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ASYMPTOTIC ANALYSIS OF FREE OSCILLATIONS FREQUENCY OF A MEDIUM CONTACTING LONGITUDINALLY REINFORCED ORTHOTROPIC CYLINDRICAL SHELL

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Abstract- In this paper, by means of the variational principle we solve a problem of free oscillation of a medium-contacting longitudinally reinforced orthotropic cylindrical shell. Based on the Ostrogradsky-Hamilton variational principle, the frequency equation of oscillations of a medium-contacting, reinforced orthotropic cylindrical shell is constructed and numerically realized. The surface loads acting by the medium on a longitudinally reinforced cylindrical shell are determined from the solutions of the system of Lame equations in displacements.

Keywords: Ribbed Shell, Variational Principle, Oscillations, Determinant, Boundary Conditions.

I. INTRODUCTION

In recent years the issues concerning the investigations of stress-strain state of medium-contacting ribbed anisotropic shells draw great attention of researchers. The papers [1, 2] were devoted to investigation of free oscillations of medium-filled isotropic cylindrical shells longitudinally reinforced and reinforced with cross system of ribs and loaded by axial compressive forces. By using the variational principle, the frequency equation of oscillations of a medium-contacting reinforced isotropic cylindrical shell is constructed and numerically realized. The free oscillations of liquid-filled ribbed isotropic cylindrical shells under axial compression was considered in the paper [3].

The shells were reinforced longitidinaly, laterally and by the cross system of ribs. The similar problem considered in [3], with regard to friction between the contract surface of the shell and medium was studied in the paper [4]. In [5] a problem of forced axially symmetric oscillations of a liquid-filled isotropic cylindrical shell reinforced and loaded by axial compressive forces was studied.

II. PROBLEM STATEMENT

It is known that among the different configuration shells the ribbed cylindrical shells are the most extended ones widely used as carrying structural elements. A ribbed shell is considered as a system consisting of the actual shell and longitudinal ribs rigidly connected with it along the contact lines. It is accepted that the stress-strain state of the sheathing may be completely determined within the linear theory of elastic thin shells based on Kirchhoff-Liave hypotheses, and for calculation of ribs the theory of curvilinear Kirchhoff-Klebsch bars are applicable.

The system of coordinates were chosen so that the coordinate lines coincide with the lines of principal curvatures of the medium surface of the sheathing. It is assumed that the ribs are located along the coordinate lines, and their edges and the edges of the sheathing lie in the same coordinate plane.

The strain state of the sheathing may be determined by three components of displacements of its median surface u, ϑ and w. The turning angle of normal elements φ_1, φ_2 with respect to coordinate lines y and x are expressed by w and ϑ with the help of dependences as:

$$\varphi_1 = -\frac{\partial w}{\partial x},\ \varphi_2 = -\left(\frac{\partial w}{\partial y} + \frac{\vartheta}{R}\right)$$

where R is the radius of median surface of the shell.

For describing the strain state of the ribs, in addition to three components of displacements of gravity centers of their cross sections (u_i, ϑ_i, w_i, respectively for the ith longitudinal bar) it is necessary to determine the torsion angle φ_{kpi}. Taking into account that according to the accepted hypotenes it holds the constancy of radial flexures along the height of cross sections, and the equality of corresponding torsion angles following from the conditions of rigid junction of ribs with the shell, we write the following relations:

$$\begin{aligned} & u_i(x) = u(x, y_i) + h_i \varphi_1(x, y_i) \\ & \vartheta_i(x) = \vartheta(x, y_i) + h_i \varphi_2(x, y_i) \\ & w_i(x) = w(x, y_i) \\ & \varphi_i = \varphi_1(x, y_i) \\ & \varphi_{kpi}(x) = \varphi_2(x, y_i) \end{aligned} \quad (1)$$

where $h_i = 0.5h + H_i^1$, h is the shell thickness, H_i^1 is the distance from the axes of the *i*th longitudinal bar to the shell surface, x_i and y_i are the coordinates of the lines of conjunction of ribs and the shell, φ_i, φ_{kpi} are the turning and torsion angles of cross sections of longitudinal bars.

For external actions it is assumed that the surface loads acting on the ribbed shell may be reduced to the constituents q_x, q_y and q_z applied to the median surface of the shell, and the boundary loads to longitudinal, tangential, lateral forces and bending moments applied to the edges of the shell T_1, S_1, Q_1, M_1 and (T_2, S_2, Q_2, M_2 on curvilinear and rectilinear edges, respectively), and to appropriate forces, bending moments and torgues applied to end cross sections of the ribs ($T_i, S_i, Q_i, M_i, M_{1i}, M_{kpi}$ for longitudinal ribs).

