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FURTHER RESULTS ON THE EFFECT OF PREDICTION HORIZON ON OBJECTIVE FUNCTIONS IN DMC CONTROL

J.M. Lopez Guede [1]   J.A. Ramos Hernanz [2]   I. Soto [3]   A. Moreno [4]

1. Systems and Automatic Control Department, University College of Engineering of Vitoria, University of the Basque Country (UPV/EHU), Vitoria, Spain, jm.lopez@ehu.es
2. Electrical Engineering Department, University College of Engineering of Vitoria, University of the Basque Country (UPV/EHU), Vitoria, Spain, josean.ramos@ehu.es
3. Electrical Engineering Department, University of Santiago, Santiago, Chile, ismael.soto@usach.cl
4. Ibermatica, Alava Technology Park, Minano, Spain, ai.moreno@ibermatica.com

Abstract- Model Predictive Control (MPC) and more specifically Dynamic Matrix Control (DMC) have shown to be appropriated control schemas to deal with systems that are unstable under classic controllers. In this paper we focus on the effect of one design parameter (prediction horizon) on its objective functions and performance. We have designed and experimental setup composed of 840 different experiments, varying several structural parameters of the DMC controllers. After analyzing all the obtained data, we conclude that the objective function to optimize and the *mse* index vary with a concrete pattern depending on the prediction horizon.

Keywords: Objective Function, Prediction Horizon, Model Predictive Control, Dynamic Matrix Control.

I. INTRODUCTION

Model Predictive Control (MPC) is a generic control technique instantiated by means of a wide set of advanced control algorithms devoted to deal with complex systems. Different instantiations of this type of advanced controllers have been used and compared with Proportional Integrative Derivative (PID) controllers [4, 13], showing a good performance.

We can found many applications in the literature, such as energy management [1], signal processing applications [9], multi-robot systems implementation [5, 6] and motor control [10], among others. They have shown their suitability for being implemented by means of neural networks [7], taking advance of their benefits. One of the most used of these algorithms is the Dynamic Matrix Control (DMC) algorithm. The main objective of this paper is to analyze the sensitivity of the objective function (and their components) of the DMC controllers under the effect of different prediction horizon values because we have neither studied nor found in the literature any study about the influence of the control horizon p on that objective function. The paper is structured as follows.

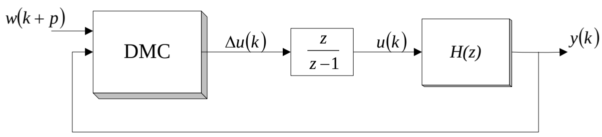

Figure 1. Block diagram of the closed loop control by means the Dynamic Matrix Controller

In the second section, we give a brief background and references about MPC. The third section focuses on DMC, where fundamental explanations about the role of the optimization function and its optimization are detailed. Besides, references to a previous work where the controlled system and its working point are detailed. In the fourth section we describe briefly the experimental design that we have carried out. The fifth, sixth and seventh sections are devoted to expose the results obtained on the optimization of objective functions and on the *mse* performance index. Finally, the last section provides our conclusions.

II. MODEL PREDICTIVE CONTROL

MPC is an advanced control technique used to deal with systems that are not controllable using classic control schemas (e.g. PID controllers). This kind of controllers works like the human brain in the sense that instead of using the past error between the output of the system and the desired value, it controls the system predicting the value of the output in a short time, in such a way the system output is as closer as possible to its desired value for these moments. MPC involves a set of techniques that share several characteristics, and the engineer has liberty to choose them. So, there are several types of predictive controllers.

The first one of these common characteristics is that there is a plant model, and it can be used a step response model, an impulse step response model, a transfer function, etc. It is used to predict the system output from the actual moment until p samples. The second one is the existence of an objective function that the controller has to optimize, while the last one is that there is a control law (in a generic sense) to minimize objective function.

A predictive controller follows a number of steps. Each sampling time, through the system model, the controller calculates the system output from now until *p* sampling times (prediction horizon), which depends on the future control signals that the controller will generate. A set of *m* control signals (control horizon) is calculated optimizing the objective function to be used along *m* sampling times. In each sampling time only the first of the set of *m* control signals is used, and at the next sampling time, all the process is executed again. For a deep insight about MPC see [2, 3, 8, 11 and 12].

