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OPTIMIZATION OF MATHEMATICAL MODELS OF MULTIFACTOR WORKING-OUT PROCESSES AND TECHNOLOGICAL PROCESSES BY EXPERIMENTAL-STATISTICAL METHODS

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Abstract- The article discusses the use of experimental-statistical methods in the absence of sufficient information about multifactorial processing operations and technological processes and the impossibility of constructing their mathematical models. The results of the researches show, that conducting experiments with classical methods often increases the probability, that the values of technological factors fall into a false optimum, and their implementation by the method of multifactor planning speeds up the determination of the optimal values of accuracy and quality.

Keywords: Working-Out, Operation, Technology, Process, Factor, Quality, Accuracy, Probability, Planning, Regression, Correlation, Coefficient, Model.

1. INTRODUCTION

With a lack of information about multifactor technological operations and technological processes, when they are rather complex and it is impossible to construct their deterministic models, it is better to use experimental statistical methods [1-7]. In this case, passive and active methods are used for conducting experiments. Passive experiments are fulfilled by changing the significances of technological factors of working-out processes by turn, and in this case the holding of a large number of experiments are required. Passive experiments use pure statistic material collected during processing details of machines and equipment at machinery enterprises. In this case, the processing of the experimental material, used to construct the mathematical models is carried out by applying of classical methods of regression and correlation.

Active experiments are carried out on the basis of a predetermined plan, and it is meant, that all factors of technological operations will change simultaneously, this, in its turn, also, allows predicting the interaction of technological factors and drastically reducing the number of conducted experiments. In the process of working-out, the results of experiments, the main principles of regression and correlation methods are used.

As a result, it is possible to obtain dependencies, which characterize the connection between the parameters of technological operations and technological processes, with accuracy and quality of the surfaces of the processed details. In all cases, the dependencies obtained in the form of mathematical models, characterize the connection between the experimental results and technological factors, specified in a form of variable parameters. Experiments are carried out by varying the values of technological coefficients of technological operations based on the given rules (by alternating) [8-15].

2. RESEARCH METHODS

The connection of technological processing factors from accuracy and quality of the processed details surfaces y is written as follows

$$y = f(X_1, X_2, X_3, ..., X_k) \qquad (1)$$

Free variables $X_1, X_2, X_3, \ldots, X_k$ are called as factors; $X_1, X_2, X_3, \ldots, X_k$ coordinate space-factor space; The geometric description of the reflection function in the factor space is called as the plane of reflection [1, 2]. Using the popular methods, it is possible to construct mathematical models of the dependences of accuracy and quality details surfaces from factors of technological operations.

Researches show, that conducting experiments using traditional methods, i.e.by changing the values of only one factor and keeping constant values of other factors, significantly increases the probability of falling into a false optimum. Applying of the methods of multifactorial planning of experiments, vice versa, accelerates the determination of optimal significances of accuracy and quality of processed details and increase the reliability of obtained results. With using the static methods, the mathematical model is set in the form of a polynomial – a part of Taylor series, where the indefinite dependence is divided into separate parts [1, 2]:

$$y = \beta_0 + \sum_{j=1}^{k}\beta_j X_j + \sum_{\substack{u,j=1 \\ u\neq j}}^{k}\beta_{uj} X_u X_j + \sum\nolimits_{j=1}^{k} B_{jj} X_j^2 + \ldots \quad (2)$$

where, $\beta_j = \frac{\partial\varphi}{\partial X_j}\Big|_{x=0}\rightarrow$, $\beta_{uj} = \frac{\partial^2\varphi}{\partial X_u \partial X_j}\Big|_{x=0}\rightarrow$, $\beta_{uj} = \frac{\partial^2\varphi}{\partial X_j^2}\Big|_{x=0}\rightarrow$.

It is known, that there are quite many variables in technological processes and in mechanical operations of processed machine details, with which is impossible to manage or to control (for example, temperature in the cutting zone, deformation of the technological system due to shear forces in cutting zone, inhomogeneity of materials of processed details, vibrations in the process of working-out and so on) and they really influence on technological factors.

It is clear, that the change in values of technological factors is of a random character and can be called random variables. Therefore, the coefficients, obtained in processing experimental results are called the sample regression coefficients b_o, b_i, b_{uj}, b_{jj}. These coefficients are theoretically called estimates of the coefficients, B_o, B_i, B_{uj}, B_{jj}. Thus, the regression equation obtained as a result of the experiment is written as follows:

$$\hat{y} = b_0 + \sum_{j=1}^{k} b_j X_j + \sum_{j=1}^{k} b_{uj} X_u X_j + \sum_{j=1}^{k} b_{jj} X_j^2 + \ldots \quad (3)$$

The coefficient b_o is called the free mathematical limit of the regression equation, b_i is called the linear effect, the coefficient b_{jj} is called the quadratic effect, and the coefficient b_{uj} is called the interaction coefficient.

