

MATHEMATICAL ANALYSIS OF REGULATED DC MOTOR OF HYBRID SYSTEM

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Abstract- The article is devoted to the mathematical analysis of a regulated DC motor of the hybrid system. The most important of the issues to be solved when creating a hybrid system is the normal performance of the functions of the two different electrical machines. The desired results can be achieved with the introduction of a hybrid electric machine capable of performing the functions of both modes. The hybrid system of the on-board power grid should work in conditions of almost two opposite modes. The characteristics of the DC motor in an open loop control system are investigated. The parameters of the DC motor in an open loop control system and mathematical expressions of mechanical characteristics are analyzed. The characteristics of the DC electric motor under constant and variable load conditions are investigated.

Keywords: Hybrid System, DC Motor, Speed Control, Armature Winding, Uninterruptable Current, Mechanical Characteristic.

1. INTRODUCTION

Currently, the consumption of expensive materials for objects with the function of free movement related to the mass production process (for example, cars, agricultural machinery, military equipment, etc.) remains relevant as one of the important issues. It should be noted that as part of the research work carried out on this issue, the DC motor (starter), which starts the internal combustion engine powered by a battery, meets the basic requirements of modern technology and is the main power unit of the hybrid system. The second important element, in addition, is an alternating current motor (generator), which is powered by mechanical energy, and also has a high frequency [1, 3].

The most important of the issues to be solved when creating a hybrid system is the normal performance of the functions of the two different mentioned electric machines. The desired results can be achieved with the introduction of a hybrid electric machine capable of performing the functions of both modes. The hybrid system of the on-board power grid should work in conditions of almost two opposite modes.

Speed control on modern machines is carried out by electrical methods. Machines change their distribution automatically, smoothly and in a wide range, based on the

control signals (tasks) in this case. The control signals are fed to the automated electric drive in accordance with the control programs stored in the memory of electronic computers. The control signals may vary depending on the external resistance affecting the excitation. The main characteristic of a controlled DC drive is determined by the boundary of the speed control range. This boundary consists of the basic ratio of the minimum rotation speed to the maximum rotation speed [2, 4]:

$$D = \omega_{\max} / \omega_{\min}$$

The maximum speed of the drive is determined by the size of the air gap between the stator and the rotors of mechanical parts (roller pads, housing, motors, etc.). Tightness is limited by the permissible limits of parameters, such as the ability of parts to transfer the generated heat to the environment.

2. ANALYSIS OF THE DC MOTOR IN AN OPEN LOOP CONTROL SYSTEM

None of the output parameters with an open adjustment system has feedback lines. The mathematical expression of the mechanical characteristics of the engine, recorded in such systems, is written as follows [1, 5, 9]:

$$\omega = \omega_0 - M / \beta \quad \text{or} \quad n = n_0 - M / \beta \quad (1)$$

where, n is the number of revolutions per minute; ω is radians per minute.

The rigidity of the mechanical characteristics is calculated as follows:

$$\beta = C_e C_m / R_1, \quad C_e = (U_n - I_n R_1) / \omega_n, \quad C_m = M_n / I_n \quad (2)$$

where, U_n is rated motor armature voltage; I_n is rated motor armature current; R_1 is armature winding resistance; M_n is rated motor torque; and C_e, C_m are coefficients of electromotive force and torque, respectively.

Motor speed control limit [3, 6, 14]:

$$D = \frac{\omega_n}{M_{nf}} S_a \times \beta = \frac{\omega_n}{M_{nf}} S_a \frac{C_e C_m}{R_0} \quad (3)$$

here, nominal angular velocity:

$$\omega_n = \frac{U_n - M_n}{C_e} - \frac{R_0}{C_e C_m} \quad (4)$$

Nominal speed drop:

$$\Delta\omega = M_n R_0 / C_e C_m \quad (5)$$

Unloading speed:

$$\omega_0 = U_n / C_n \quad (6)$$

In experimental calculations, it is sometimes taken with $C_n \approx C_e = C$, and

$$C = U_n = M_n / I_n \quad (7)$$

Given the Equation (7) in (3), we get:

$$D = \frac{U_n S_a C^2}{C C I_n R_l} = \frac{U_n / I_n}{R} S_a = \frac{R_m}{R_l} S_a \quad (8)$$

$$R_n = U_n / I_n \quad (9)$$

where, R_m and R_l are resistances of the armature windings. If $S_a = K$, then we get:

$$D = \frac{R_m}{R_l} K = R_n K; R^* = \frac{R_m}{R_l} \quad (10)$$

where, R^* is the relative resistance of the armature.

