International Journal on
"Technical and Physical Problems of Engineering"
(IJTPE)
Published by International Organization of IOTPE

ISSN 2077-3528
IJTPE Journal
www.iotpe.com
ijtpe@iotpe.com

March 2024 | Issue 58 | Volume 16 | Number 1 | Pages 374-384

THEORETICAL MODELING OF VARIATION IN SOLAR IRRADIANCE OUTSIDE THE EARTH ATMOSPHERE

M. El Malki [1] C. Hajjaj [2] H. Nebdi1 [1]

1. Laboratory of Innovation in Science, Technology and Modeling (LISTM), Faculty of Sciences, University of Chouaib Doukkali, El Jadida, Morocco, elmalki.med@ucd.ac.ma, nebdi_hamid@yahoo.fr
2. Laboratory of Applied Sciences for Environment and Sustainable Development, High School of Technology of Essaouira, University of Cadi Ayyad, Essaouira, Morocco, hajjaj.charaf@gmail.com

Abstract- All climate processes on Earth depend on solar irradiance as their primary driver. The past climate changes can be linked to variations in Total Solar Irradiance (*TSI*). Measurements are made from the ground to estimate the solar irradiance outside the atmosphere, i.e. the total solar irradiance value, but the results obtained lack the necessary precision. *TSI* measurements with high accuracy can help to identify the value of the Solar Constant and the variation of global temperatures with the purpose to distinguish between anthropogenic and natural climate changes. The measurement of the *TSI* in space has been ongoing since late 1970. Encouraging. In this work, an analytical model has been proposed based on physical laws to reproduce with a more accuracy the experimental values of solar irradiance outside the Earth's atmosphere extracted from a satellite database of measurements collected during the solar cycles 23 and 24, as well a statistical analysis has been done to compare our proposed model with the Spencer's one. Lastly, we find that our theoretical model, after evaluation and comparison with data collected by satellites, responds to the variation in Total Solar Irradiance caused by the variation in Earth-Sun distance.

Keywords: Total Solar Irradiance, Solar Constant, Solar Variability, Analytical Model, Spencer Model.

1. INTRODUCTION

The Sun is a non-polluting source of energy that sends out radiation at every moment with enormous intensity. Solar energy is a clean, abundant resource that can transform our energy supply. Unlike fossil fuels, it does not produce greenhouse gases. It offers a sustainable and affordable solution to our energy needs, while reducing our impact on the environment [1]. These are precisely why renewable energies are considered promising with considerable future potential [2]. Moreover, it is the main source of energy, in the form of electromagnetic radiation, for heating our atmosphere. The energy emitted by the sun that actually reaches the Earth's surface is known as solar radiation. It is fundamental to the conception and running of solar applications, such as solar energy systems and solar water heaters [1].

Knowing the solar radiation is essential for the calculation of various parameters of solar related systems, e.g. solar water heaters and photovoltaic modules also for the buildings construction in view of a more perfect thermal insulation appropriate to the geographical place and also for the heating of houses and premises by solar energy. The examples of use only increase over time. However, the development of these fields cannot be done without a thorough understanding of solar radiation [3]. Furthermore, the knowledge of local-global solar irradiance is significant for many applications and obtaining indispensable data in fields like solar energy, climatology, meteorology, ecology, engineering, and hydrology [4].

To benefit from the solar energy, systems to capture and store it must be realized. And the most important thing that must be known is the power emitted by the sun and the power received by our planet called the Solar Constant. This latest term (Solar Constant) refers to the average value during a year of the Total Solar Irradiance (*TSI*) [5]. Physically, the Solar Constant is the flux received by a unit area exposed perpendicularly to solar radiation expressed in W.m^{-2} [6]. Also, solar energy influences geo-biological processes on earth among other important phenomena, the climate of the earth and other planets is directly affected by it at every moment [5]. The energy received on our planet is dependent on the energy production of the sun (solar activity, number of sunspots, solar cycles ...) [7]. Due to its essential role in climate models, radiative transfer models, and solar energy applications, the *TSI* value is frequently utilized in solar radiation calculations. Several scientific researchers have tried to find an exact value of the Solar Constant [4].

