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ADVANCED THEORY METHODS BASED ON OPTIMIZATION

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Abstract- Most engineering problems have a nonlinear model. Obtaining the optimal global solution in nonlinear models is very difficult. Several methods are proposed to obtain the optimal solution to such problems. These methods include: intelligent algorithms, converting these nonlinear problems to linear problems, and using linear commercial solutions, and using nonlinear commercial solvers. In intelligent algorithms, we may arrive at inappropriate solutions. In the second method, because the original model is approximated, we may not reach the optimal solution. The advantage of the second model is the solution time because linear commercial solvers have a very short solution time. However, in the third method, due to the use of the nonlinear model, it may take longer to achieve the optimal answer than the second method. This chapter examines the applications of the second method to issues such as Economic Dispatch (ED), Optimal Power Flow (OPF), and Unit Commitment (UC).

Keywords: Mixed Integer Linear Programming (MILP), ED, OPF, UC.

1. INTRODUCTION

Dynamic ED considering valve-point effect (DED-VPE) can be a non-convex and non-derivable optimal model that this model solving is significantly hard. In this section, a combined method of MILP and internal point approach (IPM), shorten to a MILP-IPM, is considered for obtain a DED-VPE model in which lines power losses. There is too complex. Considering the non-derivable structure of the DED-VPE model, conventional solution approaches based-on derivative can no longer be utilized. By handling approach formulization, a derivable NLP is obtained that can be solved directly utilizing IPM. Although, in the event that the DED-VPE model is obtained utilizing IPM in one level, the optimization is easily stuck in a weak local optimization considering its non-convex structure and several local minimums. Hence, if the proposed model is obtained utilizing IPM in a singular level, the solution is quickly stuck in a weak local optimization because of its non-convex structure and several weak answer. In order to, achieve a good answer, a MILP model is needed to obtain the DED-VPE model without transmission losses, which provides a proper primary point for IPM to enhance the performance of the answer.

DED is one of the main models related to the ED use of power networks. This mentions the effort to divide customer demand among existing thermal power generation power plants in an economical, safe, and reliable method at a certain period. The objective function for DED can be a convex and approximate second-class polynomial. Although, in real application, the influences of wiring, which occurs as every steam acceptance amount in a turbine begins for open, make a ripple influence on the production cost figure. These influences are determined as VPEs. To implement these influences, a frequent modified sinusoid combination is contributed to input-output relation, which results in a function of non-convex, non-derivable generation costs, and several local minimums in structure.

If VPE is neglected, the estimation of the production objective function causes wrongs in the ED results. To enhance the optimization performance, a more exact DED method, DED with DED-VPE should be proposed. Although, with considering VPE, several non-convex, non-derivable, and several inappropriate minimum properties are considered, which challenges DEDVPE model solving. Over the past years, several optimization approaches are considered to obtain the DED-VPE model. Based on the intractability of the model, in most cases, existing methods are solution approaches based-on heuristic. These heuristic optimization methods are population-based approach which does not belong on performance of gradients and Hessian in the cost function. Although, they can be used to effectively calculate the DED-VPE model. Anyway, contribution approaches that combine multiple heuristic methods or deterministic methods, because random search methods are integrated, this approach has some inherent disadvantages of the approaches based on heuristic listed mentioned.

On the contrary heuristic algorithms, optimization techniques based on deterministic mathematical programming can earn optimal answers because of their proper mathematical structure and with having of strong solution methods. Thus, a recent approach for DED-VPE can be for formulate the production objective function for achieve an appropriate solution method that can be calculated utilizing a definitive approach. In [1], the cost of VPE, which is non-convex and indivisible, is partially non-derivable and then a mixed-integer second-class planning formulization (MIQP) is created for DED-VPE without due to lines power losses. Although the MIQP formulization is calculated immediately utilizing a MIP Solver, the solution procedure results in convergence immobilization and memory depletion. Accordingly, a multi-level approach, a warm start-up method, and a domain constraint plan are combined in the MIQP method. When considering the additional complexity of transmission losses, due to the mentioned procedure, more tedious regulation methods are required in the proposed method. In [2], a deterministic combination approach, which contributes MILP with internal point approach, curtailed since MILP-IPM, is considered to obtain the DED-VPE model due to losses. Because the DED-VPE model is non-differential, the conventional solution approaches based on derivative can no longer be utilized.

2. MILP MODEL FOR DED

By model formulization, an NLP method of the DED-VPE model is obtained, which can be calculated quickly utilizing PM polynomial time. Although, IPM can be a local solution approach. If the DED-VPE model is calculated utilizing IPM in one level, the optimization is easily stuck in a weak optimization based on its non-convex state and several weak minimums. To dominate this shortcoming, a MILP approach has been used to produce a suitable starting point for IPM. Hence, with calculating the NLP formulization by IPM, an optimal significant-performance answer to the DED-VPE model can be defined. Mathematical formulization of the DED-VPE model for traditional DED, the cost of generation per unit is formulated like a convex second-class polynomial [2]:

$$c_j^{quad}\left(P_{j,t}\right)=\alpha_j+\beta_i P_{j,t}+y_j P_{j,t}^2 \tag{1}$$

where, $P_{j,t}$ is the output product of power plant *j* in interval *t* and α_i, β_i and γ_i are the positive factors of power plant *i*. When VPE is assumed, a recurring modified sinusoidal function [2]:

$$c_j^{vpe}\left(P_{j,t}\right)=e_j\left|\sin\left(f_j\left(P_{j,t}-P_j^{\min}\right)\right)\right| \tag{2}$$

It can be increased for the traditional production cost, while creates the production objective function non-convex and non-derivable in the state. $P_i^{\min}$ is the lowest level of energy output of power plant *i*. The e_i and f_i are the positive factors of the VPE expense to power plant *i*. As a result, the power plant production expense to DED-VPE is represented as follows [2]:

$$e_j\left(P_{j,t}\right)=c_j^{quad}\left(P_{j,t}\right)+c_j^{vpe}\left(P_{j,t}\right) \tag{3}$$

The goal of proposed model can be for optimize the entire expense of production in a planned interval which it is solved as following [2]:

$$\min\sum_{t=1}^{T}\sum_{j=1}^{N}c_j\left(P_{j,t}\right) \tag{4}$$

while, *N* and *T* are the entire number from power plants and interval, respectively. The proposed model is optimized for the following limitations [2].

• Power equality relation:

$$\sum_{j=1}^{N}P_{j,t}=D_t+P_t^{loss},\forall t, \tag{5}$$

while D_t is the consumption in interval *t*, P_t^{loss} is the lines power losses in interval *t*, where is earned due to the B-factor approach. It can be represented as a second-class term of the power products [2]:

$$P_t^{loss}=\sum_{i=1}^{N}\sum_{j=1}^{N}P_{i,t}B_{i,j}P_{j,t},\forall t, \tag{6}$$

while, $B_{i,j}$ is the element (*i*, *j*) of the transfer power losses factor matrix (*B*). Power production constraints [2]:

$$P_j^{\min}\le P_{j,t}\le P_j^{\max},\forall i,t, \tag{7}$$

while, $P_i^{\max}$ is the upper limit of power plant *i*. Incremental level constraints [2]:

$$DR_j\le P_{j,t}-P_{j,t-1}\le UR_j,\forall j,t, \tag{8}$$

while, DR_i and UR_i can be the reduction and incremental levels of power plant *i*, respectively. Rotational reserve limitations [2]:

$$\begin{cases} SR_{j,t}\le P_j^{\max}-P_{j,t},\forall j,t,\\ SR_{j,t}\le \tau UR_j,\forall j,t,\\ \sum_{j=1}^{N}SR_{j,t}\ge R_t,\forall t, \end{cases} \tag{9}$$

while, $SR_{j,t}$ is the rotating reserve obtained with power plant *j* in *t*, R_t is the rotational reserve need of the network in interval *t*, and *τ* is the delivery time by the power plants. When renewable energy for example wind power is added, up and down spinning reserve limitations can be added to omit fluctuations.

