gamma V^*(s') \quad (4)$$

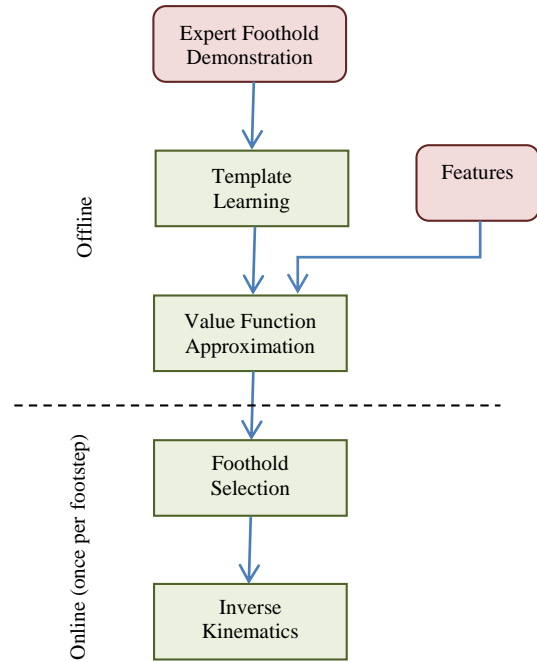


Figure 1. The software architecture of the proposed approach for foothold selection in quadruped rough terrain locomotion

In problems with small, finite state and action spaces, it is possible to learn the optimal value function exactly. However, for problems with large, continuous state or action spaces, this would be impossible, so, an approximation of $V^*(s)$ can be used. Here, an approximation of value function is learned using the algorithm proposed in [2]. This approximated value function is described as a linear combination of feature vector $\phi(s)$ of the state s :

$$\hat{V}(s) = \theta^T \phi(s) \quad (5)$$

where θ is the weight vector needed to be learned. The optimal foothold in each state can be then selected among all possible footholds based on the received immediate reward and this approximated value function.

The state of the robot is determined by the position of all feet, body position and orientation, and the swing leg index.

The reward function we used here, gave positive rewards for:

- 1- Decrease of the average distance from feet to the goal
- And negative rewards for:

- 2- Collisions, fall overs, and failures in executing the actions

The set of n possible footholds from each state can be represented by $F = \{f_1 \dots f_n\}$. Each of these footholds (f_i) is also described by a feature vector $X_i \in \mathbb{R}^d$. Also, a reward function R has been defined over footholds as:

$$R(f_i) = W^T X_i \quad (6)$$

where $W \in \mathbb{R}^d$ is the weight vector which should be learned. We learn this reward function to select a small set of templates from the template library for value function approximation. However, this reward function can also be used to rank all of the footholds in F , and the one with the highest reward selected as the target foothold [3]. We have also used this approach in our experiments to compare with our own proposed approach. Note that the reward function here is completely different from the one, used in the reinforcement learning algorithm.

B. Collecting Expert Demonstrations

An expert chooses the optimal foothold f_c from the set of n possible footholds in each state. All other footholds from that state would be considered as a suboptimal action from that state.

C. Template Learning

Using terrain templates in rough terrain quadruped locomotion was first suggested by Kalakrishnan et al. [3]. They believed the previously used terrain features [24, 25], for the foothold selection problem are insufficient for making complex decisions. One possible and also inefficient solution to this problem may be adding some non-linear features to the feature set manually. However, a much better solution has been proposed in [3]. This solution is based on the concept of the *terrain templates*. A terrain template is a discretized height map of the terrain in a small area around the foothold. Using terrain templates improves performance in terms of success rate and slip [3].

We have used the same idea, however, because in our simulations the surface friction properties and slippage probabilities have been ignored, the information encoded by the smallest scale was almost unnecessary. Thus, the templates were extracted only on the medium and large scale from all the reachable footholds in the set F , for every expert demonstration. All these extracted templates along with some other linear features described in section D, form the feature vector of the reward function (Equation (6)).