Registered: 230918 Article Info: 1753 Accepted: 240227
Reserved: 231123 Available: 240331

Among the first experiments of the measurement of this physical quantity, we cite the one carried out by Claude Pouillet in 1838 with a pyrheliometer and he found a value of 1238 W.m^{-2} [8]. Even if the radiation undergoes attenuation because of the presence of particles between the ground and the different atmospheric layers such as nitrogen, oxygen, argon, Pouillet makes an estimate of the power absorbed by the atmospheric layers using time series. After the measurements made by Langley with a bolometer, he measured the Solar Constant and found a value of about 2140 W.m^{-2} [9]. Abbott cited 13 estimates of it ranging from 1185 to 2371 W.m^{-2} that were published between 1837 and 1908. He estimated 1340 W.m^{-2} at that time [10]. He also found a relationship between the energy production of the sun and its cyclic activity [11]. Even though there were significant experimental uncertainties at the time, based on data collected from high mountains or weather balloons, over the years, he obtained values that ranged from about 1320 to 1550 W.m^{-2}. The majority of these variations were attributed to solar activity [5, 12].

Between 1928 and 1968, before the era of satellites, several attempts were made by different physicists and the values obtained varied between 1323 and 1430.5 W.m^{-2} [4]. Because of the difficulty of measuring with precision the exact value of the Solar Constant and to take into account the temporal variability of the instantaneous production of the sun (*TSI*), the scientific committee decided to make measurements outside the atmosphere at the end of the 1970s with the Nimbus satellite7, to eliminate the effect of the attenuations that the solar radiation undergoes during their journey to cross the various atmospheric layers (absorption, scattering, reflection) in order to have a more accurate value of the Solar Constant [5, 7].

Since 1978, we find the different values measured by satellites in databases until today. Several works are done to find a formula for the variation of solar irradiance outside the atmosphere, but the existing models are only empirical models like the model proposed by Spencer which is widely used by researchers in the solar energy field. On the other hand, in our best knowledge there is no analytical model which give a complete information how the solar irradiance varies over time. In this study, we suggest a theoretical model that presents a clear answer to the variation in Total Solar Irradiance due to the variation in the solar earth distance alone, not to mention the variations caused by the sun itself, and show its usefulness compared with the empirical model proposed by Spencer. We base on the data available from the SORCE observers, the model proposed by our team is based on physical laws and adequate parameters.

2. MATERIALS AND METHODS

The measurements of the Total Solar Irradiance have been obtained from a satellite mission, then we have plotted the extracted values in graphs to clearly see the regular variation according to the day number. In fact, the satellite missions offer continuous measurements from 1978 until today among the missions that have existed since that date are listed in the following Table 1 [13].

Table 1. Summary of available TSI measurements [4], [13]

Satellites	Periods (used)
Nimbus 7	1978-1993
PMOD composite	1978-2003
ACRIM composite	1978-2013
SMM	1980-1989
ERBS	1984-2003
RBIM composite	1984-2019
NOAA9	1985-….
NOAA10	1986-1987
UARS/ACRIMII	1991-1997
UARS	1991-2001
EURECA	1992-1993
Space shuttle	1992-2003
SOHO VIRGO v2	1996-1999
ACRIM III	2000-2014
SORCE	2003-2018
TCTE-TIM	2013-2018

