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A NOVEL ANALYTICAL TECHNIQUE FOR ESTIMATION OF MAGNETIC FLUX DENSITIES WAVEFORMS WITHIN 8/6 SWITCHED RELUCTANCE MOTORS

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Abstract- This paper presents a novel analytical method for estimation of the flux densities waveforms in various sections of 8/6 Switched Reluctance Motors (SRMs). For this, finite element analysis (FEA) of an 8/6 SRM is carried out to, i) identify flux tube paths in the motor in different rotor positions, ii) implement the analytical flux tube method, and iii) calculate the flux linkage and inductance characteristics of the motor. Inductance characteristic is then incorporated into the voltage equation of the stator winding to obtain dynamic excitation current waveform. It means that model of the motor is formed via non-linear Ampere's law equation that drives for each flux tube in three regions. The results obtained by the proposed analytical and finite element methods are compared and good agreements are achieved.

Keywords: Magnetic Flux Densities Waveforms, Finite Element Analysis, Flux Tube Method, Switched Reluctance Motor.

I. INTRODUCTION

Nowadays, design of SRMs is carried out with objective of high-efficiency and minimum torque ripple. In order to achieve these objectives, some requirements have been proposed [1, 2]. In [1], a high-speed four-two pole switched reluctance motor (SRM) has been tested to verify the high-efficiency and low-torque ripple characteristics of the motor. The effects of manufacturing process such as annealing electrical steel on the flux densities distribution in the SRM and its core losses estimation has been investigated [2]. It has been found that the measured core losses are increased around 37% and the torque density can be enhanced when using the annealed electrical steel [2].

Core losses reduction can help to improve total machine efficiency [3, 4]. Core losses estimation in SRMs requires predicting magnetic flux densities waveforms in the different sections of the motor. Magnetic flux density within different parts of a SRM is a function of rotor angular position and stator winding current.

Prediction of these flux densities waveforms is difficult due to the deep saturation and non-sinusoidal current waveforms in SRM circuit. There are different methods for determination of these waveforms. In [5], the magnetic flux waveform in one section of the stator yoke has been measured and used to predict the flux waveforms in other parts of SRM magnetic circuit. These predicted waveforms are employed to estimate the core losses in the motor by harmonic analysis of these waveforms. However, the error of this method is high and about 25.7%. Alternative method presents the ratio between the magnetic flux values in different sections of the magnetic equivalent circuit of SRM as a matrix form equations [6]. Although mathematical manipulation of these equations is very difficult, the accuracy of the method is reasonable. Approximate magnetic flux density waveforms can be predicted using traditional equations of a three-phase 6/4 SRM [7], however, it has neglected the saturation of the magnetic circuit and estimated core losses using this technique has nearly large error.

A two-dimensional (2D) finite element method (FEM) has been used in [8] to evaluate the magnetic flux density waveforms in different parts of the motor. These waveforms have been then fitted to 4^{th}-order polynomials and eddy current and hysteresis losses have been estimated. Disadvantages of this method include a long computation time and a huge computer memory. A linear approximation has been used in [9] to estimate the waveforms in iron parts of the motor.

This paper presents a new analytical method based on the governing equations on magnetic flux paths in three regions that developed in a non-linear and dynamic form. The waveform of the stator pole magnetic flux density at a constant speed is obtained by solution of non-linear equations iteratively. This magnetic flux density waveform is used to estimate the magnetic flux densities in different parts of an 8/6 SRM. To verify the analytical method its results are compared to a two-dimensional FE analysis of the motor.