III. DYNAMIC MATRIX CONTROL

DMC is a particular MPC algorithm that instantiates each of the generic elements that all MPC algorithms share. On one hand, regarding the system model, the plant model used by DMC algorithm is the step response model. This model uses the g_i coefficients that are obtained from the lineal system when it is excited using a step. To reduce the number of coefficients we assume that the system is stable and the output does not change after some sampling time *k*. The expression of the output of the system is given through Equation (1):

$$y(t) = \sum_{j=1}^{k} g_i \Delta u(t-i) \quad (1)$$

On the other hand, with regard to the objective function to optimize, it is mandatory to determine a prediction model. Using the step response model to model the system and maintaining the hypothesis that perturbations over the system are constants, it is possible to calculate a prediction at the instant *t* of the output until the instant $(t+p)$ under the effect of *m* control actions. That prediction is given by the Equation (2):

$$\hat{y} = G\Delta u + f \quad (2)$$

where $\hat{y}$ is the prediction of the output, *G* is a matrix that contains the system dynamics and *f* is the free response of the system. In Equation (3) there are the matrix and vectors involved in Equation (2):

$$\hat{y} = \begin{bmatrix} \hat{y}(t+1|t) \\ \hat{y}(t+2|t) \\ \vdots \\ \hat{y}(t+n|t) \end{bmatrix}_p, \quad \Delta u = \begin{bmatrix} \Delta u(t) \\ \Delta u(t+1) \\ \vdots \\ \Delta u(t+m-1) \end{bmatrix}_p \quad (3)$$

$$G = \begin{bmatrix} g_1 & 0 & \cdots & 0 \\ g_2 & g_1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ g_m & g_{m-1} & \cdots & g_1 \\ \vdots & \vdots & \ddots & \vdots \\ g_p & g_{p-1} & \cdots & g_{p-m+1} \end{bmatrix}_{p\times m}, \quad f = \begin{bmatrix} f(t,1) \\ f(t,2) \\ \vdots \\ f(t,p) \end{bmatrix}_p$$

In Equation (4) we describe how the free response of the system $f(t,k)$ is calculated:

$$f(t,k) = y_m(t) + \sum_{i=1}^{N} (g_{k+i} - g_i)\Delta u(t-i) \quad (4)$$

At this moment, it is possible to obtain the control law. Its determination is based on the existence of an objective function that uses the future outputs prediction model that has been described before. As objective function we use the function J described by Equation (6), which involves the error J' of the Equation (5) and the control effort:

$$J' = \sum_{j=1}^{p} \left[\hat{y}(t+j|t) - w(t+j)\right]^2 \quad (5)$$

$$J = J' + \sum_{j=1}^{p} \lambda \left[\Delta u(t+j-1)\right]^2 \quad (6)$$

The controller has to minimize the difference between the reference and the output prediction along a prediction horizon *p* with the *m* control actions generated in the control horizon, modulating the roughness in the variation of the manipulated variables using the *λ* parameter. Minimizing the objective function *J* described in Equation (6) we obtain the following expression, which produces *m* control actions, although only one of them is used at *t* :

$$\Delta u = \left[\left(G^T G + \lambda I\right)^{-1} G^T (w - f)\right]_m \quad (7)$$

To learn more about DMC, see [2, 3, 8, 11 and 12].

IV. EXPERIMENTAL DESIGN

In this section we describe the experimental setup that has been designed to study the effect of the prediction horizon *p* in the objective functions used by DMC controllers.

A number of different values of the parameters *p*, *m* and *λ* have been taken. The values that have been used for the prediction horizon *p* (the parameter for which the analysis is being carried out) are {$p \in$ [2, 5, 8, 11, 14, 17, 20]}. The value for the control horizon *m* is contained in the set $\{m \in N^+ \wedge m \in [1, 20]\}$. Finally, the values of the *λ* parameter are {$\lambda \in [10^{-3}, 10^{-2}, 10^{-1}, 1, 10^{1}, 10^{2}]$}. Carrying out the Cartesian product of these sets, the result is composed of 840 simulations.