One of the integral, main parts of modern automated electric drives is a semiconductor power converter [7, 8, 11]. The main and immediate tasks of power converters are devices assembled from semiconductor elements that convert single, three and multiphase sinusoidal alternating voltage (current) into a constant voltage that can be adjusted (increased or decreased). Such devices are used to regulate the voltage (current) values of anchor circuits, executive (transmitting) electric couplings, power circuits of DC motors [11, 12, 20], etc.

In order to regulate the output voltage of the thyristor converter (meaning the voltage applied to the motor), the time (phase) of opening (closing) of the thyristor can be shifted in one direction or another to an arbitrary phase angle using a control pulse. In other words, the operating time (open circuit) of one of the thyristors is multiplied through the control signal, and at the same time, the operating time of the other thyristor can be increased [12, 13, 19, 20]. In the absence of a control signal, the value of the rectified voltage at the output of the converter is calculated as follows:

$$E_0 = (E_m \sin(\pi / m)) / \pi / m$$

where, E_m is magnitude of rectified voltage, E_2 is RMS voltage. The EMF of the output voltage:

$$E_{out} = E_0 \cos \alpha$$

Voltage drop in thyristor will be: $\Delta U_T \approx (0.5 \div 1) \text{ V}$.

It should be noted that the inductive resistances of the reactor connected in series to the motor do not allow an instantaneous decrease in the current in the fans equal to zero. So, with a closed thyristor, the current becomes zero after a certain time, and in an open thyristor, the current arises again after a certain time. As a result, in the principle of operation of the converter, two thyristors instead of one simultaneously transmit current [13, 15, 16]. The current flowing through the load (motor armature) is calculated as:

$$I_1 = \frac{\sqrt{2} E_2 \sin(\pi / m)}{X_T} [\cos(\alpha + \gamma) - \cos \alpha] \quad (11)$$

Then, average voltage:

$$U_a = E_0 \cos \alpha - \left(R_L + \frac{X_T}{2\pi / m} \right) I_1 - \Delta U_T \quad (12)$$

where, ΔU_T is the voltage drop on the secondary side of the transformer; and x_T is the inductive resistance of the secondary winding.

Uninterruptable current can occur at values of the control angle $\alpha > (\pi/2 - \pi/m)$. At this time, the average value of the electromotive force (emf) [1, 5, 17, 19]:

$$E_a = \frac{1}{2\pi / m} \int_{(\pi/2)+\alpha}^{\pi} \sqrt{2} E_2 \sin(\omega t) d(\omega t) = E_0 \frac{1 + \cos(\pi / m + \alpha)}{2 \sin \pi / m} \quad (13)$$

The load current is determined by Equation (14):

$$I_s = \frac{E_0}{X_T + X_H} \left(1 - \frac{\pi}{m} \text{ctg} \frac{\pi}{m} \right) \sin \alpha \quad (14)$$

where, x_n is the inductive resistance created by the current in the armature.

The current value increases with the decrease in the operating voltage without load generated at the output of the converter. The bridge rectifier circuit has a number of advantages compared to a three-stage 1.5-period converter.

3. INVESTIGATION OF THE PARAMETERS OF THE DC MOTOR UNDER CONSTANT AND VARIABLE LOAD CONDITIONS

The use of a DC thyristor electric drive with feedback to some extent leads to a smoothing of the motor load diagram [14, 15, 18]. To determine the torque on the motor shaft according to the law of rotation frequency change, it is advisable to use a scheme describing a DC electric excitation system with negative feedback of thyristors (Figure 1). The equation of electromotive forces in the anchor circuit of a DC motor:

$$U_r = K_l \omega + ir + L \frac{di}{dt} \quad (15)$$

where, U_r is voltage at the output of the rectifier; i is motor armature current; r is active resistance of the armature circuit; L is inductive resistance of the anchor circuit; K_l is coefficient determining the size of the machine; and ω is angular velocity of the motor.

Without taking into account the value of the inductance in the armature circuit:

$$U_r = K_l \omega + ir$$

The feedback current control circuit is shown in Figure 1, characteristics of a DC motor of this circuit are given in Figure 2.