Hence, the extracted templates from expert demonstrations provide us with a very huge library of templates. This large number of templates can cause problems both in terms of overfitting and time consumption. So, to prevent these problems, it is necessary to reduce the size of the library. Following [3], it is possible to convert the problem of learning the weights in Equation (6) to a binary classification problem and discard all the templates in the library with zero weights. This way, the size of the library will reduce substantially. In our simulations, we have applied l_1 -regularized logistic regression implemented in LIBLINEAR [26].

Of course, it should be mentioned that, one of the main distinctive points of our work from [3] is in the foothold selection process. According to the approach proposed in [3], using the learned foothold ranking function (Equation (6)), the foothold with the highest reward is chosen as the target foothold. However, here, the template learning process is just used to provide the reinforcement learning algorithm with a new set of features, which are the learned templates, to approximate the value function better. Indeed, the footholds are selected based on the received immediate reward and this approximated function.

D. Features

In addition to the templates remaining in the library, some other linear features have been used to approximate the value function. These features included measures relating to robot's progress towards the goal, stability margins, body orientation and difference between maximum and minimum heights of the feet.

E. Reinforcement Learning Algorithm

To learn an approximation to the value function, the algorithm proposed in [2] has been applied. According to this approach, the task of learning an approximation to the value function has been reduced to solving an optimization problem as follows:

$$\begin{aligned} \max_{\theta, \delta} & \delta - \beta \sum | \hat{V}(s_i) - R(s_i, a_i^*) - \\ & - \gamma \hat{V}(F(s_i, a_i^*)) |^2 - \alpha \| \theta \|_2^2 \\ \text{s.t.} & R(s_i, a_i^*) + \gamma \hat{V}(F(s_i, a_i^*)) - \\ & - R(s_i, a_i) - \gamma \hat{V}(F(s_i, a_i)) \geq \delta \\ & i = 1, \dots, m \end{aligned} \quad (7)$$

where $s_i \in S$, $a_i^* = \pi^*(s_i)$ and $a_i \neq \pi^*(s_i)$ belong to the training set. This training set was obtained previously during collecting expert demonstrations. Thus, we put no much effort to create a new data set. Here, α and β are constants that determine the relative importance between the two terms.

This optimization problem is a quadratic program, which we used CVX, a package for specifying and solving convex programs [27, 28] to solve it. Having learned the parameters in Equation (5), a suitable position for the next foothold can now be easily obtained using Equation (4) replacing $V^*(s)$, with $\hat{V}(s)$.

IV. EXPERIMENTS

This section describes the simulated quadruped robot, experiment scenarios, and the results obtained.

A. The Simulated Quadruped

We have done all the experiments with a simulated quadruped robot modeled in MATLAB (Figure 2). Each leg of the robot has three degrees of freedom, two hip joints and a knee joint. We assume that the origin of robot's body is also its center of mass.

All the body properties of the simulated robot, is given in Table 1. For simplicity, the surface friction properties and slippage probabilities have been ignored and to alleviate the side effects of these assumptions, only step like obstacles have been considered. However, the collisions and fall overs are detectable.

B. Experimental Setup

In the experiments, 29 expert demonstrations were collected. Using these expert demonstrations, the terrain templates were extracted on two scales from all possible footholds from each state, including the foothold selected by the expert.

Each expert demonstration provided us with $|F|-1$ training instances as an input to the algorithm for learning an approximation to the value function. However, in order to prevent overfitting, we just used three training instances from each expert demonstration.

C. Experimental Results

We used four previously unseen terrains with varying difficulties to evaluate the performance of our proposed approach. These terrains are shown in Figure 3.