An NLP formulization for proposed model can be proposed for a non-convex and non-derivable solution model. Considering the non-derivable state of the DED-VPE model, conventional mathematical modeling-based approaches, so represented as solution approaches based on derivative, are not proper. To solve this problem, $\left|\sin\left(f_j\left(P_{j,t}-P_j^{min}\right)\right)\right|$, replace with an ancillary variable $s_{j,t}$. So, the cost function certain in Equation (4) can be represented as [2]:

$$\min \sum_{t=1}^{T}\sum_{j=1}^{N}\left(\alpha_j+\beta_j P_{j,t}+\gamma_j P_{j,t}^2+e_j s_{j,t}\right) \tag{10}$$

$$s.t. \quad s_{j,t} \geq \sin\left(f_j\left(P_{j,t}-P_j^{\min}\right)\right) \tag{11}$$

$$s_{j,t} \geq -\sin\left(f_j\left(P_{j,t}-P_j^{\min}\right)\right), \forall j,t \tag{12}$$

By considering some slack variables $0 \leq u_{j,t}^0$ and $0 \leq v_{j,t}^0$, the inequality limitations given in Equations (11) and (12) is converted to power balance limitations [2]:

$$s_{j,t}-\sin\left(f_j\left(P_{j,t}-P_j^{\min}\right)\right)-u_{j,t}^0=0 \tag{13}$$

$$s_{j,t}+\sin\left(f_j\left(P_{j,t}-P_j^{\min}\right)\right)-v_{j,t}^0=0 \tag{14}$$

From Equations (13) and (14) the following relations can be obtained [2]:

$$2s_{j,t}-u_{j,t}^0-v_{j,t}^0=0 \tag{15}$$

$$-2\sin\left(f_j\left(P_{j,t}-P_j^{\min}\right)\right)-u_{j,t}^0+v_{j,t}^0=0 \tag{16}$$

So, we have [2]:

$$s_{j,t}-\frac{u_{j,t}^0}{2}-\frac{v_{j,t}^0}{2}=0 \tag{17}$$

$$\sin\left(f_j\left(P_{j,t}-P_j^{\min}\right)\right)+\frac{u_{j,t}^0}{2}-\frac{v_{j,t}^0}{2}=0$$

Let;

$$u_{j,t}=\frac{u_{j,t}^0}{2} \text{ and } v_{j,t}=\frac{v_{j,t}^0}{2} \tag{18}$$

So we can write

$$s_{j,t}-u_{j,t}-v_{j,t}=0 \tag{19}$$

$$\sin\left(f_j\left(P_{j,t}-P_j^{\min}\right)\right)+u_{j,t}-v_{j,t}=0 \tag{20}$$

$$u_{j,t}\geq 0\ ,\ v_{j,t}\geq 0\ ,\ \forall j,t \tag{21}$$

As a result, the following derivable NLP formulization is earned for the proposed model [2]:

$$\min \sum_{t=1}^{T}\sum_{j=1}^{N}\left(\alpha_j+\beta_j P_{j,t}+\gamma_j P_{j,t}^2+e_i s_{j,t}\right) \\ \text{s.t. } (5)\text{-}(9), (19)\text{-}(21) \tag{22}$$

The execution of the MILP-IPM method can be well represented and IPM is a robust method for obtaining nonlinear optimization models and solving various power system problems successfully [3], such as OPF, state estimation, hydro-thermal coordination, and ED. Therefore, the above NLP formulization (1-22) for DED-VPE can be calculated immediately utilizing IPM. Although the NLP formulization of the DED-VPE model is obtained utilizing IPM at a singular level, based on the non-convex state of the model and several local minimums, this solution easily gets stuck in a weak local optimization. To achieve an optimal answer, a MILP approach is accepted to obtain a proper starting point for IPM. In this approach, transfer losses are not proposed and the proposed model can be formulating as MILP, which is obtained immediately and high performance utilizing the most advanced MIP solvers.

So, the best answer at a set point accuracy, which is utilized as the starting point of IPM to enhance the performance of the final ED result, which can be solved for achieve optimal solution by a numerical method. MILP formulization for the proposed model to earn the MILP formulization of DED-VPE model, breakpoints L_j+1 are selected over a production period $\left[P_j^{\min}, P_j^{\max}\right]$, include $P_j^{\min}=a_{0,j}\leq a_{1,j}\leq\ldots\leq a_{L_j,j}=P_j^{\max}$. Segment variables $P_{l,j,t}$ and binary variables $z_{l,j,t}$ $\left(l=1,\ldots,L_i\right)$ are considered such that $P_{m,j,t}=P_{j,t}$ and $P_{l,j,t}=0$ $l\neq 0$ while $P_{j,t}$ lies in segment $\left(m\in\left\{1,\ldots,L_j\right\}\right)$. Then, the production cost $c_j\left(P_{j,t}\right)$ is earned as [2].

$$\tilde{c}_j\left(P_{l,j,t}\right)\sum_{j=1}^{L_i}\left(k_{l,j}P_{l,j,t}+b_{l,j}z_{l,j,t}\right) \tag{23}$$

By some additional limitations [2].

$$\begin{cases} P_{j,t}=\sum_{l=1}^{L_i}P_{l,j,t} \\ a_{l-1,j}z_{l,j,t}\leq P_{l,j,t}\leq a_{l,j}z_{l,j,t} \\ \sum_{j=1}^{L_i}z_{l,j,t}=1, z_{l,j,t}\in\left\{0,1\right\} \end{cases} \tag{24}$$

while, L_j ، $k_{l,j}$ ،j and $b_{l,j}$ are solved as following [2]:

$$\begin{cases} L_j=ceil\left(M\,\dfrac{f_j\left(P_j^{\max}-P_j^{\min}\right)}{\pi}\right) \\ k_{l,j}=\dfrac{c_j\left(a_{l,j}\right)-c_j\left(a_{l-1,j}\right)}{a_{l,j}-a_{j-1,j}} \\ b_{l,j}=c_j\left(a_{l,j}\right)-k_{l,j}a_{l,j} \end{cases} \tag{25}$$

Mentioned, ceil (x) represents that x is approximated for the closest number more than or equivalent for x, and M is the amount of parts equivalence to every sin (x), while x pertains for [0, π]. As a result, transfer power losses are neglected, the following MILP formula is derived from the DED-VPE model [2]:

$$\min \sum_{t=1}^{T}\sum_{j=1}^{N}\tilde{c}_j\left(P_{l,j,t}\right) \\ \text{s.t. } (5), (7)-(9), (24) \tag{26}$$

where, in [4] $P_i^{loss}=0$. The non-convex and non-derivable properties that construct the proposed model unsolvable are created with VPE.

In the event that $c_j^{vpe}\left(P_{j,t}\right)$ is linearized separately, a MIQP formulization for the DED-VPE model is obtained. Although the DED-VPE model is obtained directly utilizing the MIQP method.