We get differential equations of motion and natural boundary conditions for a longitudinally reinforced orthotropic cylindrical shell on the basis of the Ostrogradsky-Hamilton variational principle. For that we write beforehand the potential and kinetic energy of the system.

The potential energy of elastic deformation of a cylindrical shell has the form:

$$\begin{aligned} \Pi_0 = \frac{hR}{2} \int_{x_1}^{x_2} \int_{y_1}^{y_2} \Bigg\{ & B_{11} \left(\frac{\partial u}{\partial x} \right)^2 - 2(B_{11} + B_{12}) \frac{w}{R} \frac{\partial u}{\partial x} + \\ & + B_{11} \frac{\partial u}{\partial x} \left(\frac{\partial w}{\partial x} \right)^2 - (B_{11} - B_{12}) \frac{w}{R} \left(\frac{\partial w}{\partial x} \right)^2 + \\ & + \frac{B_{11}}{4} \left(\frac{\partial w}{\partial x} \right)^4 + \frac{w^2}{R^2} (B_{11} + 2B_{12} + B_{22}) + \\ & + B_{22} \left(\frac{\partial \vartheta}{\partial y} \right)^2 - (B_{12} + B_{22}) \frac{w}{R} \left(\frac{\partial w}{\partial y} \right)^2 - \\ & - 2(B_{12} + B_{22}) \frac{w}{R} \frac{\partial \vartheta}{\partial y} + B_{22} \frac{\partial \vartheta}{\partial y} \left(\frac{\partial w}{\partial y} \right)^2 + \\ & + B_{12} \frac{1}{4} \left(\frac{\partial w}{\partial y} \right)^4 + 2B_{12} \frac{1}{R} \frac{\partial u}{\partial x} \frac{\partial \vartheta}{\partial y} + \\ & + B_{12} \frac{\partial u}{\partial x} \left(\frac{\partial w}{\partial y} \right)^2 + B_{12} \frac{\partial \vartheta}{\partial y} \left(\frac{\partial w}{\partial x} \right)^2 + \\ & + B_{12} \frac{1}{2R} \left(\frac{\partial w}{\partial x} \right) \left(\frac{\partial w}{\partial y} \right)^2 \Bigg\} dxdy \end{aligned} \quad (2)$$

where $B_{11} = \dfrac{E_1}{1 - \nu_1 \nu_2}$; $B_{22} = \dfrac{E_2}{1 - \nu_1 \nu_2}$; $B_{12} = \dfrac{\nu_2 E_1}{1 - \nu_1 \nu_2} = \dfrac{\nu_1 E_2}{1 - \nu_1 \nu_2}$, R is the radius of the median surface of the shell, h is the shell thickness, u, υ, w are the components of displacements of the points of the medium surface of the shell.

The expressions for the potential energy of elastic deformations of the *i*th longitudinal rib are as follows [7]:

$$\begin{aligned} \Pi_i = \frac{1}{2} \int_{x_1}^{x_2} \Bigg[& E_i F_i \left(\frac{\partial u_i}{\partial x} \right)^2 + E_i J_{yi} \left(\frac{\partial^2 w_i}{\partial x^2} \right)^2 + \\ & + E_i J_{zi} \left(\frac{\partial^2 \vartheta_i}{\partial x^2} \right)^2 + G_i J_{kpi} \left(\frac{\partial \phi_{kpi}}{\partial x} \right)^2 \Bigg] dx \end{aligned} \quad (3)$$

In Equations (2) and (3) x_1, x_2, y_1, y_2 are the coordinates of curvilinear and rectilinear edges of the shell: $F_i, J_{zi}, J_{yi}, J_{kpi}$ are area and inertia moments of the cross section of the *i*th longitudinal bar, respectively, with respect to the axis O_z and the axis parallel to the axis O_y and passing through the gravity center of the cross section, and also its inertia moment at torsion; E_i, G_i are elasticity and shear module of the material of the *i*th longitudinal bar, respectively.