As can be seen from the characteristics that $K_l K_S = 1$ (curve 3), with $K_l / K_S > 1$ (curves 2), the characteristic will be rigidly negative, and with $K_l K_S < 1$ (curve 1), the characteristic will be rigidly positive, where K_S is gain of the static torque. If the load moment increases and the gain decreases accordingly, and thus the characteristic will be nonlinear (curves 4 and 5).

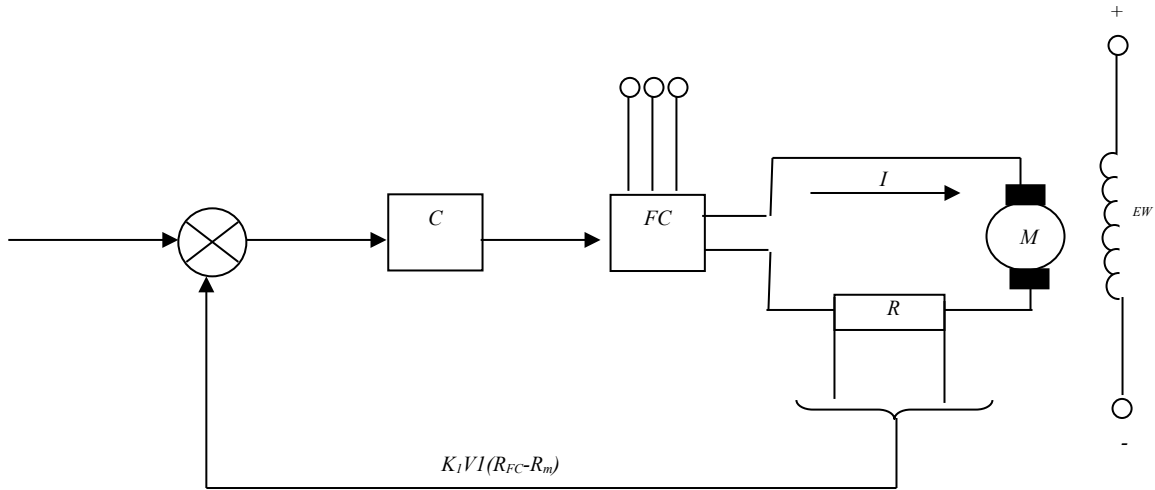


Figure 1. Feedback current control circuit

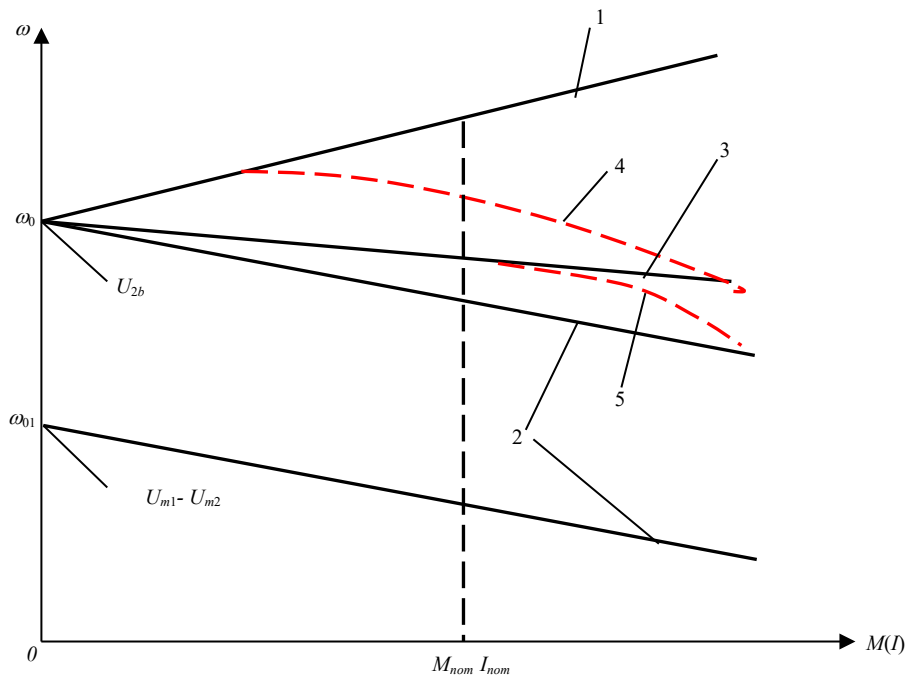


Figure 2. Mechanical characteristics of a DC motor in various speed ranges

The equations of equilibrium of torques on the motor shaft [3, 11, 13, 19]:

$$M_m = M_{sml} + M_{din}; \quad M_p = K_l$$

$$M_{sm} = M_{ave} - M_o \cos 2\omega Pt + K_l M_0 \sin 2\omega_{ave} t$$

$$M_{din} = J \frac{d\omega}{dt}$$

where, M_m, M_{sm}, M_{din} are accordingly, mechanical, static and dynamic moments on the motor shaft; $M_{ave} = M_0 + M_r$ is average torque; M_r is torque, rotating gearbox; and ω_{ap} is average angular velocity of the engine rotation;