II. ESTIMATION OF FLUX-LINKAGE AND INDUCTANCE CHARACTERISTICS OF SRM

To predict the flux-linkage, $\lambda(\theta,i)$, and inductance, $L(\theta,i)$, characteristics of an 8/6 SRM with specifications summarized in Table 1, is considered. For this, the flux tube method [7, 8] is used and implemented by MATLAB scripts. In this implementation, the rotor angular position (θ) between unaligned (0°) to aligned (30°) positionsis considered and excitation current (i) is statically change to 1.3 time of the motor rated current. Figure 1 shows the flux-linkage of the

$$i(\theta_r+2)=\frac{V_{dclink}}{R_{st}}(1-\mathrm{e}^{-\frac{R_{st}}{L_{(\theta_r+1)}}t_s})+i(\theta_r+1)\mathrm{e}^{-\frac{R_s}{L_{(\theta_r+1)}}t_s}$$

SRM using the method in which the impact of the magnetic saturation in the rotation of rotor from unaligned position to aligned position is fully clear. The $L(\theta,i)$ characteristic can be easily obtained from $\lambda(\theta,i)$ characteristic.

Table 1. Specifications of 8/6 SRM

Quantity	Values	Quantity	Values
Stator outer diameter (mm)	157	Height of rotor poles (mm)	19.2
Core diameter (mm)	73	Air-gap length (mm)	0.35
Shaft diameter (mm)	17	No. of winding turns/ phase	419
Stack length (mm)	38	Bus DC link voltage (V)	300
Stator poles arc (°)	21	Rated speed (rpm)	1425
Rotor poles arc (°)	23	Stator and rotor lamination material:	
Height of stator poles (mm)	28.7	V800 - 50A	

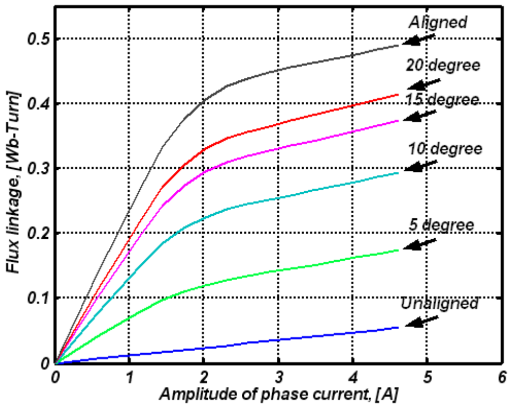


Figure 1. Flux-linkage characteristic of 8/6 SRM at different rotor positions using flux tube method

III. CALCULATION OF EXCITATION CURRENT AGAINST ROTOR DISPLACEMENT

During the beginning of commutation, the voltage equation of phase being commutated can be written as:

$$-V_{dclink}=R_s i+\frac{d}{dt}\left(L_d i\right) \tag{1}$$

where V_{dclink} is the dc link voltage applied to the phase being commutated, R_s is the stator resistance, i is the current being carried by the phase at the time of commutation and L_d is the difference in the aligned and unaligned inductance. Suppose the SRM rotates at rated speed and dc link voltage is applied to the stator winding at rotor position θ_r.

Therefore, rotor travels any degree at t_s=1/6N_r. The current at position θ_r+1 is calculated as follows [8]:

$$i(\theta_r+1)=\frac{V_{dclink}}{R_{st}}(1-e^{-\frac{R_{st}}{L_{\theta_r}}t_s}) \tag{2}$$

where R_{st} is the resistance of the stator phase winding and $L_{\theta r}$ is the minimum inductance of the stator winding at rotor position of θ_r that can be obtained from $L(\theta,i)$ characteristic. The current equation at rotor position θ_r+2 is then calculated as follows [8]:

$$i(\theta_r+2)=\frac{V_{dclink}}{R_{st}}(1-e^{-\frac{R_{st}}{L_{(\theta_r+1)}}t_s})+i(\theta_r+1)e^{-\frac{R_s}{L_{(\theta_r+1)}}t_s} \tag{3}$$

where $L_{(\theta r+1)}$ is the inductance of the stator winding at rotor position θ_r+1 and current $i(\theta_r+1)$ that can be extracted also from $L(\theta,i)$ characteristic precisely. This trend continues for rotor position θ_r+2 and higher angles up to the rated current. At this rotor condition, current remains constant up to the phase commutation angle, θ_c, and at this angle, the reversed dc bus voltage is applied to the phase winding. Therefore, the winding current begins to drop and the current equation at position θ_c+1 is expressed as follows [8]:

$$i(\theta_c+1)=-\frac{V_{dclink}}{R_{st}}(1-e^{-\frac{R_{st}}{L_{\theta c}}t_s})+i_{peak}e^{-\frac{R_{st}}{L_{\theta_c}}t_s} \tag{4}$$

where i_{peak} is the peak excitation current. The current at rotor position θ_c+2 is then calculated by substituting θ_r in Equation (3) by θ_c as follows [8]: This trend continues up to tending current to zero and completing commutation.

IV. MODELING SRM BY EXTENDED MAGNETIC FLUX TUBE METHOD

To model the 8/6 SRM, three regions for the positions of the rotor pole against stator pole are considered. These regions consist of: 1) unaligned position up to beginning of stator and rotor poles overlap, 2) beginning of overlapping up to full overlapping, and 3) full overlapping position up to aligned position of the stator and rotor poles [10].

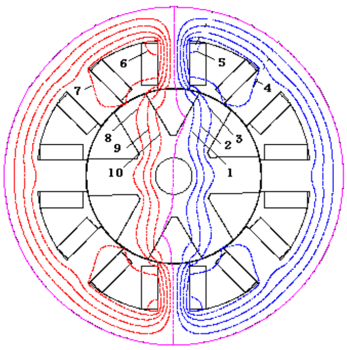
Two-dimensional (2D) FE analysis of the motor is carried out using Flux2D 10.2 software package [11] in order to identify flux tubes in the motor cross-section. Figure 2 shows the flux tubes at each region. From this 10, 9 and 8 flux tubes are identified from FE analysis to model region I, region II and region III, respectively.

To determine the flux density variations in magnetic flux paths in the regions, Ampere's equation is written considering the path of the each flux tube. To drive the equations, the length of the tubes must be calculated in each section of the SRM cross-section. Geometrical lengths in some sections may be expressed as a function of the rotor position. In addition, the excitation current versus rotor positions must be incorporated in Ampere's equation to accurately calculate the magnetic flux intensity of the flux tubes in three regions.

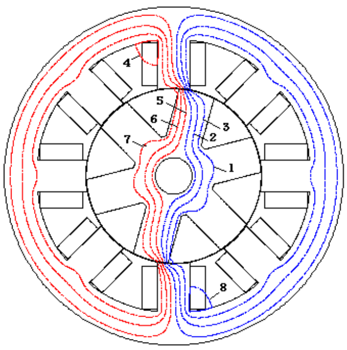
To demonstrate the above mentioned method, consider the magnetic flux tube1 in region III. Figure 2(c) shows that the magnetic flux tube1 passes through the stator pole (sp), air-gap (g), rotor pole (rp), rotor core (rc) and stator yoke (sy). Therefore, the Ampere's equation of the path is as follows:

$$N_{ph}i(\theta) = 2H_{sp}(\theta, i(\theta))l_{sp}(\theta) + 2H_{rp}(\theta, i(\theta))l_{rp}(\theta) + \\ + \frac{B_g(\theta, i(\theta))A_g(\theta)}{P_g(\theta)} + H_{sy}(\theta, i(\theta))l_{sy}(\theta) + H_{rc}(\theta, i(\theta))l_{rc}(\theta) \quad (5)$$

where N_{ph}, H and l are the number of winding turns, magnetic field intensity and length of the flux tubes in each sections of the motor, respectively. The P_g and A_g are the permeances and area of the magnetic flux tube in the air-gap and $i(\theta)$ recalls the excitation current versus rotor position that calculated in section III.