Total generation cost $c_j\left(P_{j,t}\right)$, segmented, and the MILP formula provided in (26), which can be directly and efficiently solved using an advanced MIP solver solved. This is usually due to the MILP method is more efficient because heuristics MILP techniques are better developed than nonlinear ones. Also, compared to the MIQP formulization, the MILP formula can provide a proper estimation of the objective function. While the same breakpoints are set. We inform that the second derivative is a function of the expense of power certain in Equation (3) on the interval $\left(\left(P_j^{min}+\frac{k\pi}{f_j}\right),\left(P_j^{min}+\frac{(k+1)\pi}{f_j}\right)\right)$ [2]:

$$c''_j\left(P_{j,t}\right)=2\gamma j-e_j f_j^2\left|\sin\left(f_j\left(P_{j,t}-P_j^{\min}\right)\right)\right| \quad (27)$$

In (27), $\left|\sin\left(f_j\left(P_{j,t}-P_j^{min}\right)\right)\right|>0$, γ_i can be a small positive actual number and $e_j f_j^2$ can be extremely bigger than γ_i. It can be just a little area (about 0.04-0.2) in each terminal of the time period $\left(\left(P_j^{min}+\frac{k\pi}{f_j}\right),\left(P_j^{min}+\frac{(k+1)\pi}{f_j}\right)\right)$, where, $c''_j\left(P_{j,t}\right)$ is slightly greater than 0, while for the remainder of the time period, $c''_j\left(P_{j,t}\right)<0$ [5]. Thus, any period among two neighbor valve points is considered concave. For each $P_{l,j,t}\in\left(a_{l-1,j},a_{l,j}\right)$, $P_{l,j,t}=P_{j,t}$ we have [2]:

$$\begin{aligned}
&\tilde{c}_j\left(P_{l,j,t}\right)=k_{l,j}P_{l,j,t}+b_{l,j}=k_{l,j}\left(P_{l,j,t}-a_{l,j}\right)+c_j\left(a_{l,j}\right)=\\
&=\frac{c_j\left(a_{l,j}\right)-c_j\left(a_{l-1,}\right)}{c_i\left(a_{l,i}\right)-c_i\left(a_{l-1,i}\right)}\left(P_{l,j,t}-a_{l,j}\right)+c_j\left(a_{l,j}\right)=\\
&=\frac{c_j^{quad}\left(a_{l,j}\right)-c_j^{quad}\left(a_{l-1,j}\right)}{c_j\left(a_{l,j}\right)-c_j\left(a_{l-1,j}\right)}\left(P_{l,j,t}-a_{l,j}\right)+c_j^{quad}\left(a_{l,j}\right)+\\
&+\frac{c_j^{vpe}\left(a_{l,j}\right)-c_j^{vpe}\left(a_{l-1,j}\right)}{c_j\left(a_{l,j}\right)-c_j\left(a_{l-1,j}\right)}\left(P_{l,j,t}-a_{l,j}\right)+c_j^{vpe}\left(a_{l,j}\right)= \quad (28)\\
&=\frac{\left(P_{l,j,t}-a_{l,j}\right)}{a_{l,j}-a_{l-1,j}}c_j^{quad}\left(a_{l,j}\right)+\frac{\left(a_{l,j}-P_{l,j,t}\right)}{a_{l,j}-a_{l-1,j}}c_j^{quad}\left(a_{l-1,j}\right)+\\
&+c_j^{quad}\left(a_{l,j}\right)+\tilde{c}_j^{vpe}\left(P_{l,j,t}\right)=\\
&=\alpha c_j^{quad}\left(a_{l,j}\right)+\beta c_j^{quad}\left(a_{l,j}\right)+\tilde{c}_j^{vpe}\left(P_{l,j,t}\right)
\end{aligned}$$

while

$$\tilde{c}_j^{vpe}\left(P_{l,j,t}\right)=\frac{c_j^{vpe}\left(a_{l,j}\right)-c_j^{vpe}\left(a_{l-1,j}\right)}{c_j\left(a_{l,j}\right)-c_j\left(a_{l-1,j}\right)}\times\left(P_{l,j,t}-a_{l,j}\right)+c_j^{vpe}\left(a_{l,j}\right) \quad (29)$$

The linear formulation $c_j^{vpe}\left(P_{l,j}\right)$ is in the period $a_{l-1,i}-a_{l,i}$ and [2]:

$$\begin{cases}\alpha=\dfrac{P_{l,j,t}-a_{l-1,j}}{a_{l,j}-a_{l-1,j}}\\ \beta=\dfrac{a_{l-1,j}-P_{l,j,t}}{a_{l,j}-a_{l-1,j}}\\ \alpha+\beta=1,\alpha>0,\beta>0\end{cases} \quad (30)$$

For the exact convexity of $c_j^{quad}\left(P_{l,j}\right)$, it can be earned [2]:

$$c_j\left(P_{l,j}\right)\geq\tilde{c}_j\left(P_{l,j,t}\right)>c_j^{quad}\left(P_{l,j}\right)+\tilde{c}_j^{vpe}\left(P_{l,j,t}\right) \quad (31)$$

while, "≥" shows that "⩾" holds approximately each where. Therefore, toward to the MIQP formulization, the MILP formulization is provided a good estimation of the production objective function. The MILP-IPM method to solving the proposed model can now be represented as.

Level one: Calculate the MILP formulization given in Equation (26) utilizing the MILP approach to earn an optimal global answer at a predetermined accuracy for DED-VPE without lines power losses.

Level two: Calculate the NLP formula certain in Equation (22) utilizing IPM, while the primary start is equivalent to the answer earned in level one, to find a proper-performance weak answer for DED-VPE by power losses transfer is obtained.

In level one, a MILP formula for DED-VPE without lines power losses is calculated, providing an answer that can be utilized as a proper primary point in level two. By ignoring the losses, the DED-VPE model can be formulated to earn a MILP formulization that is directly and efficiently solved using the most advanced MIP solvers. Therefore, through the enumeration algorithm, an optimal global answer can be achieved at a presented tolerance. This leads the solved answer in level one is optimal solution. While lines power losses are proposed to be several of the limitations of the lines power losses will be contributed to the main problem. So, more products can be required to achieve the power equality relations. In a DED model, lines power losses in each interval are small toward to the relevant demand. Thus, due to the "the most" economic answer solved in level one, the output of some power plants is adjusted via IPM in level two to satisfy the novel limitations, so a high-performance answer can be earned. Recently, much consideration is paid to the model of DED about prohibited operating regions. The considered MILP-IPM method can also be solved to help the model of DED-POZs due to the relevant MILP problem and the NLP problem.

3. LOGICALLY CONSTRAINED OPF SOLVER-BASED MINLP APPROACH

Logical limitations, which are a special type of discrete or numeral limitations include logical, negation, and conditional terms, are proposed to be the state of feasible solution models, and operational power networks are no exception. However, there are logical limitations to most power system control models, these limitations are often overlooked due to the simplicity of operation and computational transaction capability due to the disjoint functioning regions adjacent to the production power plants. This may make it easier to find an optimal answer. Although, an exact modeling must meet entire of operating limitations. In the event that it may result to an answer with an unfavorable result. Therefore, a proper method or tradeoff, between method tolerance and computational performance, should be checked to neutralize the mentioned disadvantage. The ACOPF, even in theoretical studies, because of the limitation of active and reactive load flow, and due to logical limitations converts to a harder of a nonconvex-nonlinear model. Hence, to obtain an action problem, the influence of the valve must be considered. In addition, due to the VAr shunt compensator and in particular, the thyristor-controlled series capacitor (TCSC) and thyristor-controlled phase shifter (TCPS) with improved efficiency, voltage fluctuations, and loading, action as an important task in model operating and planning.

Combining such devices with high nonlinear properties in addition to integer variables with logical limitations leads to a complex NLP model of complex integers. Consideration of logical limitations and flexible AC transmission systems (FACTS) leads to a significant increment in computational complexity. This increment in computational complexity can be one of the main motivations for the widespread use of heuristic-based methods to obtain OPF-based operational models. In [19], the most used Evolutionary Computation based models and their applications on OPF are reviewed and also some unused Evolutionary Computation based models for OPF are also presented. In [20], tried for demonstrate reactive power optimization model, miscellaneous targets, voltage stability indexes types and formulization of them, reviewing recent studies in this filed and comparison between them for studding efficiency of them.