In this work, we have exploited the database taken from the 'Laboratory for Atmospheric and Space Physics at the University of Colorado in Boulder' [14]. The solar irradiance measurements of the studied database are spread over 17 years (from 25/02/2003 to 25/02/2020). This database allowed us to plot the variation of solar irradiance outside the atmosphere in relation to the D-day number n. We note that during the years 2003-2019, the solar irradiance value outside the atmosphere varies between two extreme values 1315 and 1408 W.m^{-2} with an average of around 1360 W.m^{-2}. To describe these measurements, there are several empirical models among them we mention the empirical formula given in Equation (1) and proposed by Duffie and Beckman [14, 15]:

$$I_{OA} = I_{OAmean}(1 + 0.033\cos(\frac{360n}{365})) \tag{1}$$

where, I_{OA} is Total solar irradiance outside atmosphere, I_{OAmean} is the average value of the *TSI*. Also, we find the empirical model presented in Equation (2) proposed by Spencer [15, 17]. This is the most widely used and cited one by researchers in the field:

$$\begin{aligned} I_{OA} = I_{OAmean}\,(1.000110 + 0.034221\cos B + \\ +0.001280\sin B + 0.000719\cos 2B + \\ +0.000077\sin 2B) \end{aligned} \tag{2}$$

where, $B = (n-1)\times\frac{360}{365}$.

This empirical model does not represent the variation of solar irradiance outside the atmosphere in a physical way, i.e. as a function of the physical parameters. For this, we propose in the following an analytical model based on physical laws. By plotting the different values of irradiance outside the atmosphere extracted from the SORCE (Solar Radiation and Climate Experiment) database according to the D-day number n, we obtain the graph in Figure 1.

We notice that the variation of the total solar irradiance follows a law of variation almost the same for the different years studied. For this, we propose a new theoretical model that minimizes the uncertainty of the variation obtained by Spencer and gives more accurate information compared to the old models. To achieve this objective, we rely on the database taken by different satellites from 2003 to 2020; then we exploit these measurements in the form of graphs and we compare the results using statistical indicators. We find that the value of the total solar irradiance outside the atmosphere varies in an almost regular way depending on the number of the day, we know that the distance to the sun varies over time and therefore the quantity of energy received by the earth also changes, respectively.

Based on physical laws such as the Stefan-Boltzmann law and the Wien displacement law, and introducing the emissivity of the sun to find a good theoretical model adequate to the satellite measurements. To study the reliability of the proposed model, the use of statistical indicators is essential. In fact, there are several indicators in the literature, in the present work we have chosen the most used by researchers to investigate the performance of the proposed model such *MBE*, *RMSE*, *nRMSE*, *TS*, *SD* and Pearson's correlation coefficient (ρ) [16, 17]. The model error is defined as the difference between the measured value and the predicted one (calculated theoretically), as follows:

$$e_{xi} = x_{meas_i} - x_{pred_i} \tag{3}$$

where, x_{meas_i} and x_{pred_i} are the *i*th measured and predicted value, respectively. The *MBE* represents the arithmetic mean of the error. A positive *MBE* represents an overestimation of the predicted data and a negative *MBE* an underestimation of the measured value [16, 18]:

$$MBE = \frac{1}{N}\sum_{i=1}^{N} e_{xi} \tag{4}$$

where, N is the total number of the measurements in the dataset. The *RMSE* provides information about the short-term performance of a predicted dataset by making a term-by-term comparison of the actual deviation between the measured data and those predicted by the prediction model. This indicator is always positive, and the ideal value should be zero [16, 18]:

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N} {e_{xi}}^2} \tag{5}$$

Standardized performance evaluation techniques facilitate comparative analysis of prediction models. The normalized root mean square error *nRMSE* generally provides better information about the quality of the prediction model than the *MBE* or *RMSE* [16, 18]:

$$nRMSE = \left(\frac{1}{N}\sum_{i=1}^{N}\frac{e_{xi}}{x_{mesi}}\right)100 \tag{6}$$