The voltage at the output of the thyristor converter in the mode of continuous currents [6, 9, 10, 16]:

$$U_r = \frac{1 + \cos \alpha}{2} U_0 \tag{16}$$

Analyzing the changes in the small limits of the control angle of the thyristor converter of the output voltage at zero value U_0 , we obtain [6, 18, 20]:

$$U_r \approx U_0 - K M_l \tag{17}$$

After the transformations, we obtain a system of equations expressing the operation of a DC electric drive with thyristor feedback along the motor armature:

$$\frac{d\omega}{dt} + \frac{K_l K_u}{J(r + K_1 K_2 K_3 r_1)} \omega = \frac{K_m (\omega_0 - K_2 K_3 U)}{J(r + K_1 K_2 K_3 r_1)} \tag{18}$$

$$-\frac{M_{or}}{J} = \frac{K M_0}{J} \sin 2\omega_0 p t + \frac{M_0}{J} \cos 2\omega_0 p t$$

where, K_S is gain of the dynamic moment.

Comparing the Equations (17) and (18) we get:

$$\frac{d\omega}{dt} + \frac{K_l K_u}{Jr} \omega = \frac{K_m U'_0}{Jr} - \frac{M_{or}}{J} - \frac{KM_0}{J} \sin 2\omega_0 pt + \frac{M_0}{J} \cos 2\omega_0 pt \quad (19)$$

It can be concluded that negative feedback on the motor current armature leads to a softening of the mechanical characteristics of the motor [15, 18, 19, 20]. Meanwhile, maintaining a constant average cyclic speed is performed by sliding. After integrating the differential linear Equation (19), the general velocity equation will look like this:

$$\omega = \frac{U_c - K_2 K_3 U_s}{K_l} - \frac{M_{or}(r + K_1 K_2 K_3 r_1)}{K_1 K_m} - \frac{KM_0}{J} + \frac{K_1 K_m}{J(r + K_1 K_2 K_3 r_1)} \sin 2\omega_{or} - 2\omega_{or} \cos \omega_{or} \quad (20)$$

$$\omega = \left[\frac{K_1 K_m}{J(r + K_1 K_2 K_3 r_1)} \right]^2 + [2\omega_{or}]^2 \quad (21)$$

We perform a replacement and get:

$$\cos \beta = \frac{\left[\frac{K_1 K_m}{J(r + K_1 K_2 K_3 r_1)} \right]}{\left[\frac{K_1 K_m}{J(r + K_1 K_2 K_3 r_1)} \right]^2 + [2\omega_{or}]^2} \quad (22)$$

After the trigonometric transformations, the Equation (23) is written in the following form:

$$\sin \beta = \frac{2\omega_{or}}{\sqrt{\left[\frac{K_1 K_m}{J(r + K_1 K_2 K_3 r_1)} \right]^2 + [2\omega_{or}]^2}} \quad (23)$$

$$\text{tg} \beta = \frac{2\omega_{or} J(r + K_1 K_2 K_3 r_1)}{K_1 K_m} \quad (24)$$

$$\beta = \text{arctg} \frac{2\omega_{or} J(r + K_1 K_2 K_3 r_1)}{K_1 K_m} \quad (25)$$

Equation (20) is written in the following form:

$$\omega = \frac{U_0 - K_2 K_3 U_{or}}{K_l} - \frac{M_{or}(r + K_1 K_2 K_3 r_1)}{K_1 K_m} - \frac{KM_0(\cos \beta \sin 2\omega_{or} - \sin \beta \cos 2\omega_{or})}{J \sqrt{\left[\frac{K_1 K_m}{J(r + K_1 K_2 K_3 r_1)} \right]^2 + [2\omega_{or}]^2}} + \frac{M_0(\cos \beta \sin 2\omega_{or} + \sin \beta \cos 2\omega_{or})}{J \sqrt{\left[\frac{K_1 K_m}{J(r + K_1 K_2 K_3 r_1)} \right]^2 + [2\omega_{or}]^2}} \quad (26)$$