Regarding the system that has been used to carry out the experimentation, the main part of the argumentation on its utilization has been intentionally omitted due to space issues. Its detailed description, the determination of the working point can be found in [4]. Its dynamics is described through Equation (8) and its response though Figure 2, while controlled by means of a discrete PID controller tuned using the Ziegler-Nichols method. There we can see that its response is unstable when the system is excited by a unitary step.

$$H(z) = \frac{1}{z-0.5} \quad (8)$$

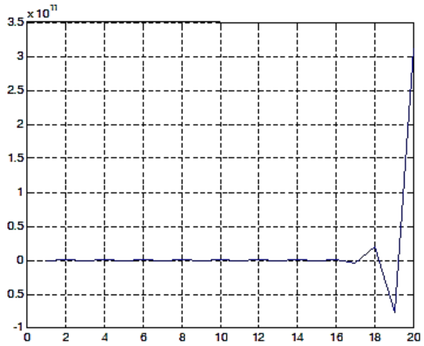

Figure 2. Unstable response of the closed loop system while controlled by a discrete PID controller, when it is excited by a unitary step

V. RESULTS ON THE OBJECTIVE FUNCTION J' SENSITIVITY

In this section we describe the results that we have reached on the sensibility of the J' objective function under the controlling action of DMC controllers with different prediction horizon *p* values. A number of figures have been obtained varying the *p* parameter, as can be seen through Figures 3-9. After analyzing those figures, we can conclude that in general, the shape of the objective function J' is maintained for concrete values of *m* and *λ* parameters. The range of values goes from 10^{-6} to more than the unit. For a given values of the *m* and *λ* parameters, the value of the objective function J' increases and becomes worst as the prediction horizon *p* increases.

This is because the predictions of the future output of the controlled system are less accurate when they are more distant in time. We can also see that the *λ* parameter is an important one, because except with very small values of *p*, the *λ* parameter is determinant. Finally, we can see in all figures that using values of the control horizon $m \le 3$, the minimization of the objective function J' is not as good as desirable, even for very small values of the *λ* parameter.

VI. RESULTS ON THE OBJECTIVE FUNCTION J SENSITIVITY

In this section we describe the results that we have reached on the sensibility of the J objective function under the controlling action of DMC controllers with different prediction horizon *p* values, and can be analyzed in Figures 10-16. There is an obvious pattern: the shape of the curves is identical to the curves of the J' objective function (previous section), but modified in the value by an offset due to the control effort. That is the second term of the Equation (6), and it reflects the effort (and probably the energy) that the DMC controller has to use to exert its controlling action. It is also related to the variations of the controlling signal, and usually it is desired that the control action is smooth. As we can see in all the figures it looks like that value is a constant value with regard to the variation of the *p* parameter, fact that we can assess realizing that the control effort is not dependent on the *p* parameter.

VII. RESULTS ON *mse* INDEX

In this section we are going to enumerate the effects on the *mse* performance index values, taking into consideration Figures 17-23 of the Appendix 3. The following are the main conclusions. In general, we can see that with increasing values of the *p* parameter, the value of the *mse* index becomes lower for each combination of *m* and *λ* parameters values. The *mse* index value ranges from about 10^{-7} with to about 2×10^{-1}, with different combinations of *m* and *λ* parameters values.

The worst results are produced when *λ* parameter is very high, but if the control horizon *m* parameter increases, the results are relatively better if becomes high. As the *p* parameter value increases, the *mse* index value becomes relatively low, however it increases again when $p \ge 10$. The most likely reason for this effect is that it is convenient to predict the output of the system, but if we try to predict it at a very distant sample time, that prediction will be less accurate.

Also, we can notice that it makes no sense to use a control horizon *m* larger than the prediction horizon.

VIII. CONCLUSIONS

We have started this paper reviewing the scope and the application field of MPC techniques in the first section, giving a short background and referencing some previous related works in the second section. In the third section we have also described mathematically the objective function that is usually used in DMC control. Later we have described the experimental design that has been carried out with a total of 840 experiments in the fourth section. The results of objective functions J' and J, and the index *mse* are analyzed in the fourth fifth and sixth sections respectively. The results discussed show that the objective function is clearly influenced by the prediction horizon *p*, and that the control effort is constant and independent of that value.