Table 1. Body properties of the simulated quadruped

Property	Value
Front legs 1st hip minimum joint angle	-0.6 (rad)
Front legs 1st hip maximum joint angle	0.6 (rad)
Front legs 1st hip joint angle when the legs are retracted	0.0 (rad)
Front legs 2nd hip minimum joint angle	-3.5 (rad)
Front legs 2nd hip maximum joint angle	2.4 (rad)
Front legs 2nd hip joint angle when the legs are retracted	1.22 (rad)
Front legs knee minimum joint angle	-3.1 (rad)
Front legs knee maximum joint angle	1.0 (rad)
Front legs knee joint angle when the legs are retracted	-3.1 (rad)
Hind legs 1st hip minimum joint angle	-0.6 (rad)
Hind legs 1st hip maximum joint angle	0.6 (rad)
Hind legs 1st hip joint angle when the legs are retracted	0.0 (rad)
Hind legs 2nd hip minimum joint angle	-2.4 (rad)
Hind legs 2nd hip maximum joint angle	3.5 (rad)
Hind legs 2nd hip joint angle when the legs are retracted	-1.22 (rad)
Hind legs knee minimum joint angle	-1.0 (rad)
Hind legs knee maximum joint angle	3.1 (rad)
Hind legs knee joint angle when the legs are retracted	3.1 (rad)
Robot body length	30 (cm)
Robot body width	18 (cm)
Robot body height	26 (cm)
Robot usable leg length	13 (cm)

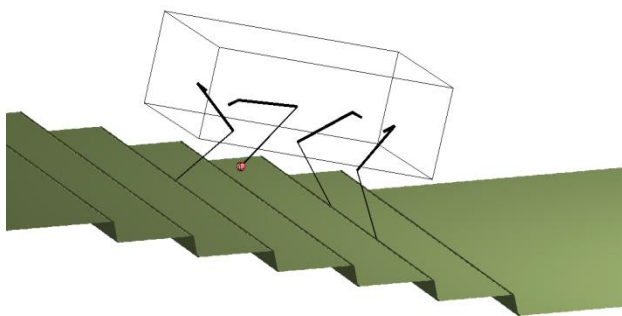


Figure 2. The simulated quadruped, crossing one of the testing terrains

We measured the locomotion performance using various metrics to demonstrate the efficiency of the proposed algorithm, the simulation results of this

algorithm were compared with the simulation results of the methods proposed in [2] and [3]. Of course, according to our observations, using the approach proposed in [3], the robot fails to show an acceptable performance even on flat terrains. In fact, the robot is not even able to walk for a long time on flat terrains, and falls over because of the unsuitable footholds. Thus, the results from this approach were not included in the paper. This poor performance of the robot is due to inadequate number of expert demonstrations to learn a good foothold ranking function.

However, our approach performs much better using the same number of expert demonstrations. Figures 4 and 5 show the obtained results from 10 runs using our proposed approach and the approach proposed in [2]. Note that whenever the robot fell over, it was placed again to traverse the rest of the terrain from that point.

The average number of collisions and fall overs are also included in Tables 2 and 3 respectively, to show the overall performance of these two methods.