(a)



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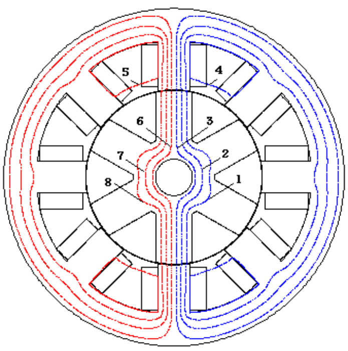


(c)

Figure 2. Identification of flux tubes at 8/6 SRM in, (a) region I, (b) region II, and (c) region III

The relationship between the magnetic field intensity of various sections within the motor cross-section is expressed based on the area of the magnetic flux tube path. In this paper, the magnetic field intensities in Equation (6) can be written in term of the magnetic field intensity in the stator pole. Ampere's equation must be derived for each magnetic flux tubes in the regions proportional the path of the magnetic flux tube in the SRM cross-section. Consequently, system with 26 non-linear equations is formed that models the SRM for calculation of magnetic field intensity in the stator pole.

The model solves using an iteration method with very low convergence criterion. Having the magnetic field intensity in the stator pole at any rotor position, the magnetic flux density of stator laminations is obtainable from magnetization (*BH*) characteristic. Figure 3 shows the magnetic flux and excitation current waveforms using the proposed method and FE method. As seen the result of the suggested method is very close to that of FEM. Difference between these two values particularly during the decreases of the stator excitation current is caused by difference of the used currents in the two methods, which is clearly shown in Figure 3.

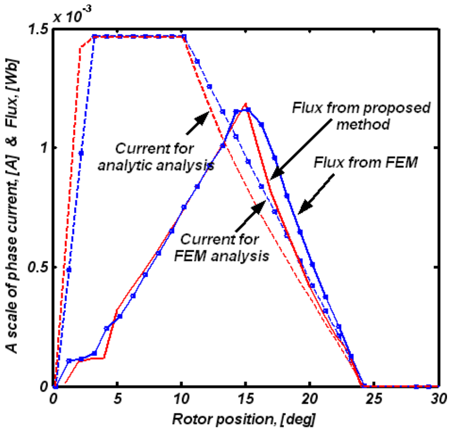


Figure 3. Comparison of stator pole flux and excitation current waveforms of 8/6 SRM for proposed analytical and FE methods

V. CALCULATION OF FLUX DENSITIES WAVEFORM WITHIN 8/6 SRM

Having the magnetic flux density in the stator pole, magnetic flux densities waveforms in other portions of the SRM such as stator yoke, rotor pole and rotor core are predictable. These portions are shown in Figure 4. Figures 5(a) and 5(b), represents the magnetic flux density of a single stator (sp1) and rotor poles (rp1), over a full revolution of the 8/6 SRM, respectively. It can be observed that the magnitude and the frequency of the stator pole flux density waveform are higher than that of the rotor pole. The diametrically opposite poles would have a similar waveform except that the polarity would be reversed. Other stator and rotor poles would have the similar waveforms but offset by 15° in 8/6 SRM.

The magnetic flux densities of the stator yoke vary from portion to portion considering the activated phase. Therefore, when the waveforms are calculated in the 8 stator poles, the waveforms can be predicted in Sy1, Sy2, Sy3 and Sy4 portions of the stator yoke. In addition, the corresponding diametrically opposite sections has reverse polarities. The magnetic flux densities for four different sections are plotted in Figure 6 for a full rotation of rotor.

Finally, when the waveforms are calculated in the 6 rotor poles, the waveform can be predicted in the rotor core. The magnetic flux density, experienced by one of six portions of the rotor core over a full revolution, has been represented in Figure 7. The other sections have the identical waveforms but are offset by 60°.