These methods may work well in solving optimal answers for special models or methods. Although achieving a proper answer for other models and methods, while logical limitations are considered, can need fundamental changes. Hence, the most successful method, to calculate OPF models, while utilized to OPF models by logical limitations and FACTS devices, their reliability is seriously questioned. This indicates the need to provide a reliable method for logical constrained models. Until it can be no solver-based approach for logically constrained models ACOPF with or without VAr compensators. However, for the logical limitation ED(LCED) model, while is a simple OPF subject, multiple solver-based methods are considered, and later in [6], the authors have enhanced the MIQP method considering a Big-M due to MIQP method and an unambiguous distance-based MIQP method, respectively. These methods can achieve the optimum ED global answer. Although, their inability to deal by non-smooth and nonlinear statements expressions still is a refutable flaw that inhibits their use in LC-ACOPF subjects. To consider nonlinear expressions, include transmission losses, a new development has been introduced, first in [7] and later in [8]. Such a development may pose significant problems for commercial solvers.

However, to solve this model, in [7], a semidefinite method and in [8], a decomposition method is utilized. However, the above methods are not able to solve the constrained practical models, but by representing the importance of the solved models, they have created new insights in this field of research. Even in some available linear problems for ACOPF models, due to linear complexity, while is highly dependent on estimation methods and logical limitations are ignored. Therefore, the main motivations for providing MINLP subject-based methods that may fill the available research gap may be represented as follows: (a) the popularity and proper results of methods based on solver in other regions; and (b) the deficiency of a proper answer-based method for LCOPF-based subjects.

In [9], there are many shortcomings in logically constrained OPF solver-based (LCOPF). However, in the literature, heuristic-based methods are utilized to solve LCOPF subjects by logical items include conditional expressions, logical-and, logical-or, etc., they require multiple examinations and regulations to find a reliable a reliable answer. Hence, based on the rapidity and precision in achieving a proper answer, a solver-based method is very important in practice. To remedy this shortcoming, we present a solution method in this section to modify the logical constraints to solver-based MINLP terms. In particular, the review of logical limitations in terms of cost function has been considered, thus facilitating methods of pre-solving and exploring commercial solvers. This results in greater computational performance. Using this modified approach, two sub power and sub function based MINLP methods, namely SPMINLP and SF-MINLP, are presented, respectively. The solutions not only represent the performance of the considered methods in achieving a good optimum answer.

To demonstrate the widespread usage of considered method, three mathematical formulas related to three different reconfigurations. The OPF subject is regulated as follows by considering separate operational areas as logical constraints [9].

$$\min \sum_{j\in\Omega_g} F_j\left(P_{gj}\right) \tag{32}$$

s.t.

$$P_{g_j} - P_{D_j} + g_j^{sh} v_j^2 - \sum_{ij\in\Omega_l} p_{ij}^d - \sum_{ji\in\Omega_l} p_{ji}^r = 0,\ j\in\Omega_b \tag{33}$$

$$Q_{g_j} - Q_{D_j} + b_j^{sh} v_j^2 - \sum_{ij\in\Omega_l} q_{ij}^d - \sum_{ji\in\Omega_l} q_{ji}^r = 0,\ j\in\Omega_b \tag{34}$$

$$\begin{cases} \underline{P}_{g_j} = \underline{P}_{g_{j1}} \le P_{g_j} \le \overline{P}_{g_{j1}}, or \\ \underline{P}_{g_{ik}} \le P_{g_i} \le \overline{P}_{g_{ik}}, \forall 2 \le k \le \left(z_j - 1\right), or \\ \underline{P}g_{j_{z_i}} \le P_{g_j} \le \overline{P}_{j_{z_i}} = \overline{P}_{g_j} \end{cases} \tag{35}$$

$$\underline{Q}_{g\,j} \le Q_{g\,j} \le \overline{Q}_{g\,j},\ j\in\Omega_g \tag{36}$$

$$\underline{v}_j \le v_j \le \overline{v}_j,\ j\in\Omega_b \tag{37}$$

$$\left| fl_{ij}\left(v,\theta,tp\right)\right| \le \overline{fl}_{ij},\ ij\in\Omega_l \tag{38}$$

where, $F_i(.)$ is further approximated by a second-class function such as Equation (39) [9].

$$F_j\left(P_{g_j}\right)=a_j\left(P_{g_j}\right)^2+b_jP_{g_j}+c_j \quad (39)$$

However, multiple valves lead to the ripples and therefore it is inevitable to meet the valve point influences on the cost function. The point influence of the valve is modeled as a modified sine expression as in Equation (40) [9].

$$F_j\left(P_{g_j}\right)=a_j\left(P_{g_j}\right)^2+b_jP_{g_j}+c_j+\left|e_j\sin\left(f_j\left(\underline{P}_{g_j}-P_{g_j}\right)\right)\right| \quad (40)$$

The active and reactive output balance limitations are described in Equations (33) and (34), respectively. A set of Equation (35) of separate operational zones are available. It set starts the subject of logical limitations for the OPF subject. Reactive power generation and bus voltage constrains are indicated with Equations (36) and (37), respectively. In Equation (38), the power flow of branches, f_{ij} can be represented by Equation (41), or in several texts just the active load flow of branches is considered [9].

$$fl_{ij/ji}=\sqrt{\left(p_{ij/ji}^{d/r}\right)^2+\left(q_{ij/ji}^{d/r}\right)^2} \quad (41)$$

In this method, LTCT and VAR shunt compensator are proposed in LCOPF. With combining LTCT systems in the output network, the forward and backward flows are proposed as following Equations (42)-(45) [9].

$$p_{ji}^d=\left(tp_{ij}v_j\right)^2g_{ij}-\left(tp_{ij}v_j\right)v_i\left[g_{ji}\cos\left(\theta_{ji}\right)-b_{ji}\sin\left(\theta_{ji}\right)\right] \quad (42)$$

$$P_{ji}^r=v_j^2g_{ji}-\left(tp_{ji}v_i\right)v_j\left[g_{ji}\cos\left(\theta_{ji}\right)-b_{ji}\sin\left(\theta_{ji}\right)\right] \quad (43)$$

$$q_{ji}^d=-\left(tp_{ij}v_j\right)^2\left(b_{ij}+\frac{b_{ij}^{ch}}{2}\right)- -v_jv_i\left[g_{ji}\sin\left(\theta_{ji}\right)-b_{ji}\cos\left(\theta_{ji}\right)\right] \quad (44)$$

$$q_{ji}^r=-v_j^2\left(b_{ji}+\frac{b_{ji}^{ch}}{2}\right)+ +\left(tp_{ji}v_j\right)v_i\left[g_{ji}\sin\left(\theta_{ji}\right)+b_{ji}\cos\left(\theta_{ji}\right)\right] \quad (45)$$

where, each tap must meet the high and low limits, as in Equation (46) [9].

$$\underline{tp}_{ij}\le tp_{ij}\le\overline{tp}_{ij} \quad (46)$$

To combine the compensating influences of the shunt VAR, as shown, the reactive equality limitation Equation (34) is corrected as in Equation (47) [9].

$$Q_{gj}+Q_{Cj}-Q_{Dj}+b_j^{sh}v_j^2-\sum\nolimits_{ij\in\Omega_l}q_{ij}^d- -\sum\nolimits_{ji\in\Omega_l}q_{ji}^r=0,\quad j\in\Omega_b \quad (47)$$

while, every VAR shunt compensator has its own constraints, as in Equation (48) [9].