The potential energy of external surface and edge loads applied to the sheathing is determined as a work performed by this loads when taking the system from one deformed state to initial underformed state and is represented in the form:

$$\begin{aligned} A_0 = & -\int_{x_1}^{x_2} \int_{y_1}^{y_2} \left(q_x u + q_y \vartheta + q_z w \right) dxdy - \\ & - \int_{y_1}^{y_2} \left(T_1 u + S_1 \vartheta + Q_1 w + M_1 \varphi_1 \right) \Big|_{x=x_1}^{x=x_2} dy - \\ & - \int_{x_1}^{x_2} \left(S_2 u + T_2 \vartheta + Q_2 w + M_2 \varphi_2 \right) \Big|_{y=y_1}^{y=y_2} dx \end{aligned} \quad (4)$$

In the similar way, the potential energies of external edge loads applied to the and faces respectively of the *i*th longitudinal bar, are determined by the following expressions (it accepted that only edge loads are applied to the ribs):

$$A_i = -\left(T_i u_i + S_i \vartheta_i + Q_i w_i + M_i \varphi_i + M_{1i} \varphi_{zi} + M_{kpi} \varphi_{kpi} \right) \Big|_{x=x_1}^{x=x_2} \quad (5)$$

The total potential energy of the system equals the sum of potential energies of elastic deformations of the shell and longitudinal ribs, and also potential energies of all external loads:

$$\Pi = \Pi_0 + \sum_{i=1}^{k_1} \Pi_i + A_0 + \sum_{i=1}^{k_1} A_i \quad (6)$$

The kinetic energy of the shell and longitudinal ribs are written in the form of:

$$K_0=\rho_0 h\int_{x_1}^{x_2}\int_{y_1}^{y_2}\left[\left(\frac{\partial u}{\partial t}\right)^2+\left(\frac{\partial \vartheta}{\partial t}\right)^2+\left(\frac{\partial w}{\partial t}\right)^2\right]dxdy$$

$$K_i=\rho_i F_i\int_{x_1}^{x_2}\left[\left(\frac{\partial u_i}{\partial t}\right)^2+\left(\frac{\partial \vartheta_i}{\partial t}\right)^2+\left(\frac{\partial w_i}{\partial t}\right)^2+\frac{J_{kpi}}{F_i}\left(\frac{\partial \varphi_{kpi}}{\partial t}\right)^2\right]dx \quad (7)$$

where t is time coordinate, ρ_0,ρ_i is the density of the material from which the shell is made, the *i*th longitudinal bar. The kinetic energy of the longitudinally reinforced shell is:

$$K=K_0+\sum_{i=1}^{k_1}K_i \quad (8)$$

The equations of the motion of the ribbed shell are obtained on the basis of Ostrogradsky-Hamilton's principle of stationary action:

$$\delta W=0 \quad (9)$$

where $W=\int_{t'}^{t''}\tilde{L}dt$ is the Hamilton action, $\tilde{L}=K-\Pi$ is the Lagrange function, t' and t'' are the given arbitrary times.

Assuming that the shell is reinforced by an infinitely great number of ribs, by limit passage $k_1\to\infty$ and with respect to that variation and differentiation operations are permutation, Equation (9) may be reduced to the form:

$$\left[\left(a_1+\gamma_c^{(1)}\right)\frac{\partial^2}{\partial\xi^2}+\frac{\partial^2}{\partial\theta^2}\right]u+\left(1+a_{12}\right)\frac{\partial^2\vartheta}{\partial\xi\partial\theta}-\left(a_{12}\frac{\partial}{\partial\xi}+\delta_c^{(1)}\frac{\partial^3}{\partial\xi^3}\right)w-\rho_1\frac{\partial^2 u}{\partial t_1^2}=\frac{R^2q_x}{G_{12}h}$$

$$\left(1+a_{12}\right)\frac{\partial^2 u}{\partial\xi\partial\theta}+a_2\frac{\partial^2\vartheta}{\partial\theta^2}-a_2\frac{\partial w}{\partial\theta}-\frac{\partial^2\vartheta}{\partial t_1^2}=\frac{R^2q_y}{G_{12}h} \quad (10)$$

$$-\left(a_{12}\frac{\partial}{\partial\xi}+\delta_c^{(1)}\frac{\partial^3}{\partial\xi^3}\right)u-a_2\frac{\partial\vartheta}{\partial\theta}+a_2w+a^2\left[a_1\frac{\partial^4}{\partial\xi^4}+2\left(a_{12}+2\right)\frac{\partial^4}{\partial\xi^2\partial\theta^2}+a_{12}\frac{\partial^4 w}{\partial\theta^4}+\eta_c^{(1)}\frac{\partial^4}{\partial\xi^4}\right]w+\rho_1\frac{\partial^2 w}{\partial t_1^2}=\frac{R^2}{G_{12}h}q_z$$

where,

$$a^2=\frac{h^2}{12R^2},\quad \bar{\rho}_c=\frac{\rho_c}{\rho_0},\quad \Delta=\frac{\partial^2}{\partial\xi^2}+\frac{\partial^2}{\partial\theta^2},$$