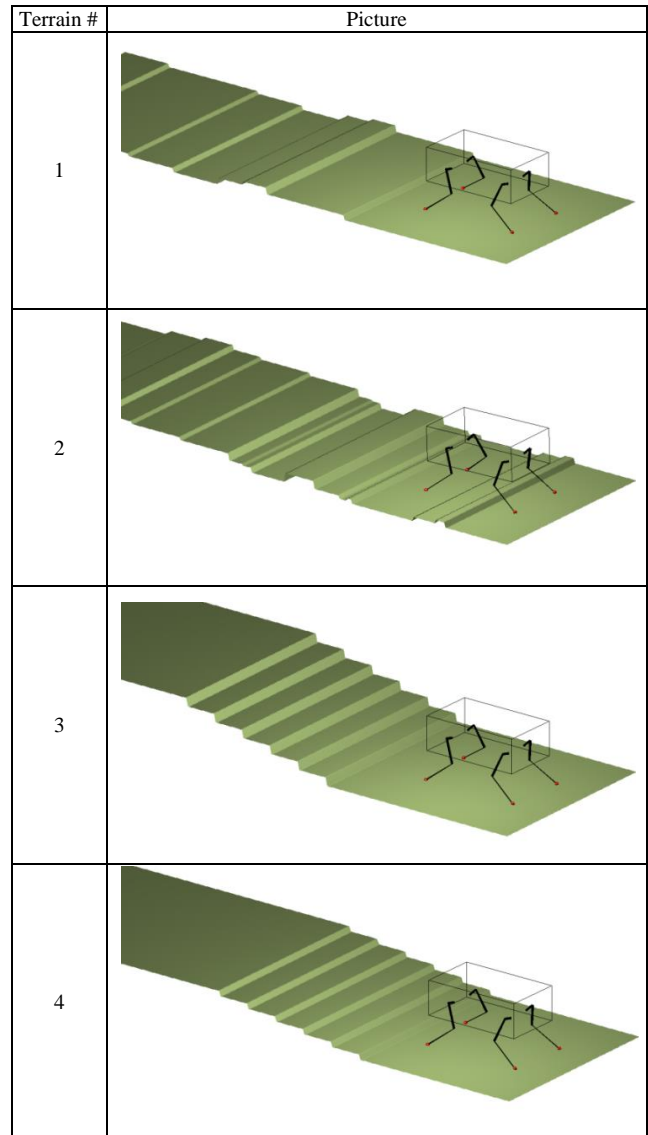


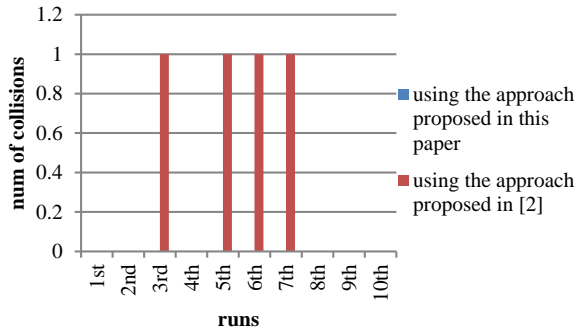
Figure 3. The terrains on which the simulated quadruped was tested

Table 2. Average number of collisions over four different types of terrain

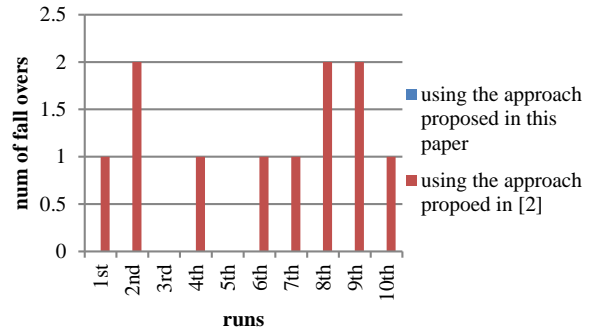
Terrain	Avg. number of collisions using the proposed approach	Avg. number of collisions using the approach in [2]	Decrement
1	0	0.4	100%
2	0.7	0.9	22.22%
3	0.8	1.3	38.46%
4	0.7	0.9	22.22%

Table 3. Average number of fall overs over four different types of terrain

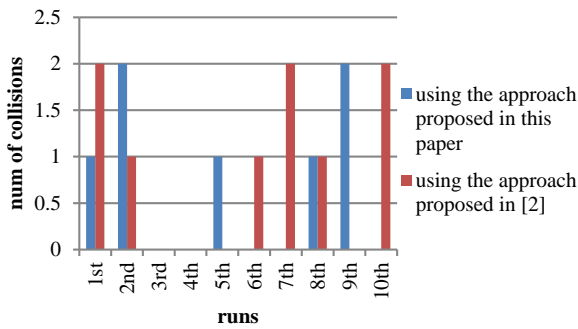
Terrain	Avg. number of fall overs using the proposed approach	Avg. number of fall overs using the approach in [2]	Decrement
1	0	1.1	100%
2	0.1	0.6	83.33%
3	0.7	1.5	53.33%
4	0.3	0.7	57.14%