$$\underline{Q}_{Cj}\le Q_{Cj}\le\overline{Q}_{Cj} \quad (48)$$

LTCTs and VAR shunt compensators, on the other hand, are discrete controllers with defined step sizes. The following devices are represented utilizing integer decision-making variables [9].

$$tp_{ij}=\underline{tp}_{ij}+n_{ij}\tau_{ij},\quad \forall\ n_{ij}\in Z\ge 0 \quad (49)$$

$$Q_{Cj}=\underline{Q}_{Cj}+n_{ij}\tau_{ij},\quad \forall\ n_{ij}\in Z\ge 0 \quad (50)$$

In this method, the influences of special FACTS devices consist TCSC and TCPS is considered. Due to TCPS, which is represented by a phase shift transformer by a decision-making variable φ_{ij}, the forward and backward reactive powers are described as following, Equation (51)-(54) [9].

$$P_{ji}^d=\frac{v_i^2g_{ji}}{\cos^2\phi_{ji}}-\frac{v_jv_i}{\cos\phi_{ji}} \left[g_{ji}\cos\left(\theta_{ji}+\phi_{ji}\right)+b_{ji}\sin\left(\theta_{ji}+\phi_{ji}\right)\right] \quad (51)$$

$$P_{ji}^r=v_i^2g_{ji}-\frac{v_jv_i}{\cos\phi_{ji}}\times \times\left[g_{ji}\cos\left(\theta_{ji}+\phi_{ji}\right)-b_{ji}\sin\left(\theta_{ji}+\phi_{ji}\right)\right] \quad (52)$$

$$q_{ji}^d=-\frac{v_i^2}{\cos^2\phi_{ji}}\left(b_{ji}+\frac{b_{ji}^{ch}}{2}\right)-\frac{v_jv_i}{\cos\phi_{ji}} \left[g_{ji}\sin\left(\theta_{ji}+\phi_{ji}\right)-b_{ji}\cos\left(\theta_{ji}+\phi_{ji}\right)\right] \quad (53)$$

$$q_{ji}^r=-v_i^2\left(b_{ji}+\frac{b_{ji}^{ch}}{2}\right)+\frac{v_jv_i}{\cos\phi_{ji}} \left[g_{ji}\sin\left(\theta_{ji}+\phi_{ji}\right)+b_{ji}\cos\left(\theta_{ji}+\phi_{ji}\right)\right] \quad (54)$$

Considering TCSC with the control variable x_{ij}^c, the forward and backward active and reactive load flow are represented as following, Equations (55)-(58), respectively, considering Equations (59) and (60) [9].

$$P_{ji}^d=v_j^2g_{ji}-v_jv_i\left[g_{ji}\cos\left(\theta_{ji}\right)+b_{ji}\sin\left(\theta_{ji}\right)\right] \quad (55)$$

$$P_{ji}^r=v_i^2g_{ji}-v_jv_i\left[g_{ji}\cos\left(\theta_{ji}\right)-b_{ji}\sin\left(\theta_{ji}\right)\right] \quad (56)$$

$$q_{ji}^d=-v_j^2\left(b_{ji}+\frac{b_{ji}^{ch}}{2}\right)-v_jv_i\left[g_{ji}\sin\left(\theta_{ji}\right)-b_{ji}\cos\left(\theta_{ji}\right)\right] \quad (57)$$

The conductance, g_{ij}, and susceptance, b_{ij}, of lines with TCSC are solved as following.

$$g_{ji}=\frac{r_{ji}}{r_{ji}^2+\left(x_{ji}-x_{ji}^c\right)^2} \quad (58)$$

$$b_{ji}=-\frac{r_{ji}-x_{ji}^c}{r_{ji}^2+\left(x_{ji}-x_{ji}^c\right)^2} \quad (59)$$

Therefore, considering the above expressions and also considering the following two limitations, Equations (60) and (61), the LCOPF method is obtained with FACTS, TCSC and TCPS devices.

$$x_{ji}^c\le x_{ji}^c\le\overline{x}_{ji}^c,\ ji\in\Omega_l \quad (60)$$

$$\underline{\phi}_{ji}\le\phi_{ji}\le\overline{\phi}_{ji},\ ji\in\Omega_l \quad (61)$$

Because available commercial NLP solvers are not able to solve logically constrained subjects, logical limitations must be recreating to solver-friendly statements, and made possible by MIP formulizations. Hence, because of the highly nonlinear and non-convex state of LCOPF subjects, available solver-based methods are not suitable for ED models, as these methods create problems in the MINLP solution procedure. In such methods, incrementing the number of variables, with allocating control variables to the maximum and minimum constraints related limitations, is not the alone obstacle that determines that unsuitable to LCOPF-based subjects. Although, the potential lack of available MINLP solvers is another impediment. The original idea of the considered MINLP methods stems from the fact that an MINLP solver exhibits higher efficiency if the control variables are consisted in the cost function. Thus, to handle the above deficiencies, two MINLP methods for LCOPF models are considered in which logical limitations are reconstructed with the objective function term. The margins of every sub-power are represented as Equation (62).

$$\underline{P}_{jk} \le P_{jk} \le \overline{P}_{jk} : \forall j \in \Omega_g, k \in \{1,\ldots,z_i\} \quad (62)$$

Hence, to determine one sub-power area, the binary control variables u_{jk} such as Equations (63) and (64) are assigned to these regions. Constraint in Equation 64 ensures that alone one of the control variables is determined to one, and as a result, alone one of the sub-power areas is determined for obtain the product of power plant *i* in Equation (63).

$$P_{gj} = P_{j1}u_{j1} + P_{j2}u_{j2} + \cdots + P_{jz_{j-1}}u_{jz_{j-1}} + \\ +P_{jz_j}u_{jz_j} = \sum_{k=1}^{z_j} P_{jk}u_{jk} \quad (63)$$

$$\sum_{k=1}^{zj} u_{jk} = 1, \forall u_{jk} \in \{0,1\} \quad (64)$$

By placing in Equation (63) in the objective function, Equations (39) or (40), a sub-power-based MINLP method is solved.

$$F_j\left(P_{gj}\right) = a_j\left(\sum\nolimits_{k=1}^{zj} P_{jk}u_{jk}\right)^2 + b_j\left(\sum\nolimits_{k=1}^{z_j} j_{ik}u_{jk}\right) + c_j \quad (65)$$

$$F_j\left(P_{gj}\right) = a_j\left(\sum\nolimits_{k=1}^{z_j} P_{jk}u_{jk}\right)^2 + b_j\left(\sum\nolimits_{k=1}^{z_j} P_{jk}u_{jk}\right) + \\ c_j + \left|e_j \times \sin\left(f_j \times \left(\underline{P_{gj}} - \left(\sum\nolimits_{k=1}^{zj} P_{jk}u_{jk}\right)\right)\right)\right| \quad (66)$$

Control variables are consisted in the cost function, while determines that compatible by MINLP pre-answer procedures and commercial solver. Thus, the SP-MINLP model is obtained with correcting the mathematical formulizations. These expressions (a) replace P_{gi} in Equations (32) and (33) with Equation (63) and (b) considering Equations (62) and (64). By assigning an objective function to each separate operating zone, another MINLP method can be solved. The objective function in Equation (39) of power plant *i* is determined as a goal due to a logical sub-function in Equation (67) [9].