The coefficients of the regression Equation (3) by the least squares method, is determined by the condition.

$$\varphi = \sum_{i+1}^{n}(y_{i-}\hat{y}_i)^2 = \min \quad (4)$$

where, N is the volume of samples, taken from the general total of the studied parameters. The difference between the sample size N and the number of compounds l taken for that sample is called the number of degrees of freedom f of the sample.

$$f = N - 1 \quad (5)$$

While searching for regression equations, the number of ratios is equal to the number of determined coefficients.

3. RESEARCH RESULTS AND THEIR DISCUSSION

Table 1, was shown the number of coefficients, which must be determined to obtain various degrees of equations (polynomials) for free factors from 2 to 5 [2]. The Table 1 shows, that the number of necessary determined coefficients increases with an increase of degrees of free factors and with raising degrees of equations.

The type of regression equation is chosen by experimental tests. While analyzing the dependence on one variable parameter, an empirical regression line is set up to determine the type of regression equation (Figure 1).

Table 1. The number of coefficients that need to be determined to obtain equations (polynomials) of different degrees due to 2 to 5 free factors

Number of factors (of independent parameters)	Degrees of equations			
	First degree	Second degree	Third degree	Fourth degree
	Number of coefficients			
2	3	6	10	15
3	4	10	20	35
4	5	15	35	70
5	6	21	56	126

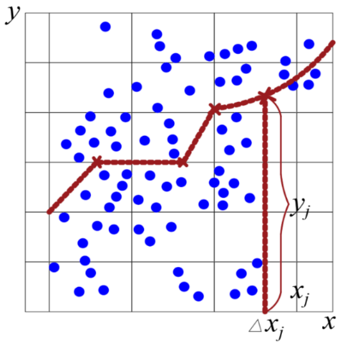

Figure 1. Correlation area

For this, the whole range of change of the correlation area X is divided into equal intervals $\triangle X$. All points in the given interval $\triangle X_j$ belong to $\triangle X_j$ its middle. For this purpose, a special average $\bar{y}_j$ is calculated for each interval;

$$\hat{y}_i = \frac{\sum_{i+1}^{nj} X_{ij}}{n_j} \quad (6)$$

where, n_j, is the number of points in interval $\triangle X_j$ and;

$$\sum_{i+1}^{k} n_j = N \quad (7)$$

where, k is the number of divided intervals, N is the sample volume.

At the next stage, the points $X_j, \bar{y}_j$ continuously are connected with a straight line. The obtained broken line is called the empirical regression line with respect to y in x. According to the view of regression empirical line, the regression equation $\hat{y} = f(x)$ is chosen.

4. LET'S LOOK THROUGH AN EXAMPLE OF ONE PARAMETER LINEAR REGRESSION

Let's suppose, that is necessary to determine the equations of linear regression with using the method of the least squares.

$$\hat{y}_i = b_0 + b_1 x \quad (8)$$

By sample volume N.

For this case, the system of normal equations is given in the following form [2].

$$\sum_{i=1}^{N} y_1 - \sum_{i=1}^{N}(b_0 + b_1 X_1) = 0$$

$$\sum_{i=1}^{N} y_1 X_1 - \sum_{i=1}^{N}(b_0 + b_1 X_i) X_i = 0 \quad (9)$$

Coefficient b_1 is calculated by the help of the determinant

$$b_1 = \frac{\begin{vmatrix} N & \sum_{i=1}^{N} y_i \\ \sum_{i=1}^{N} X_i & \sum_{i=1}^{N} X_i y_i \end{vmatrix}}{\begin{vmatrix} N & \sum_{i=1}^{N} X_i \\ \sum_{i=1}^{N} X_i & \sum_{i=1}^{N} X_i^2 \end{vmatrix}} = \frac{N\sum_{i=1}^{N} X_i y_i - N\sum_{i=1}^{N} X_i - N\sum_{i=1}^{N} y_i}{N\sum_{i=1}^{N} X_i^2 - (\sum_{i=1}^{N} X_i)^2} \quad (10)$$

The coefficient b_o can b_1 determined from Equation (8), as it's known $b_o = \overline{y} - b_1\overline{x}$, where, $\overline{x}, \overline{y}$ are the average values of x and y, respectively. The last expression indicates the presence of a correlation dependence between the coefficients b_o and b_1.