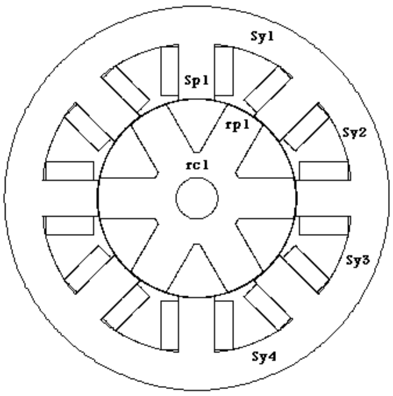
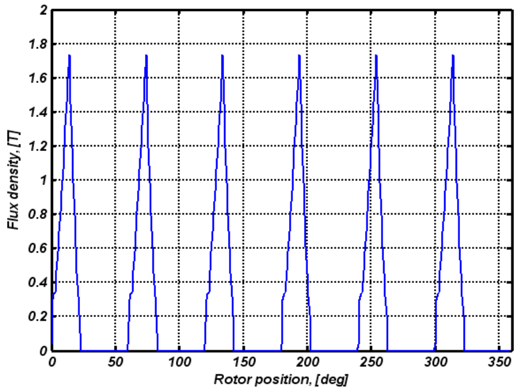
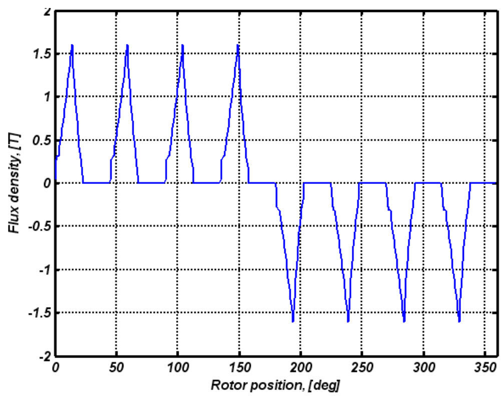


Figure 4. Various sections of 8/6 SRM

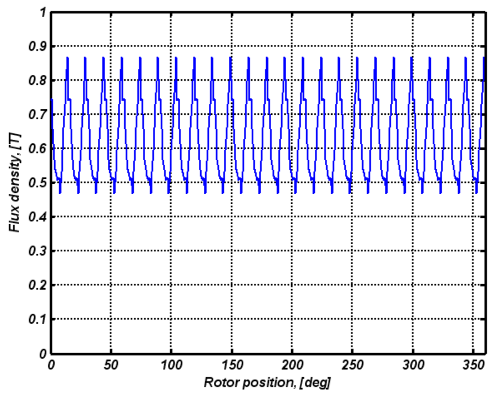


(a)

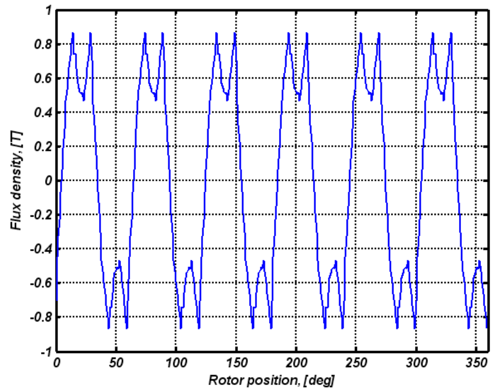


(b)

Figure 5. Flux density waveform in (a) a single stator pole, and (b) a single rotor pole, of 8/6 SRM over a full revolution of rotor



(a)



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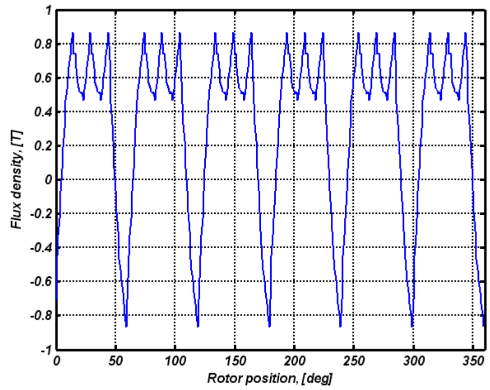