$$L_1=x_2-x_1,\quad \xi=\frac{x}{R},\quad \gamma_c^{(1)}=\frac{E_c}{E}\left(1-\nu^2\right)\bar{\gamma}_c^{(1)},$$

$$\eta_c^{(1)}=\frac{E_c\left(J_{yc}+h^2F_c\right)k_1}{2\pi R^3hE}\left(1-\nu^2\right),\quad \rho_1=1+\bar{\rho}_c\bar{\gamma}_c^{(1)},$$

$$a_i=\frac{E_i}{G_{12}\left(1-\nu_{12}\nu_{21}\right)},\quad a_{12}=a_1\nu_{21}=a_2\nu_{12},\quad \bar{\gamma}_c^{(1)}=\frac{E_ck}{2\pi Rh},$$

$$\delta_c^{(1)}=\frac{h_c}{R}\bar{\gamma}_c^{(1)},\quad \theta=\frac{y}{R},\quad t_1=\omega_0 t,\ \omega_0=\sqrt{\frac{G_{12}}{\left(1-\nu^2\right)\rho_0R^2}}.$$

The surface loads q_x,q_y and q_z, acting by the medium on the longitudinally reinforced shell are determined from the solutions of the system:

$$\left(\lambda_s+2\mu_s\right)\frac{\partial\theta}{\partial r}-\frac{2\mu_s}{r}\frac{\partial\omega_x}{\partial\theta}+2\mu_s\frac{\partial\omega_\theta}{\partial x}=0$$

$$\left(\lambda_s+2\mu_s\right)\frac{1}{r}\frac{\partial\theta}{\partial\theta}-2\mu_s\frac{\partial\omega_r}{\partial x}+2\mu_s\frac{\partial\omega_x}{\partial x}=0 \quad (11)$$

$$\left(\lambda_s+2\mu_s\right)\frac{\partial\tilde{\theta}}{\partial x}-\frac{2\mu_s}{r}\frac{\partial}{\partial r}\left(r\omega_\theta\right)+\frac{2\mu_s}{r}\frac{\partial\omega_r}{\partial\theta}=0$$

where $s_x,\ s_\varphi,\ s_r$ are longitudinal, torsional and radial components of the displacement vector of the medium, respectively; $\lambda_s,\ \mu_s$ are the Lame coefficients for the medium; x,r,θ are longitudinal, normal and circumferential coordinates.

The volumetric expansion $\tilde{\theta}$ and rotation components $\omega_x,\omega_\theta,\omega_r$ are determined from the expressions:

$$\theta=\frac{\partial s_r}{\partial r}+\frac{s_r}{r}+\frac{1}{r}\frac{\partial s_\varphi}{\partial\varphi}+\frac{\partial s_x}{\partial x},\quad 2\omega_x=\frac{1}{r}\left[\frac{\partial\left(rS_\varphi\right)}{\partial r}-\frac{\partial s_r}{\partial\varphi}\right]$$

$$2\omega_\varphi=\frac{\partial s_r}{\partial x}-\frac{\partial s_x}{\partial r},\quad 2\omega_r=\frac{1}{r}\frac{\partial s_x}{\partial\varphi}-\frac{\partial s_\varphi}{\partial x}$$

In its turn, by means of the functions $s_x,\ s_\varphi,\ s_r$ the stresses are expressed as

$$\sigma_{rx}=\mu_s\left(\frac{\partial s_x}{\partial r}+\frac{\partial s_r}{\partial x}\right)$$

$$\sigma_{r\theta}=\mu_s\left[r\frac{\partial}{\partial r}\left(\frac{s_\theta}{r}\right)+\frac{1}{r}\frac{\partial s_r}{\partial\theta}\right] \quad (12)$$

$$\sigma_{rr}=\lambda_s\left(\frac{\partial s_x}{\partial x}+\frac{1}{r}\frac{\partial\left(rs_r\right)}{\partial r}+\frac{1}{r}\frac{\partial s_\theta}{\partial\theta}\right)+2\mu_s\frac{\partial s_r}{\partial r}$$

III. METHOD OF SOLUTION

Assume that the contact between the shell and medium is sliding, i.e. for $r=R$

$$w=s_z \quad (13)$$

$$q_x=-\sigma_{rx}=0,\ q_\theta=-\sigma_{r\theta}=0,\ q_z=-\sigma_{rr} \quad (14)$$