$$\begin{cases} F_{j1}\left(P_{j1}\right) = a_j P_{i1}^2 + b_1 P_{j1} + c_j \\ F_{jk}\left(j_{ik}\right) = a_j P_{jk}^2 + b_1 P_{jk} + c_j \ , \quad \forall 2 \le k \le \left(z_j - 1\right) \\ F_{jz_i}\left(P_{jz_i}\right) = a_j P_{jz_i}^2 + b_1 P_{jz_i} + c_j \end{cases} \quad (67)$$

And the objective function due to the point effects of valve in Equation (40), after the changes are shown by Equation (68) [9].

$$\begin{aligned} &F_{j1}\left(P_{j1}\right) = a_j P_{j1}^2 + b_1 P_{j1} + c_j + \left|e_j \times \sin\left(f_j \times \left(\underline{P_j} - P_{j1}\right)\right)\right| \\ &F_{jk}\left(P_{jk}\right) = a_j P_{jk}^2 + b_1 P_{jk} + c_j + \left|e_j \times \sin\left(f_j \times \left(\underline{P_j} - P_{jk}\right)\right)\right| \\ &\forall 2 \le k \le \left(z_j - 1\right) \\ &F_{jz_i}\left(P_{jz_j}\right) = a_j P_{jz_j}^2 + b_1 P_{jz_j} + c_j + \left|e_j \times \sin\left(f_j \times \left(\underline{P_j} - P_{jz}\right)\right)\right| \end{aligned} \quad (68)$$

By modifying the above logical sub-functions, Equations (67) and (68), to an equal MINLP method, the sub-function-based MINLP (SFMINLP) to solve LCOPF models is obtained as follows [9].

$$\min \sum_{j \in \Omega_g} \sum_{k=1}^{z_i} \left(F_{jk}\left(P_{jk}\right)\right) u_{jk} \quad (69)$$

To ensure that alone one of the control variables u_{ik} may be equivalence to one, Equation (69) is considered, and as a result, alone one of sub-functions is chosen [9].

$$\sum_{k=1}^{zj} u_{jk} = 1, \forall u_{jk} \in \{0,1\} \quad (70)$$

Although, to solve a MIP model, another definition such as Equation (71) is required, while P_{gj} in Equations (1)-(33) should be changed with Equation (72) [9].

$$\underline{P}_{jk} \le P_{jk} \le \overline{P}_{jk} : \forall j \in \Omega_g, k \in \{1,\ldots,z_j\} \quad (71)$$

$$P_{gj} = \sum_{K=1}^{z_i} P_{jk}u_{jk} \quad (72)$$

Therefore, by creating some expressions in the proposed mathematical formulations, the SF-MINLP method is earned. These expressions include: (a) substituting the cost function in Equation (32) with a goal based on the sub-function in Equations (67) or (68) (b) placing Equation (72) in Equation (33) and (c) considering Equations (70) and (71).

4. APPROXIMATED MILP FORMULATIONS FOR UC PROBLEMS

UC of deciding which producing power plants should be committed/decommitted in a scheduling interval. The generation amount at which the power plants must act must also be defined to minimize the specific objective function. The committed power plants usually have to earn the anticipated demand and reserve limits of the system as well as the set of technical limitations. This model is of great practical importance due to the performance of the programs earned has a great economic effect on power production utilities. For this reason, and due to the high intricacy of the problem, much research has been done.

Even after years of intense research, it is like a challenging research subject. The considered optimization methods for UC involve very different patterns. These are very detailed, from precise methods and Lagrangian relaxation. In the past, the combined state of the model and its period multiple properties are inhibited methods from succeeding: they have led to severely weak methods that were only able to solve small subjects of the model and have virtually no practical interest in them. Heuristic methods, include those due to preference lists, have also not been entirely successful because they frequently result in poor performance answers. Metaheuristics techniques have very promising results when reviewed. The performance of their results was better than the results obtained using completely consistent methods, and proper answers are earned very rapidly. Although, some difficulties can be created when utilizing metaheuristic techniques.

The second problem is the shortage of data that metaheuristic techniques create in terms of answer performance. Several suggestions are made to fix these bugs. But it is still an open line of research. An open subject about the optimization of the solution is made by an independent power plant centralized commitment system operator. Only if the optimal models are solved can the power plants be guaranteed to obtain optimal ED. Thus, the schedule and improvement of optimization methods that create favorable solutions for UC subjects are of fundamental significance.

The performance increment of MIP solvers encourages the use of these solvers. Some research has already led to the definition of alternative, more efficient, and concise MILP formulas. In [10], a MIP formula for second-class optimization of UCP is proposed, and also an approach due to a LP formulization is proposed. Instead of proposing a second-class item of the cost function, the piecewise linear estimation of the fuel cost and refreshes that in an iterative procedure. Performance updates due to answers earned in past iterations. The solution method improved in this work has been experimented with in different samples. For each of them, the novel method converges with repetition to the optimal answer.

Different types of modeling alternatives that reflect different subjects, include fuel, multi-zone, and diffusion limitations, are created. Safety limitations and market-based subjects have recently come to the fore. Decentralized production planning has also considered new subjects in this area and in several markets, this model has been decreased to alone-power plant optimization. Although, for some decentralized markets, the conventional model is still very like to centralized markets. The original difference is the objective function, which maximizes overall welfare instead of minimizing production costs. Therefore, the methods used to manage centralized generation will also be efficient in calculating many of the subjects of decentralized market production. In [10], a centralized model of UCP is considered.

Refer to [11] for standard second-class mathematical formulizations. In [12], the model of short-term UC in hydropower production is a large-scale, NLP model that is hard to calculate. The nonlinear cost function of the model can be estimated using piecewise linear functions. Thus, UC can be estimated to a MILP [14-17]. Using an efficient MILP solver in the resulting formulizations, proper performance answers can be solved in a relatively low time. A method for approximating the non-linear function of the "perspective cuts" is called, is provided. In many cases, a MILP-based heuristic achieves analogous or slightly better answers in less time when using a novel method instead of standard piecewise linear. Also, "dynamic" formulas, in which the estimation is repeatedly improved, give even better solutions if the estimation is properly controlled.

5. MINLP MODELING

The UC objective function represents the entire power generation cost for minimization as follows [12]:

$$\sum_{j\in P} c^j\left(p^j,u^j\right)=\sum_{j\in P} s^j\left(u^j\right)+ +\sum_{t\in T}\left(a_t^j\left(\left(p_t^j\right)^2+b_t^j p_t^j+c_t^j u_t^j\right)\right) \tag{73}$$

The power generation cost is typically expressed with a convex $(a_t^j>0)$ second-class form severable in power variables. Fixed generation costs are indicated by the $c_t^i u_t^i$. UC constraints are divided to three categories: local limitations for thermal power plants, local limitations for hydro power plants, and global limitations.

• Local limitations for thermal power plants: for every $i\in P$ [12].

$$\overline{p}_{\min}^j u_t^j \le p_t^j \le \overline{p}_{\max}^j u_t^j \tag{74}$$

$$p_t^j \le p_{t-1}^j + u_{t-1}^j \Delta^j + \left(1-u_{t-1}^j\right)\bar{l}^j \tag{75}$$

$$p_{t-1}^j \le p_{t-1}^j + u_{t-1}^j \Delta_-^j + \left(1-u_t^j\right)\bar{u}^j \quad t\in \mathrm{T} \tag{76}$$

$$u_t^j \ge u_r^j - u_{r-1}^j \quad t\in \mathrm{T}, r\in\left[t-\tau_+^j, t-1\right] \tag{77}$$

$$u_t^j \ge u_r^j - u_{r-1}^j \quad t\in \mathrm{T}, r\in\left[t-\tau_-^j, t-1\right] \tag{78}$$

$$u_t^j \in \{0,1\} \quad t\in \mathrm{T} \tag{79}$$

The constant τ_+^j indicates that in order to avoid extreme mechanical stresses based on start-up / shutdown methods which in the long run spoil the condition of the power plant, several intervals after the start-up interval should remain online. Similarly, τ_-^i shows how many intervals after the *i* interval of the off period should stay offline. The interval "0" is utilized to show the primary condition of the power system. Knowledge of the complete status consider of each power plant before starting the current operation, i.e., UC u_0^j and its produced power u_0^i. In order to limit the minimum on / off time in Equations (77) and (78) and also to calculate time-dependent startup costs, that is important for recognize how long every power plant is on or off before the 0 period. have been.