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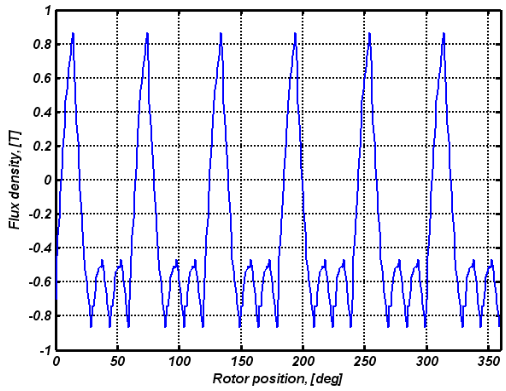

(d)

Figure 6. Flux density waveform in stator yoke of 8/6 SRM over a full revolution of rotor (a) Sy1 section, (b) Sy2 section, (c) Sy3 section, and (d) Sy4 section

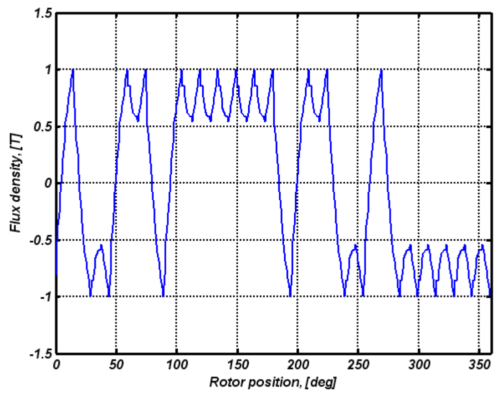

Figure 7. Flux density waveform in one portion of rotor core of 8/6 SRM over a full revolution of rotor

VI. CONCLUSIONS

In this paper, a novel method proposed to predict the flux density in different portions of the 8/6 SRM. In this method Ampere's equation for all magnetic flux paths in three regions followed a non-linear form which include the current dynamic. A SRM model was introduced based on the non-linear relationships which are functions of the geometry and magnetic characteristic of the motor, rotor position and stator winding excitation current.

To solve these non-linear equations, an iterative method with the minimum convergence conditions was used. FEM was applied in some stage to evaluate the results. Comparison of the results based on the analytical and FE methods indicated a good coloration between the two methods.

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BIOGRAPHIES

Mojtaba Babaei received his B.Sc. degree from K.N. Toosi University of Technology, Tehran, Iran and the M.Sc. degree from Shahed University, Tehran, Iran in 2002 and 2005, respectively. He received the Ph.D. degree in Electrical Engineering from Science and Research Branch, Islamic Azad University, Tehran, Iran in 2011. Since 2011, he has been working as an Assistant Professor at the Faculty of Engineering, Yadegar-e-Imam Khomeini Branch, Islamic Azad University, Tehran, Iran. His research interests include design, modeling, analysis and control of electrical machines and drive systems. He is author of a number of papers and technical reports in the above-mentioned areas.

Jawad Faiz received the M.Sc. degree in Electrical Engineering from the University of Tabriz, Tabriz, Iran in 1975 graduating with first class honors. He received the Ph.D. degree in Electrical Engineering from the University of Newcastle upon Tyne, Englandin 1988. Early in career, he served as a Faculty Member in the University of Tabriz for 10 years. After obtaining Ph.D. degree, he rejoined the University of Tabriz where he held the position of Assistant Professor from 1988 to 1992, Associate Professor from 1992 to 1997, and has been a Professor since 1998. Since February 1999 he has been working as a Professor at School of Electrical and Computer Engineering, Faculty of Engineering, University of Tehran, Tehran, Iran. He is currently the Director of the Center of Excellence on Applied Electromagnetic systems. He has received a number of awards including the first basic research award from the Kharazmi International Festival in 2007, the silver Einstein medal for academic research from the UNESCO, the first rank medal in Research from the University of Tehran in 2006, the Elite Professor Award from the Iran Ministry of Science, Research and Technology in 2004. He is the author of 206 papers in international journals and 222 papers presented in International conferences. He is a senior member of the IEEE and member of Iran Academy of Sciences. His teaching and research interests are switched reluctance and VR motors design, design and modeling of electrical machines and drives, transformer modeling and design and fault diagnosis in electrical machinery.