APPENDICES

Appendix 1. Influence on J' index

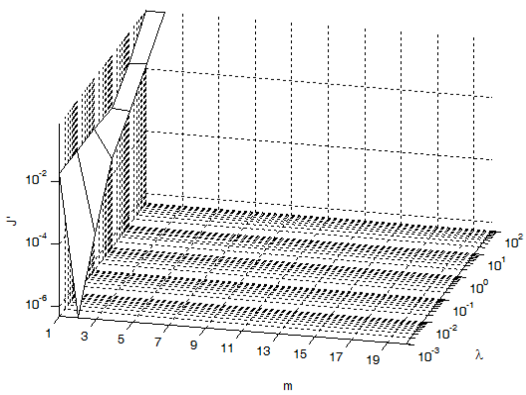

Figure 3. *J'* with *p* = 2

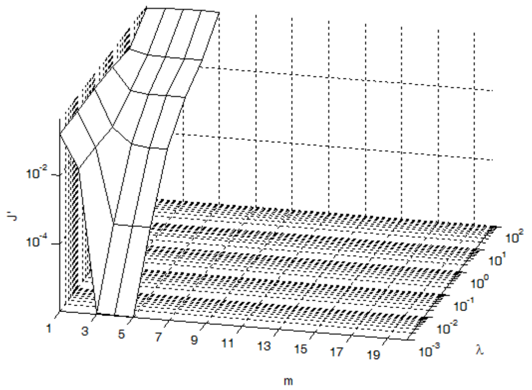

Figure 4. *J'* with p = 5

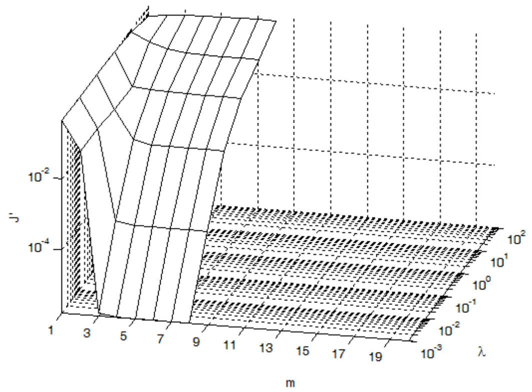

Figure 5. *J'* with p = 8

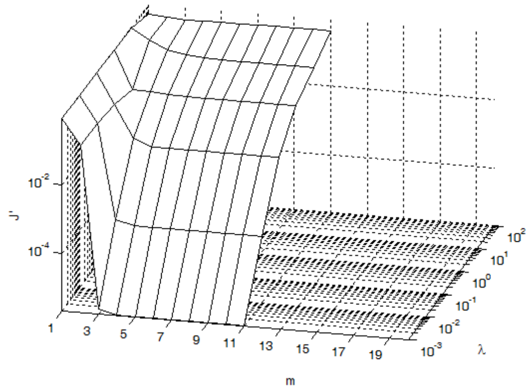

Figure 6. *J'* with p = 11

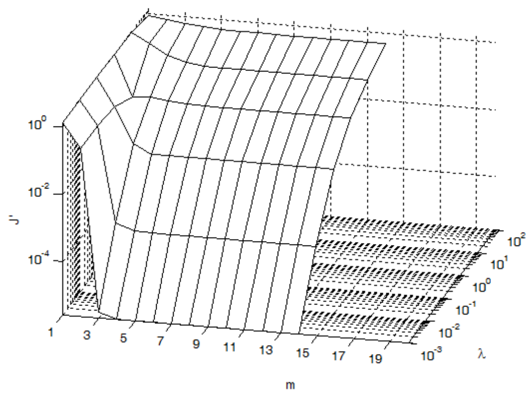

Figure 7. *J'* with p = 14

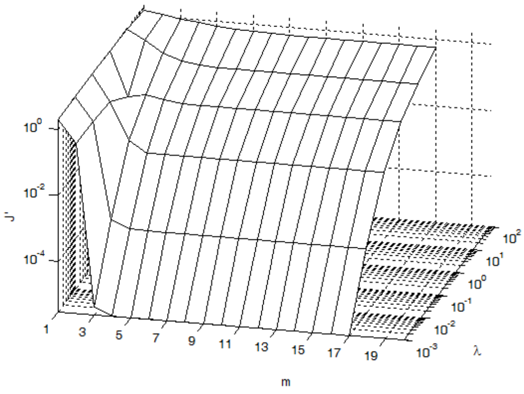

Figure 8. *J'* with p = 17

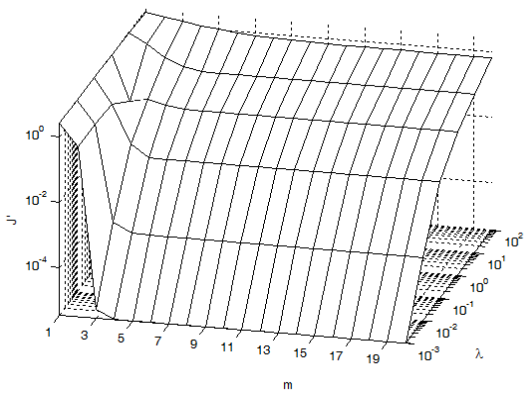

Figure 9. *J'* with p = 20

Appendix 2. Influence on *J* index

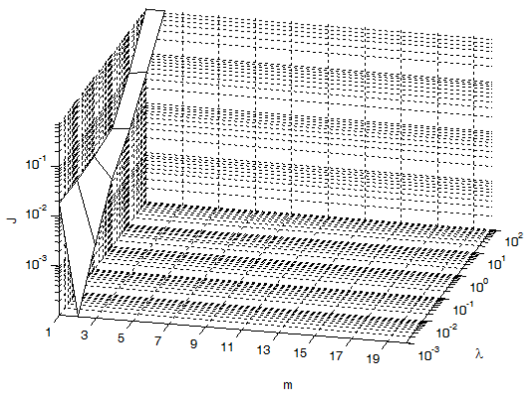

Figure 10. *J* with p = 2

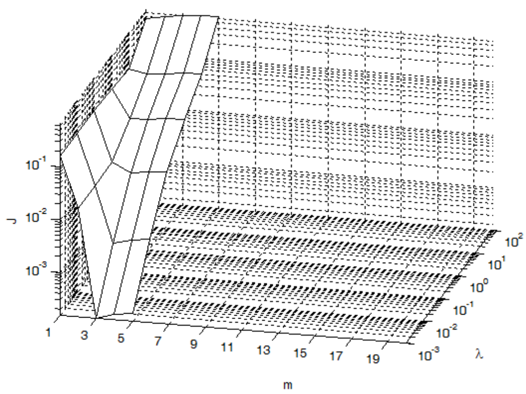

Figure 11. *J* with p = 5

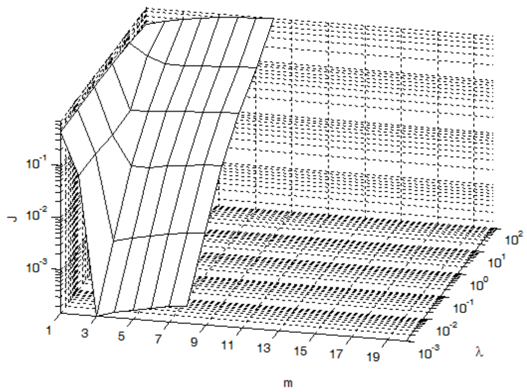

Figure 12. J with p = 8

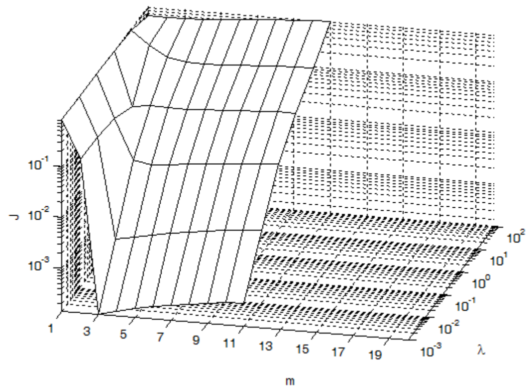

Figure 13. J with p = 11

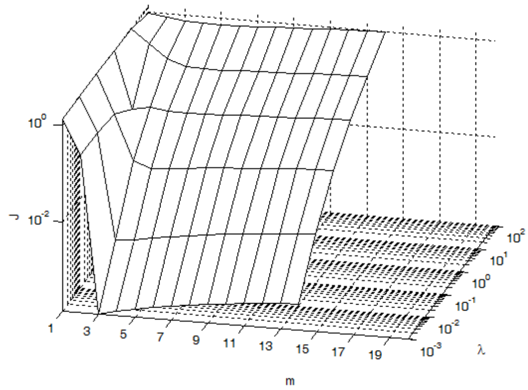

Figure 14. J with p = 14

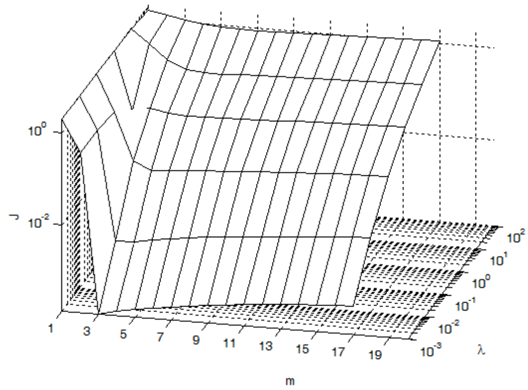

Figure 15. J with p = 17

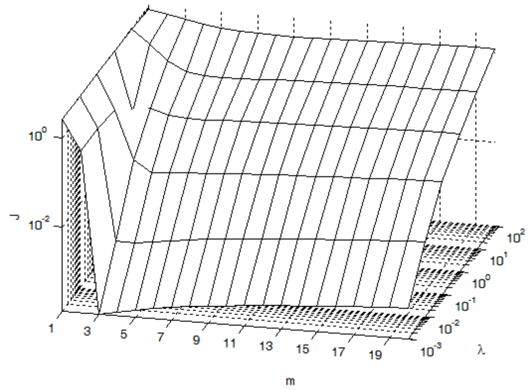

Figure 16. J with p = 20

Appendix 3. Influence on *mse* Index

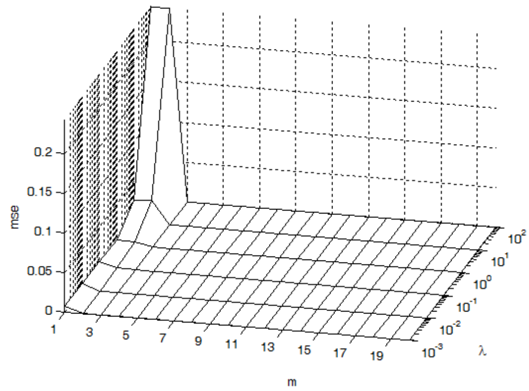

Figure 17. *mse* with p = 2