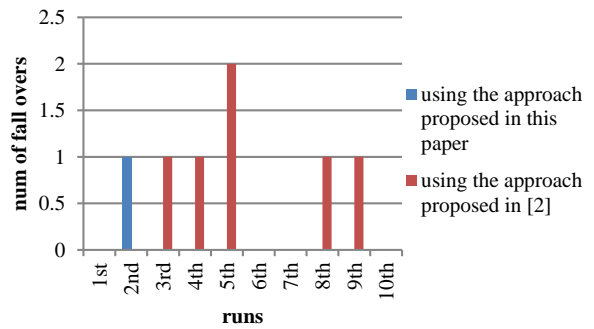
(a)



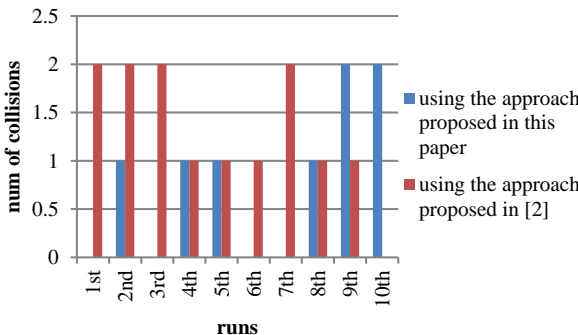
(a)



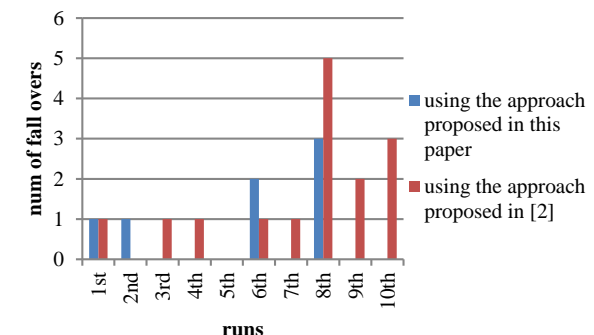
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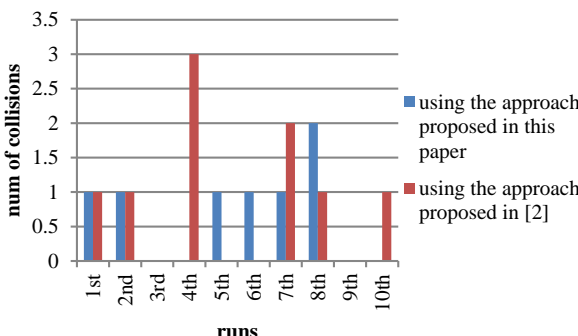
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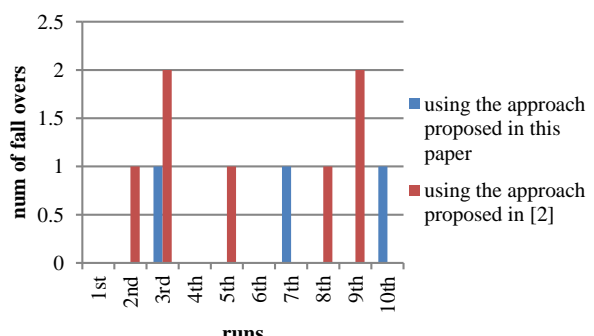
(c)



(c)



(d)



(d)

Figure 4. Number of occurred collisions traversing (a) terrain 1, (b) terrain 2, (c) terrain 3, (d) terrain 4

Figure 5. Number of occurred fall overs traversing (a) terrain 1, (b) terrain 2, (c) terrain 3, (d) terrain 4

Table 4. Average number of collisions per foothold on successful runs over four different types of terrain