In what follows, the hingely supported shells are considered, i.e. for $\xi=0$ and $\xi=\xi_1\left(\xi_1=L_1/R\right)$ the following boundary conditions are fulfilled:

$$\vartheta=w=0$$

$$T_1=M_1=0$$

We look for the components of the displacement vector of the points of median surface of the shell in the form of:

$$
\begin{aligned}
u &= A\cos n\theta \cos\frac{m\pi}{\xi_1}\xi\sin\omega t\\
\vartheta &= B\sin n\theta \sin\frac{m\pi}{\xi_1}\xi\sin\omega t\\
w &= C\cos n\theta \sin\frac{m\pi}{\xi_1}\sin\omega t
\end{aligned}
\tag{15}
$$

where A, B, C are unknown constants, ω is a sought-for frequency. Using the solution of system (4), by displacing the points of the median surface of the shell (15) and contact conditions (13), (14), we can determine the contact pressure q_z. We represent this expression in the form of:

$$q_z = q_z^{(0)} C\cos n\varphi \sin kx \sin\omega t$$

where, in the case of small inertial actions of the medium on the oscillation process of the system, $q_z^{(0)}$ has the form of:

$$
\begin{aligned}
q_z^{(0)} &= -\mu_s \Delta^{-1}\Big\{\Big(2(1-2\nu_s)I_n(k^*) + 2k^* I_n'(k^*)\Big)k^{*2}\times\\
&\times\left[2k^{*2}\left(k^{*2}-n^2\right)\frac{I_n'(k^*)}{I_n(k^*)} + 2n^2k^*\right] -\\
&-2\Big(k^* I_n'(k^*) - \left(k^{*2}+n^2\right)I_n(k^*)\Big)k^{*3}\times\\
&*\left[2(3-2\nu_s)k^*\frac{I_n'(k^*)}{I_n(k^*)} - 2n^2\right] +\\
&+2n\Big(I_n'(k^*) - k^* I_n'(k^*)\Big)k^{*3}\times\\
&\times\left[2(3-2\nu_s)k^*\frac{I_n'(k^*)}{I_n(k^*)} - 2n^2\right]\Big\}
\end{aligned}
\tag{16}
$$

When the inertial actions of the medium on the oscillation process of the system is significant, $q_z^{(0)}$ has the form of:

$$
\begin{aligned}
q_z^{(0)} &= \frac{E_s}{1+\nu_s}I_n(\gamma_l^*)\left[\frac{I_n(\gamma_t^*)}{I_n(\gamma_l^*)}\left(-\gamma_l^*\frac{I_n'(\gamma_l^*)}{I_n(\gamma_l^*)} + \gamma_l^{*2} + n^2 - \frac{\nu}{1-2\nu_s}\mu_l^{*2}\right)\right.\cdot\\
&\cdot\frac{-n^2k^{*2}\mu_t^* + \dfrac{R^4k^{*3}\gamma_t^{*2}}{\mu_t^*}\dfrac{I_n'^2(\gamma_t^*)}{I_n^2(\gamma_t^*)}}{-\dfrac{k^{*3}\gamma_l^*\gamma_t^{*2}}{\mu_t^*}\dfrac{I_n'(\gamma_l^*)}{I_n(\gamma_l^*)}\dfrac{I_n'^2(\gamma_t^*)}{I_n^2(\gamma_t^*)}} +\\
&+\frac{2nk^*\gamma_l^*\mu_t^*\dfrac{I_n'(\gamma_l^*)}{I_n(\gamma_l^*)} + \dfrac{2nk^{*3}\gamma_t^*}{\mu_t^*}\dfrac{I_n'(\gamma_t^*)}{I_n(\gamma_t^*)}}{-\dfrac{k^{*3}\gamma_l^*\gamma_t^{*2}}{\mu_t^*}\dfrac{I_n'(\gamma_l^*)}{I_n(\gamma_l^*)}\dfrac{I_n'^2(\gamma_t^*)}{I_n^2(\gamma_t^*)}}\cdot\\
&\cdot\left(-n^2 + n\gamma_t^*\frac{I_n'(\gamma_t^*)}{I_n(\gamma_t^*)} + \frac{\nu_s}{1-2\nu_s}n\gamma_t^*\left(\gamma_t^* - \gamma_t^*\frac{I_n'(\gamma_t^*)}{I_n(\gamma_t^*)}\right)\right) +\\
&+\left(\frac{k^*\gamma_t^*}{\mu_t^*}\frac{I_n'(\gamma_t^*)}{I_n(\gamma_t^*)} + \gamma_t^{*2} + n^2 - \frac{\nu_s}{1-2\nu_s}\frac{2k^*\gamma_t^*}{\mu_t^*}\right)\cdot\\
&\left.\cdot\frac{\left[-\dfrac{k^{*3}\gamma_l^*\gamma_t^{*2}}{\mu_t^*}\dfrac{I_n'(\gamma_l^*)}{I_n(\gamma_l^*)}\dfrac{I_n'^2(\gamma_t^*)}{I_n^2(\gamma_t^*)}\right]}{2k^{*2}\gamma_l^*\gamma_t^*\dfrac{I_n'(\gamma_l^*)}{I_n(\gamma_l^*)}\dfrac{I_n'(\gamma_t^*)}{I_n(\gamma_t^*)} - 2n^2k^{*2}}\right].
\end{aligned}
\tag{17}
$$