• Local limitations for hydro cascade power plant: for each $h\in H$ and $i\in H(h)$ [12].

$$0 \le q_t^i \le \overline{q}_{\max}^i \quad t \in \mathrm{T} \tag{80}$$

$$\overline{v}_{\min}^i \le v_t^i \le \overline{v}_{\max}^i \quad t \in \mathrm{T} \tag{81}$$

$$v_t^i - v_{t-1}^i = \overline{w}_t^i - w_t^i - q_t^i + \sum_{k \in \beta(i)} \left(q_{t-t_{ki}}^k + w_{t-t_{ki}}^k \right) \quad t \in \mathrm{T} \tag{82}$$

To the balance in Equation (82) to be well established, we consider that the science of the capacity of every storage in the interval $t = 0$ and also water discharged and spilled in entire previous intervals $t = 1$.

• Global limitations: the system-wide limitations connecting the various power plants during themselves are [12].

$$\sum_{j \in p} p_t^j + \sum_{h \in H} \sum_{i \in H(h)} \alpha^j q_t^j = \overline{d}_t \quad t \in T \tag{83}$$

The power to discharged water performance is considered fixed, for prohibit nonlinearities. This is a large-scale MINLP that exists in the cost function. This formulization is "basic" and in some respects, for example, related to the modeling of hydro power plants is less accurate than several previous forms. We also model shutdown and start-up operation to limit the output of thermal power plants to any specified amount (more than or equivalence $\overline{p}_{min}^j$) in the first hour and last hour of operation.

Our choice to simplify the assumptions of equilibrium between the original sights of UC process models and the easily of the method seems to be generally allowed in the literature. For example, rotating reserve limitations, or in the "standard" formulization can easily be included in the formulation, but they are not used. Especially, more complex methods of hydro cascades, for example, the NLP influences and / or non-zero technical minimums for discharged water, is used more in integer variables in formulizations. Similarly, the valve points of thermal power plants or the cavity points of water power plants can be easily scheduled.

Because the considered method is autonomous of entire these details, it can be easily utilized in this formulization and many other UC formulizations. The UC model considered here, however, has historically had a common centralized decision-making environment in the past. But the free market is suitable for use in today's market. Both at the step while GenCos require to minimize their generation plan once to their demand curve by market generations and in the methods of calculating the optimal bidding strategies.

To create UC tractable with efficient MILP solvents, the nonlinear term of the cost function needs to be linearized. Because the nonlinear nature is the same for every interval and thermal power plant, for symbolic simple ness in this part, we assume indicators and constants fixed and reduction them. The problem is to best express the second-class cost function [12]:

$$f(p,u) = ap^2 + bp + cu \tag{84}$$

while, k + 1 points $\overline{p}^0, \overline{p}^1, \dots, \overline{p}^k$ are selected in the distance $[\overline{p}_{\min}, \overline{p}_{\max}]$ and convex (for example $\overline{p}^0 = \overline{p}_{\min}$ and $\overline{p}^k = \overline{p}_{\max}$). This solutions in a MILP that differs from UC only by the following details (for each i and t) (assume $f(p) = f(p,1) = ap^2 + bp + c$).

• k new variables δ_l are considered together by limitations [12].

$$p = \sum_{l=1} \delta_l + \overline{p}_{\min}^u \, 0 \le \delta_l \le \overline{p}^l - \overline{p}^{l-1} l = 1, \dots, k \tag{85}$$

• The cost factor of in the cost function is variated for $f(\overline{p}_{\min})$.

• Every variable δ_l is determined a linear cost F_l indicating the linear function by amount 0 when $\delta_l = 0$ and amount $f(\overline{p}^l) - f(\overline{p}^{l-1})$ until $\delta_l = \overline{p}^l - \overline{p}^{l-1}$, i.e.,

For example $F_l = \dfrac{f(\overline{p}^l) - f(\overline{p}^{l-1})}{\overline{p}^l - \overline{p}^{l-1}} = a(\overline{p}^l - \overline{p}^{l-1}) + b$

the MILP estimation of the second-class function is solved by substituting Equation (84) with the following equation [12]:

$$f(\overline{p}_{\min})u + \sum_{j=1}^{k} F_l \delta_l$$

Subject to the main limitations of the model plus additional limitations Equation (85). It will offer for that approximate MILP formula from UC like the criterion piece-wise formulization. There are several options to linearize. For example, this is simple to create a low estimation that is quite simple in both functions and derived amount at points "in the middle" of time periods.

The top and bottom estimations, both created in this method that operate only in p space. So, that shown in space (p,u), as seen in

Although past linearization is perfectly normal, it is certainly not the best feasible estimation for the UC cost function. In fact, a different probability has been suggested in [13]. Random selection of k points $\overline{p}^1, \dots, \overline{p}^k$ in the period $[\overline{p}_{min}, \overline{p}_{max}]$ is a different method of creating MILP that estimates UC.

• Each term in Equation (84) is omitted from the cost function and changed by a novel variable z. Other statements in the cost function that do not contain p and u, for example, items responding to variable start-up costs, remain intact.

• Limitations of the form [12]

$$z \ge (2a\overline{p} + b)p + (c - a\overline{p}^2)u \tag{86}$$

By $\overline{p} = \overline{p}^h$, $h = 1, \dots, k$ are contributed for the formula, respectively. It offers for the mentioned like the sight-cut (estimate) formulization of UC. It selects can be explaining with a complex theory evaluation that cannot be repeated here due to clarity. Here we will only briefly explain the fundamental structural ideas, to make it clear that the above choice is theoretically preferable to others.

The function $f(p,u)$ is basically only at the places (p,u) of the domain (disconnected) $\mathcal{D} = [0,0] \cup [\overline{p}_{min}, \overline{p}_{max}] \times \{1\}$. Although, standard branch and boundary methods typically calculate the steady relaxation of the proposed formula. While u is considered to get amounts in [0, 1] instead of [0, 1] to obtain minimum limits on the desired amount of the model. So, it creates sense to check. While UC convex relaxation formulization creates the best possible.

If such a question does not fully accept the simple answer to the UC model, but if one person limits oneself to the "basic blocks", one can answer it. Actually, convex envelope of $f(p,u)$ on *D* means convex features [12].

$$h(p,u)=\begin{cases}0 & \text{if} \quad p=0,\ u=0\\ \dfrac{ap^2}{u}+bp+cu & \text{if} \quad u\overline{p}_{\min}\le p\le u\overline{p}_{\max}\\ +\infty & \text{otherwise}\end{cases} \qquad (87)$$

This function in convex assessment, a perspective function $g(p,u)=uf(p/u)$ of $f(p)$, is strongly according to a well-known object. In this drawing $g(p,u)$, a cone is marked with $f(p)$ marked from the main and having a "lower shape", as shown in Figure 1, *epi h* is the part from the cone relating to $u\le 1$. Since $0<u<1$, $h(p,u)\ge f(p,u)$ is immediately confirmed to all $(p,u)\in\mathcal{D}$, so that h can be a proper cost function to steady relaxation from $f(p,u)$. In fact, preliminary calculations show that the maximum $h(p,u)-f(p,u)$ in $\mathcal{D}$, $ap_{\max}^2/4$, which is obtained in $[\overline{p}_{\max}/2,1/2]$. Although, *h*(*p*, *u*) has a serious drawback for use as an objective function: even that can be even a "more nonlinear" function than $f(p,u)$.