Terrain	Avg. collision per foothold on successful runs using the proposed approach	Avg. collision per foothold on successful runs using the approach proposed in [2]
1	0	3.33×10^{-2}
2	1.85×10^{-2}	5.33×10^{-2}
3	3.33×10^{-2}	1.11×10^{-2}
4	2.85×10^{-2}	4.66×10^{-2}

Table 5. Success rate over four different types of terrain

Terrain	Success rate using the proposed approach (out of 10 runs)	Success rate using the approach proposed in [2] (out of 10 runs)
1	100%	20%
2	90%	50%
3	60%	20%
4	70%	50%

We also measured the locomotion performance using two other metrics, which are success rate and average collisions experienced by the robot at each foothold. A run was considered as a successful run if the robot crossed the terrain and reached the goal without falling over. We also averaged the number of collisions per foothold. However, we averaged this number only over successful runs, in order to render the statistic meaningful. The obtained results in terms of these two metrics are shown in Tables 4 and 5.

According to the obtained results and as expected, no matter which method is used, the robot shows a poorer performance on terrains with higher obstacles and more irregularities. However, the overall performance of our approach is much better in terms of all the metrics.

The main advantage of the proposed method in this paper to the method proposed in [3] is the low number of expert demonstration it needs. Also using terrain templates in the learning process, has led into a more accurate approximation of the value function, as it results in less fall overs and collisions compared to the method used in [2].

As mentioned before, the proposed approach in this paper uses simple inverse kinematics to place the swing leg in its desired foot location and performs one step look ahead rather than multiple steps. However, it can be seen that despite these simple adopted methods, the obtained results are quiet fair.

V. CONCLUSIONS AND FUTURE WORKS

In this paper we have presented a software system for selecting suitable footholds in quadruped rough terrain locomotion. Novel features of our approach include using learned terrain templates as a new set of features for learning an approximation to the value function, which is further used to select a suitable foothold in each state. Evaluations have shown that, our proposed approach performs much better than using the foothold ranking function learned in the template learning phase in selecting footholds, in terms of both collisions and fall overs. Also, because we have made no assumptions about the structure of the robot in our proposed approach, it is applicable to any other legged robots for rough terrain locomotion.

In future work we intend to apply the proposed approach on a real quadruped robot to evaluate its performance in real world and also complete our controller using new methods to learn robot gait patterns.

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BIOGRAPHIES



Nafiseh Sharghivand received her B.Sc. degree in Information Technology Engineering from University of Tabriz, Tabriz, Iran in 2011. Currently, she is pursuing her M.Sc. degree in Computer Engineering while working as a researcher at Multi-Agent Learning,

Modeling, and Simulation, Research Laboratory of University of Tabriz. Her current interests include machine learning and its applications in legged locomotion.



Seyed Naser Razavi received his B.Sc. degree in Computer Engineering in 2001 from Petroleum University of Technology, Tehran, Iran, and his M.Sc. and Ph.D. degrees in Artificial Intelligence from Iran University of Science and Technology, Tehran, Iran, in 2004

and 2011, respectively. Currently he is working as an Assistant Professor of Electrical and Computer Engineering Department in University of Tabriz, Tabriz, Iran. He is the Director of the Multi-Agent Learning, Modeling, and Simulation, Research Laboratory of University of Tabriz. His current researches focus on machine learning and swarm intelligence and specifically multi-agent learning and its applications. In addition, he has cooperated with Systems and Transportation Laboratory in University of Technology of Belfort, Belfort, France, in several research projects since 2008.



Mohammad Reza Feizi Derakhshi received his B.Sc. degree in Computer Engineering in 1997 from Isfahan University, Isfahan, Iran, and his M.Sc. and Ph.D. degrees in Artificial Intelligence from Iran University of Science and Technology, Tehran, Iran, in 2000

and 2007, respectively. Currently he is working as an Assistant Professor of Electrical and Computer Engineering Department in University of Tabriz, Tabriz, Iran. His current researches focus on natural language processing, semantic web and document processing, optimization algorithms and data bases.