Equation (26) is written in the following form:

$$\omega = \frac{U_0 - K_2 K_3 U_{or}}{K_l} - \frac{M_{or}(r + K_1 K_2 K_3 r_1)}{K_1 K_m} + \frac{M_0(\cos \beta \sin 2\omega_{or} + \sin \beta \cos 2\omega_{or})\varepsilon}{J \sqrt{\left[\frac{K_1 K_m}{J(r + K_1 K_2 K_3 r_1)} \right]^2 + [2\omega_{or}]^2} \cos \varepsilon \quad (27)$$

From the Equation (27) it can be seen that the change in the speed of rotation with respect to the average speed is expressed as follows:

$$\omega_{or} = \frac{U_0 - K_2 K_3 U_{or}}{K_l} - \frac{M_{or}(r + K_1 K_2 K_3 r_1)}{K_1 K_m} \quad (28)$$

Negative feedback on the motor shaft relative to the value of the static moment occurs according to the harmonic law with sliding up to a certain angle [11, 15, 16, 18]. The amplitude of the engine rotation speed is inversely proportional to the average value of the engine rotation speed, proportional to the values of $M_0(r + K_1, K_2, K_3, r_1)$ and the moment of inertia of the rotating parts of the engine. Let's write down a formula expressing the angular acceleration of the engine speed:

$$\omega = \frac{U_0 - K_2 K_3 U_{or}}{K_l} - \frac{M_{or}(r + K_1 K_2 K_3 r_1)}{K_1 K_m} + \frac{M_0(\cos \beta \sin 2\omega_{or} + \sin \beta \cos 2\omega_{or})\varepsilon}{J \sqrt{\left[\frac{K_1 K_m}{J(r + K_1 K_2 K_3 r_1)} \right]^2 + [2\omega_{or}]^2} \cos \varepsilon \quad (29)$$

To find the differential equation of the engine torque, we write the equilibrium equation as follows:

$$\omega = \frac{U_d}{K_l} - \frac{ir}{K_l} \quad (30)$$

$$\text{or } \omega = \frac{U_d}{K_l} - \frac{M_m}{K_l K_m}$$

Considering Equation (5):

$$\omega = \frac{U_0 - K_2 K_3 U_s}{K_l} - \frac{M_m(r + K_1 K_2 r_1)}{K_1 K_m} \quad (31)$$

Differentiating Equation (31):

$$\frac{dM_m}{dt} + \frac{K_l K_m}{J(r + K_1 K_2 r_1)} M_m = \frac{K_l K_m}{J(r + K_1 K_2 r_1)} \times [M_{or} - M_0 \cos 2\omega_{or} + KM_0 \sin 2\omega_{or}] \quad (32)$$

Similarly:

$$\frac{dM_m}{dt} + \frac{K_l K_m}{J(r + K_1 K_2 r_1)} M_m = \frac{K_l K_m}{J(r + K_1 K_2 r_1)} \times [M_{or} - M_0 \cos 2\omega_{or} + KM_0 \sin 2\omega_{or}] \quad (33)$$

It can be seen from the ratio (33) that the change in the torque on the motor shaft when negative feedback on the motor armature current is connected is determined by the law of harmonics with an offset by the angle of the static torque on the shaft. The amplitude of the torque change on the motor shaft is proportional to the torque M_0 and proportional to the value of the average speed of rotation of the engine ($r + K_1, K_2, K_3, r_1$) and the moment of inertia of the rotating masses of the engine.

4. CONCLUSION

The article presents a mathematical analysis of a hybrid system with an adjustable DC motor. The important problems that need to be solved when creating a hybrid system are considered, the desired results can be achieved with the introduction of a hybrid electric machine capable of performing the functions of both modes. The hybrid system of the on-board power grid should work in

conditions of almost two opposite modes. The characteristics of a DC motor in an open-loop control system are investigated. The parameters of a DC motor in an open-loop control system and mathematical expressions of mechanical characteristics are analyzed. The characteristics of a DC electric motor under constant and variable load conditions are investigated.

With an increase in the overall gain of the system, the rigidity of the engine characteristics can be reduced to the required value. That is, the adjustment range can be increased to the desired value by increasing the overall gain of the system. The stable operation of the drive largely depends on the feedback parameters. When the feedback is included in the control signal, the system operation calms down and static errors are reduced, and cases of abrupt rotation of the drive are eliminated.

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