➢ Example: Electricity price and demand information are shown in Figures 1 and 2, respectively. The information on the generators is also presented in Table 2. Solve the UC subject first with the MINLP model presented in Section 5. Then solve this problem with the MILP method (piecewise-linear estimations) presented in Section 5 and compare the results.

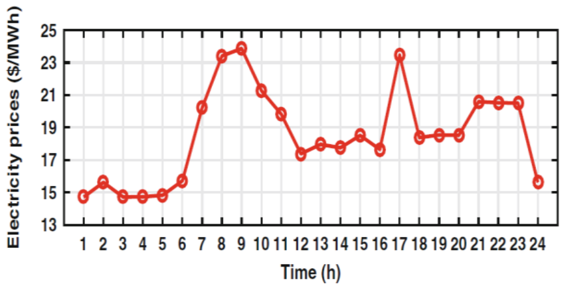

Figure 1. Hourly electricity prices [18]

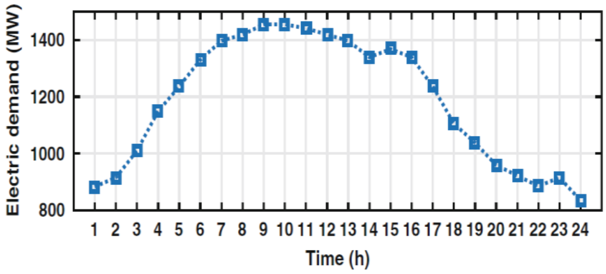

Figure 2. Hourly electric demand [18]

Table 1. Results of solution methods

Solution method	Cost function ($)	Execution time (s)
MINLP	488845	0.16
MILP (5 piece)	483181.786	0.016
MILP (20 piece)	483138.294	0.031
MILP (100 piece)	483123.3761	0.14
MILP (1000 piece)	483122.6487	1.872
MILP (1500 piece)	483122.6460	3.65

Table 2. Generators information and limits [18]

gi	*ai*	*Bi*	*Ci*	Costs *D*	Costs	RU	RD	UT	*DTi*	SD	SU	$P_{gi}^{\min}$	$P_{gi}^{\max}$	*U0i*	*Uini*	*S0i*
1	0..014	12.1	82	42.6	42.6	40	40	3	2	90	110	80	200	1	0	1
2	0.028	12.6	49	50.6	50.6	64	64	4	2	130	140	120	320	2	0	0
3	0.013	13.2	100	57.1	57.1	30	30	3	2	70	80	50	150	3	0	3
4	0.012	13.9	105	47.1	47.9	104	104	5	3	240	250	250	520	1	1	0
5	0.026	13.5	72	56.6	56.9	56	56	4	2	110	130	80	280	1	1	0
6	0.021	15.4	29	141.5	141.5	30	30	3	2	60	80	50	150	0	0	0
7	0.038	14.0	32	113.5	113.5	24	24	3	2	60	80	50	150	0	0	0
8	0.039	13.5	40	42.6	42.6	22	22	3	2	45	55	30	110	0	0	0
9	0.039	15.0	25	50.6	50.6	16	16	0	0	35	45	20	80	0	0	0
10	0.051	14.3	15	57.1	57.1	12	12	0	0	30	40	20	60	0	0	0

There are ten power plants in this system, the goal of which is optimal planning for these ten power plants during 24 hours. In the proposed model, there are limitations regarding the increase and decrease rate, the minimum time of staying on and off, along with the determination of the initial state of the power plants. Also, in addition to the fuel cost of power plants, the cost of start-up and shut-down power plants is also considered in the objective function. Figures 3 and 4 show the generators production planning for the nonlinear and linear models, respectively. Based on the results, no production in the MINLP model occurred in 24 hours. The MILP method also has different answers for different pieces for linearization. To improve the approximation, we need to increase the number of linearization slices. But it should be noted that the increase in the number of pieces increases the execution time. The greater the number of pieces, the closer the answers get to the global optimal value. But a large number of pieces will increase the execution time. Therefore, a tradeoff must be made between the execution time and the quality of the response. In Table 1, the value of the cost function and execution time for different methods is expressed. Based on the solutions, the MILP model has a superior performance from the MINLP model in terms of the cost function amount.

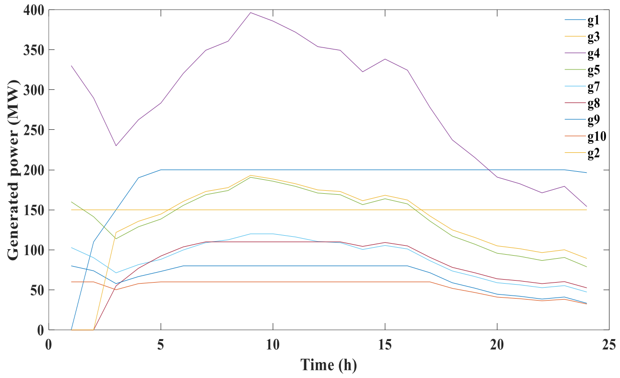

Figure 3. The generators production planning for the MINLP model

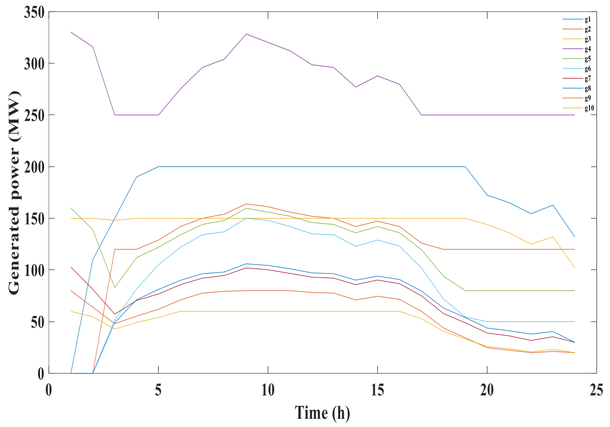

Figure 4. The generators production planning for the MILP model

6. CONCLUSION

In this chapter, modeling and solving methods of three issues of ED, OPF and UC are examined. Heuristic techniques to solve these problems have not been successful because they often result in poor performance results. Metaheuristic techniques yielded very promising results when reviewed. However, some drawbacks can be highlighted when using metaheuristic techniques. The main drawback of metaheuristic techniques is their dependency on parameterization.

Parameter setting is time-consuming and the complicated setting method needs in-depth knowledge of the implemented method. The second problem is the lack of information that meta-heuristic techniques offer in terms of solution quality. But mathematical models based on nonlinear solvers are the second way to implement these problems. But because nonlinear solvers often get stuck in local optimization and also greatly increase the solving time, they are not very efficient in solving such problems. Due to the dramatic increment in the performance of MIP and MILP solvers, linear formulations have been proposed for the above three problems. Instead of proposing a nonlinear expression, the linear method assumes an estimation of these functions.

By increasing the accuracy of the estimation used in solving nonlinear problems, in addition to the optimality of the answers obtained, the solution time is greatly reduced. Also, two MINLP and MILP methods are implemented in Gam's software. The solutions represent the better efficiency of the MILP approach in calculating in terms of cost of the DED problem. But in terms of execution time, the nonlinear model performs better than the linearized model. The superiority of the linear model over the non-linear model is the execution time. Therefore, if the execution time is important for solving the problem, we use the linearized model, and otherwise, we use the non-linear model.

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