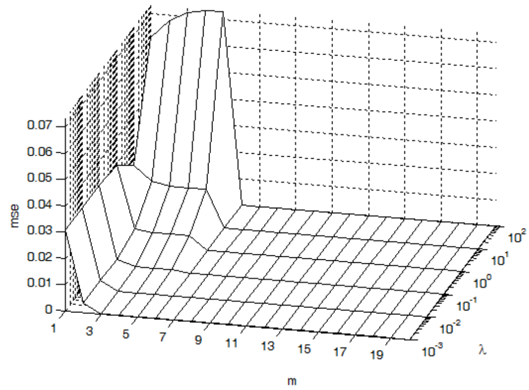

Figure 18. *mse* with p = 5

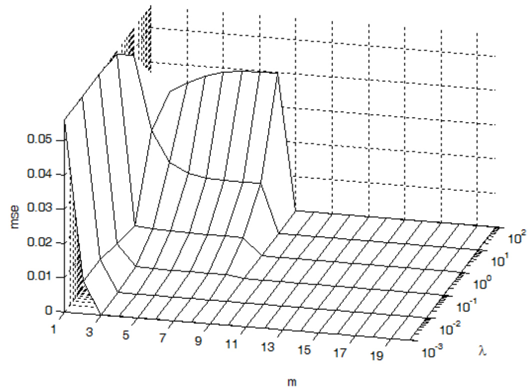

Figure 19. *mse* with p = 8

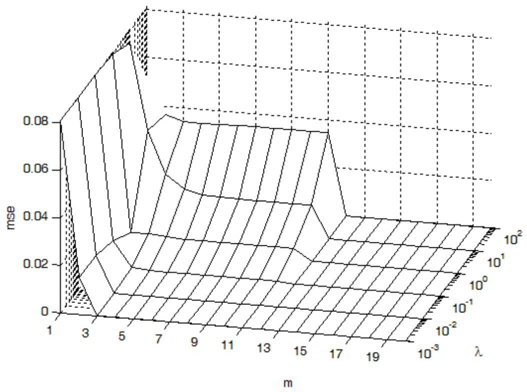

Figure 20. *mse* with p = 11

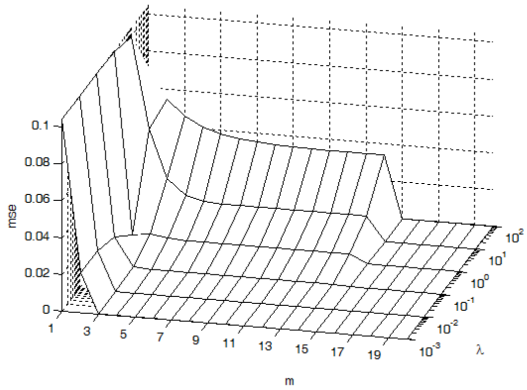

Figure 21. *mse* with p = 14

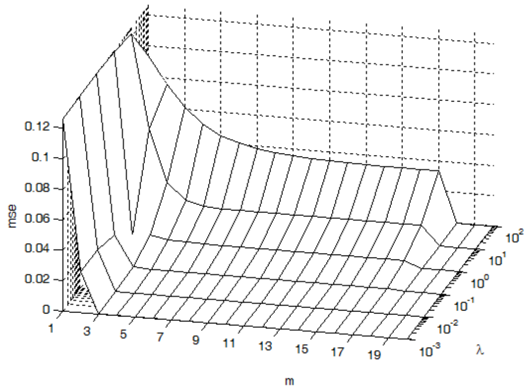

Figure 22. *mse* with p = 17

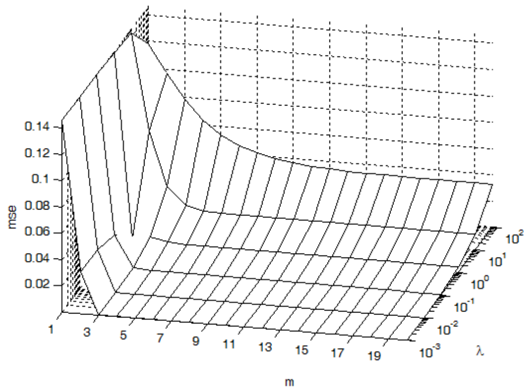

Figure 23. *mse* with p = 20

NOMENCLATURES

g_i : The output coefficients of the system when it is excited using a step.
f: The free response of the system.
$\hat{y}$: The prediction of the output
G: The dynamic matrix of the DMC controller
λ: The parameter of the DMC controller related to its embodiment.
p: The prediction horizon
m: The control horizon
t: The time instant
$u(t)$: The whole input of the controlled system at time t
J : The objective function
J' : The objective function without the control effort
w : The reference signal
I : The identity matrix

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BIOGRAPHIES