where I_n is modified *n*th order Bessel function of first kind, k, n, γ_e, γ_t are wave numbers, $\gamma_e^2 = k^2 - \mu_e^2$, $k^* = kR$, $\gamma_t^2 = k^2 - \mu_t^2$, $\gamma_l^* = \gamma_l R$, $\gamma_t^* = \gamma_t R$, $\mu_t^* = \mu_t R$, $\mu_l^* = \mu_l R$.

Completing the equations of motion of the shell (10), medium (11) by contact conditions (13), (14) we arrive at a contact problem on oscillations of the medium-filled shell reinforced with longitudinal ribs. In other words, a problem of oscillations of an orthotropic shell with medium and reinforced with longitudinal ribs is reduced to joint integration of equations of theory of shells, medium subject to indicated conditions on their contact surface.

IV. RESULTS AND CONCLUSIONS

For finding the approximate expressions of $q_z^{(0)}$, we will use asymptotic formulas for logarithmic derivative of the Bessel function I_n $(x << n;\ n >> 1)$:

$$\frac{I_n'(x)}{I_n(x)} \approx -\frac{n}{x} + \frac{x}{2n} \tag{18}$$

Using formulas (16), (17) and (18) for $q_z^{(0)}$ we find: in the case of small inertial actions of the medium on oscillations process of the system.

$$q_z^{(0)} \approx \tilde{\chi} n E_s^* \tag{19}$$

and when the inertial actions of the medium on the oscillations process of the system are significant.

$$\tilde{q}_z^{(0)} \approx -\tilde{\chi} n E_s^* - 2\frac{\rho_s^*}{n}\lambda \tag{20}$$

where in (19) and (20):

$$\lambda = \frac{\omega^2}{\omega_0^2},\ \rho_s^* = \frac{\rho_s}{h_* \rho_0},\ \tilde{\chi} = \frac{1-\nu_{12}\nu_{21}}{2(1+\nu_s)},$$

$$E_s^* = \frac{E_s}{G_{12}h_*},\ h_* \frac{h}{R}\, E_s \big/ G_{12} << 1$$

After substitution of (15) and (19) in (10), we get a homogeneous system of linear algebraic equations that contains $A, B, C,$ as unknowns, and whose nontrivial solution is possible only in the case when its determinant equals zero:

$$\det\left\|b_{ij}\right\| = 0\ \ (i, j = 1, 2, 3) \tag{21}$$

where,

$$b_{11}=-\left(a_1+\gamma_c^{(1)}\right)k^{*2}-n^2+\rho_1\lambda;\ b_{12}=\left(1+a_{12}\right)k^*n;$$

$$b_{13}=-a_{12}k^*+\delta_c^{(1)}k^{*3};\ b_{21}=\left(1+a_{12}\right)k^*n;\ b_{22}=\lambda-a_2n^2;$$

$$b_{23}=na^2;\ b_{31}=a_{12}k^*+\delta_c^{(1)}k^{*3};\ b_{32}=-a_2n;$$

$$b_{33}=a_2+a^2\left(a_1k^{*4}+2\left(a_{12}+2\right)n^2k^{*2}+a_{12}n^4+\eta_c^{(1)}k^{*4}\right)-$$

$$-\rho_1\lambda+R^2\tilde{\chi}nE_s^*$$

In the open form, equation (21) has the form:

$$\lambda^3+\alpha_1\lambda^2+\alpha_2\lambda+\alpha_3=0 \quad (22)$$

where,

$$\alpha_1=\rho_1^{-1}\left(\tilde{b}_{11}-\tilde{b}_{33}-a_2n^2\right);$$

$$\alpha_2=\rho_1^{-2}\left[a_2n^2\tilde{b}_{33}+\left(\delta_c^{(1)2}k^{*4}-a_{12}^2\right)k^{*2}-\rho_1n^2a_2^2-\right.$$