To assess the effect of linear dependence (9), the sampling rate r^* of correlation coefficient is calculated:

$$r^* = \frac{\sum_{i=1}^{N} (X_i - \overline{X})(y_i - \overline{y})}{(N-1)\sigma_x \sigma_y} \quad (11)$$

where, $\sigma_x \sigma_y$ are sample standard deviations. From Equations (9) and (10) we obtain:

$$r^* = \frac{b_1 \sigma_x}{\sigma_y} = b_1 \sqrt{\frac{\sum_{i=1}^{N} X_i^2 - (\sum_{i=1}^{N} X_i)^2}{\sum_{i=1}^{N} y_i^2 - (\sum_{i=1}^{N} y_i)^2}} \quad (12)$$

After determining the regression equation, it is important to analyze the results. In accordance with this analysis, the significance of all coefficients is compared with errors of reflection, and is checked and determined the equation of adequacy. The research is called regression analysis. For conducting the regression analysis, the following conditions must be fulfilled [2]:

1. The input parameter X is measured with very narrow undetected errors. The presence of errors in the determination of y is explained by the presence in the process of undefined variables, which are not included in the regression equation.
2. The output parameters are the results of observations of $y_1, y_2, y_3, ..., y_n$, normally distributed free random variables.
3. Experiments carried out with a sample of volume N under such conditions, which should be provided that each experiment is repeated m times and that the variances of the sample $D_1^2, D_2^2, D_3^2, ..., D_n^2$ would be provided.

Determining of the uniformity of deviations includes in itself the following operations:

1) The average value is determined from the results of parallel experiments:

$$\overline{y}_i = \frac{\sum_{u=1}^{m} y_{iu}}{m}, \; i = 1, 2, 3, ..., N \quad (13)$$

2) Sample dispersions are determined:

$$D_i^2 = \frac{\sum_{u=1}^{m} (y_{iu} - \overline{y}_i)^2}{m-1}, \; i = 1, 2, 3, ..., N \quad (14)$$

3) Find the sum of the dispersion:

$$\sum_{u=1}^{m} D_i^2 \quad (15)$$

4) The ratio is compiled:

$$G_{\max} = \frac{D_{\max}^2}{\sum_{u=1}^{N} D_i^2} \quad (16)$$

where, $D_{\max}^2$ is the maximum value of the sample dispersion.

If the variance is uniform, then

$$G_{\max} < G_p(N, m-1) \quad (17)$$

where, $G_p(N, m-1)$ $-p$ is the table value of the Cokhren test at the significance level.

If the sample variance is uniform, then the reflection variance is calculated:

$$D_{reflection}^2 = \frac{\sum_{u=1}^{N} D_i^2}{N} \quad (18)$$

The number of rates of freestyled of this variance is:

$$f = N(m-1) \quad (19)$$

The reflection variance is used to assess the significance of the coefficients of the regression equation (8). The value of the significance level of the coefficients is assessed according to the student's criteria

$$t_j = \frac{|b_j|}{\sigma b_j} \quad (20)$$

where, b_j is the jth coefficient of the regression equation, but σ_{b_j} is the standard deviation of the jth coefficient.

If t_j is greater than the tabular value $t_p(f)$ for the chosen significance level p and the number of degrees of freestyled f, the coefficient b_j differs significantly from zero, σ_{b_j} determined by the law of accumulation of errors:

$$\sigma_{b_j} = \sqrt{\sum_{i=1}^{N} \left(\frac{\partial b_j}{\partial y_j} \right) D_i^2} \quad (21)$$

If $D_1^2 = D_2^2 = D_3^2 = D_4^2 = ... = D_N^2 = D^2$, we get;

$$\sigma_{b_0} = \sqrt{\frac{D_{reflection}^2 \sum_{i=1}^{N} X_i^2}{N\sum X_i^2 - (\sum X_i)^2}} \quad (22)$$

$$\sigma_{b_1} = \sqrt{\frac{D_{reflection}^2 N}{N\sum X_i^2 - (\sum X_i)^2}} \quad (23)$$

Insignificant coefficients are subtracted from the regression equation. The rest odds are recalculated. This process is fulfilled, because the coefficients are correlated with each other.

Adequacy of the equation is checked by Fisher criteria:

$$F = \frac{D_{residual}^2}{D_{reflection}^2} \quad (24)$$

where, $D^2_{reflection}$ is reflection variance; $D^2_{residual}$ is residual variance.

$$D^2_{res} = \frac{m\sum_{i=1}^{N}(\hat{y}_i - \overline{y}_i)^2}{N-1} \tag{25}$$

If the ratio of Equation (25) exceeds the value of the table $F_p(f_1, f)$, the equation is considered adequate.

5. CONCLUSIONS

As a result of research, it was found, that the mathematical models obtained using the results of experiments in all cases can characterize the relationship of technological factors with the indicators of accuracy and quality obtained as a result of processed details. It is clarified, that mathematical models (polynomials) obtained by dint of using statistical methods may be given as a part of Taylor series and indefinite dependencies can be broken down into separate parts.

It was determined, that the number of relations must be equal to the number of determined coefficients in the compiling of regression equations. The type of regression equation is necessary to determine according to results of experiments. Researches show, that conducting experiments using traditional methods, i.e.by changing the values of only one factor and keeping constant values of other factors, significantly increases the probability of falling into a false optimum. Applying of the methods of multifactorial planning of experiments, vice versa, accelerates the determination of optimal significances of accuracy and quality of processed details and increase the reliability of obtained results.

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