Jose Manuel Lopez Guede was born in Eibar, Spain, 1976. He received the M.Sc. degree in 1999 and the Ph.D. degree in 2012, both in Computer Sciences from University of the Basque Country, San Sebastian, Spain. Since 2002 he is working at the same university. His current position is Lecturer at Systems Engineering and Automatic Control Department at the University College of Engineering of Vitoria Gasteiz, Spain.

Jose Antonio Ramos Hernanz was born in Vitoria Gasteiz, Spain, 1966. He received B.Sc. degree in Electrical Engineering from University College of Engineering of Vitoria Gasteiz, Spain, in 1991 and M.Sc. degree in Industrial Organization Engineering from Faculty of Engineering of Bilbao, Spain in 2003 both belong to University of the Basque Country, Bilbao, Spain. Since 1991 he is working at University College of Engineering of Vitoria Gasteiz, Spain. He is Full Lecturer at Electrical Engineering Department of the same university. He was Academic Secretary of Department of Electrical Engineering at University of the Basque Country (10/1998-02/2002), and Vice-Dean of Educational Planning and Faculty at University College of Engineering of Vitoria Gasteiz (11/2004-03/2009). His main research interests are electric power quality, photovoltaic energy, and bird protection on power lines.

Ismael Soto is an Electrical Engineer, Master in Electronics and Ph.D. in Telecommunications. He Currently works in the Department of Electrical Engineering, University of Santiago de Chile, as full Professor. He is also responsible for laboratory research GETIC, from which has contributed to the generation of engineers and PhD. In the last years the emphasis has been in the Digital Signal Processing, prioritizing transmission applications, the modelling for the optimization of coding and cryptography to be used on wireless communications and nanotechnology.

Aitor Moreno received the Computer Engineering degree in 1995 at the University of Deusto and the Master's Degree in Advanced Artificial Intelligence in 2008. Since September 2008, he manages projects related to the implementation of control systems based on neural networks, genetic algorithms, fuzzy logic systems and expert systems to the analysis of large volumes of information. He has published several scientific papers and informative concerning the application of Artificial Intelligence in real situations, and has participated in national and other events as speaker.