$$\left.-\tilde{b}_{11}\tilde{b}_{33}-a_2n^2\tilde{b}_{11}\rho_1-\rho_1\left(1+a_{12}\right)^2n^2k^{*2}\right];$$

$$\tilde{b}_{11}=-\left(a_1+\gamma_c^{(1)}\right)k^{*2}-n^2;$$

$$\tilde{b}_{33}=a_2+a^2\left(a_1k^{*4}+2\left(a_{12}+2\right)n^2k^{*2}+a_{12}n^4+\eta_c^{(1)}k^{*4}\right)+$$

$$+R^2\tilde{\chi}nE_s^*;$$

$$\alpha_3=\rho_1^{-2}\left[a_2n^2\tilde{b}_{11}\tilde{b}_{33}-b_{21}b_{32}b_{13}-b_{12}b_{23}b_{31}-\right.$$

$$\left.-a_2n^2k^{*2}\left(\delta_c^{(1)2}k^{*4}-a_{12}^2\right)-n^2a_2^2\tilde{b}_{11}+\tilde{b}_{33}\left(1+a_{12}\right)^2n^2k^{*2}\right]$$

When the inertial actions of the medium on the oscillations process of the system are significant, the equation with respect to λ takes the form:

$$\lambda^3+\beta_1\lambda^2+\beta_2\lambda+\beta_3=0 \quad (23)$$

where ,

$$\beta_1=\left[\rho_1\left(\rho_1+2\rho_s^*/n\right)\right]^{-1}\cdot$$

$$\cdot\left[\tilde{b}_{11}\rho_1\left(\rho_1+2\rho_s^*/n\right)-\rho_1\tilde{b}_{33}-a_2n^2\rho_1\left(\rho_1+2\rho_s^*/n\right)\right];$$

$$\beta_2=\left[\rho_1\left(\rho_1+2\rho_s^*/n\right)\right]^{-1}\cdot$$

$$\cdot\left[a_2n^2\tilde{b}_{33}+\left(\delta_c^{(1)2}k^{*4}-a_{12}^2\right)k^{*2}-\rho_1n^2a_2^2-\tilde{b}_{11}\tilde{b}_{33}-\right.$$

$$\left.-a_2n^2\tilde{b}_{11}\left(\rho_1+2\rho_s^*/n\right)-\left(\rho_1+2\rho_s^*/n\right)\left(1+a_{12}\right)^2n^2k^{*2}\right];$$

$$\beta_3=\rho_1^2\left[\rho_1\left(\rho_1+2\rho_s^*/n\right)\right]^{-1}\alpha_3$$

Solving equation (22) in the domain of real numbers, we find

$$\lambda=\sqrt[3]{-\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}+\sqrt[3]{-\frac{q}{2}-\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}-\frac{\alpha_1}{3} \quad (24)$$

$$p=\alpha_2-\frac{\alpha_1^2}{3};\ q=\frac{2\alpha_1^3}{27}-\frac{\alpha_1\alpha_2}{3}+\alpha_3$$

Replacing in (24) $\alpha_1,\alpha_2,\alpha_3$ by β_1,β_2,β_3, respectively, we can find a real root of Equation (23). The numerical values of λ are found. For the problem parameters were accepted:

$R=0.16$ m; $h=0.00045$ m; $\nu_2=0.19$; $\nu_1=0.11$; $L_1=0.8$ m; $h_c=0.1375\times10^{-1}R$; $\rho_0=\rho_c=7800$ kq/m^3; $E_c=6.67\times10^9\,\Pi a$; $\rho_s=2800$ kq/m^3; $m=8$;

$$\frac{J_{yi}}{2\pi R^3h}=0.8289\times10^{-6};\quad \frac{J_{zi}}{2\pi R^3h}=0.13\times10^{-6};$$

$$\frac{J_{kpi}}{2\pi R^3h}=0.5305\times10^{-6}$$

The dependence of the frequency parameter on wave formation n in the circumferential direction found from equations (22) and (23) be means of (24) are depicted in Figure 1.

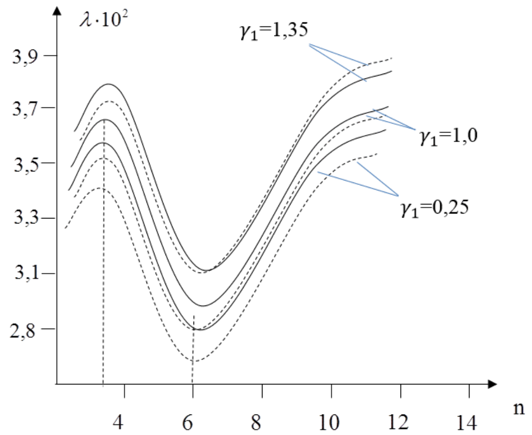

Figure 1. Dependence of the frequency parameter on wave formation n in the circumferential direction

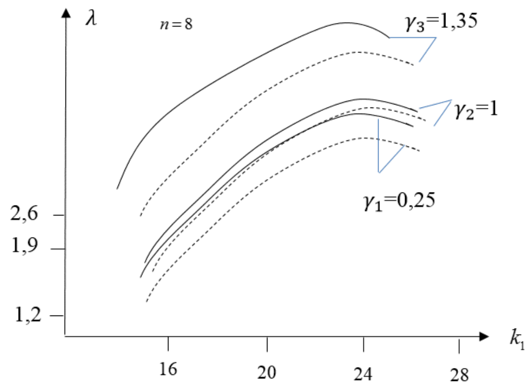

Figure 2. Dependence of the frequency parameter on the amount of longitudinal ribs

The values of a frequency parameter for a medium where the inertial properties are not taken into account correspond to the solid curve. The dashed line corresponds to the case when inertial properties are taken into account. Computations shows that both in an isotropic shell [1-5] and for a orthotropic shell, account of inertial properties of the medium reduces to decrease of the values of the frequency parameter λ, the growth of the parameter $\gamma\left(\gamma=\frac{E_1}{E_2}\right)$ to increase.

Dependence of the frequency parameter on the amount of longitudinal ribs for different values of γ is depicted in Figure 2. It is seen that with the increase in the amount of longitudinal ribs the frequency parameter of the oscillations of the constructions under investigation al first increases, and then begins to decrease. The latter is connected with the fact that with increasing the amount of longitudinal ribs, the influence of their inertial properties on the oscillations process is significant. Furthermore, with the strengthening of the anisotropy property of the shell's material, the frequency parameter of the oscillations of the studied constructions increases.

REFERENCES

[1] S.G. Suleymanova, "Free Oscillations of a Cylindrical Shell with a Filler Longitudinally Reinforced and Loaded with Axial Compressive Forces", Proceedings of IMM, Azerbaijan National Academy of Sciences, Baku, Azerbaijan, XXVII, pp. 135-140, 2007.

[2] F.S. Latifov, S.G. Sulerymanova, "A Problem of Free Oscillations of Medium Filled Cylindrical Shells Reinforced with Cross System of Ribs and Loaded with Axial Forces", International Scientific and Technical Journal of Mechanics of Machines, Mechanisms and materials, United Institute of Machine Building, Belarussian National Academy of Sciences, Minsk, Belarus, No. 1, pp. 59-62, 2009.

[3] F.S. Latifov, A.A. Aliyev, "Free Oscillation of Liquid Filled Ribbed Cylindrical Shells under Axial Compression", International Scientific and Technical Journal of Mechanics of Machines, Mechanisms and materials, United Institute of Machine Building, Belarussian National Academy of Sciences, Minsk, Belarus, No. 2, pp. 61-63, 2009.

[4] F.S. Latifov, I.M. Jafarova, "Free Oscillations of a Filled Shell Reinforced with Cross System of Ribs under Axial Compression with a Regard to Friction", Natural Science and Technology, Moscow, Russia, No. 5, Vol. 43, pp. 38-44, 2009.

[5] F.S. Latifov, O.Sh. Salmanov, "A Problem of Forced Axisymmetric Oscillations of a Liquid Filled Cylindrical Shell Reinforced and Loaded with Axial Compressive Forces", International Scientific and Technical Journal of Mechanics of Machines, Mechanisms and materials, United Institute of Machine Building, Belarussian National Academy of Sciences, Minsk, Belarus, No. 4, Vol. 5, pp. 45-48, 2008.

[6] F.S. Latifov, R.A. Iskanderov, S.B. Mikailov, "Oscillations of Laterally Reinforced Orthotropic Cylinders with Flowing Liquid in Medium", Problems of Computation Mechanics and Strength of Constructions, "O. Gonchar" Dnepropertrovsk State University, Issue 21, pp. 132-139, 2013.

[7] I.Ya. Amiro, V.A. Zarutskii, "Theory of Ribbed Shells", Methods for Calculating the Ribs, Scientific Thought, Kiev, Ukraine, p. 367, 1980.

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