TS is a widely used test to evaluate whether a prediction model is significantly different from the measured data or not. The smaller the *TS* value is, the more accurate the predicted data will be [18]:

$$TS = \sqrt{\frac{(N-1)MBE^2}{RMSE^2 - MBE^2}} \tag{7}$$

The Standard Deviation presents the difference between the predicted and measured data. This indicator is always positive and the ideal value is zero [18]:

$$SD = \sqrt{\frac{N\left(RMSE^2 - MBE^2\right)}{(N-1)}} \tag{8}$$

The Pearson criterion is based on the Pearson correlation coefficient, ρ. If two data sets X and X' are highly correlated, the Pearson coefficient is equal to 1 (direct correlation) or -1 (inverse correlation). However, a Pearson coefficient close to 0 implies a weak or zero correlation [21]:

$$\rho = \frac{\sum_{i=1}^{n}\left(X_i - \bar{X}\right)\left(Y_i - \bar{Y}\right)}{\sqrt{\sum_{i=1}^{n}\left(X_i - \bar{X}\right)^2}\sqrt{\sum_{i=1}^{n}\left(Y_i - \bar{Y}\right)^2}} \tag{9}$$

Additionally, to model a phenomenon in physics, we follow some of the mechanisms of mathematical modeling of physical concepts.

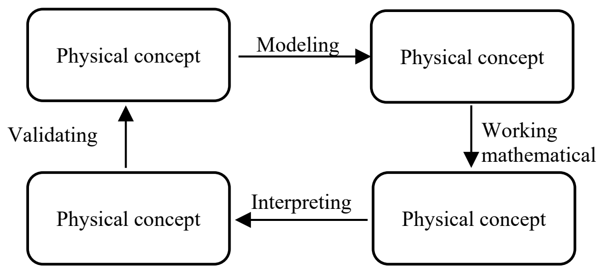

Figure 1.Clarification of the interaction between mathematical and physical concepts [22]

A given physical concept is mathematically modeled using a suitable mathematical element, such as a function. Then, using mathematical and statistical mechanisms, we obtain mathematical results and interpret and compare them with real or measured data to understand physical phenomena and discuss the validity of the model [22]. In the following section, we highlight and apply these concepts to our study.

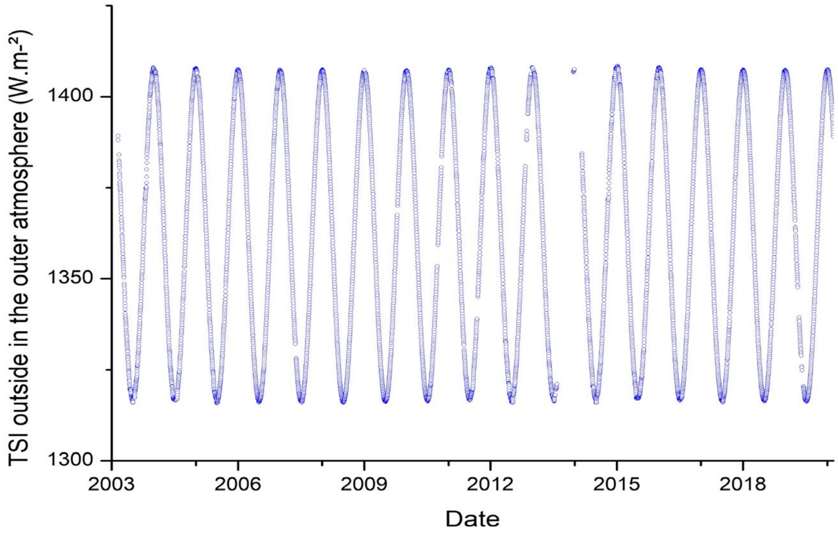

Figure 1. Variations of the total solar irradiance in the outer atmosphere as a function of the D-day number from 2003 to 2019

3.THEORETICAL DETERMINATION OF SOLAR CONSTANT

In this section, we present the proposed analytical model of the total solar irradiance, which is based on physical law. We consider that the Sun is a black body, hence the Stefan-Boltzmann law (the energy emitted by a black body per unit of time and unit of area) for black bodies is written as [23]:

$$\varepsilon = \sigma T_S{}^4 \tag{10}$$

The Sun emits energy in all directions in space. The emitted solar energy is therefore distributed on a sphere centered on the sun, so the total emissivity of the sun is:

$$\varepsilon_s = \varepsilon S \tag{11}$$

where, S is the surface of the sun; we obtain the following equation:

$$\varepsilon_s = 4\pi R_s{}^2 \sigma T_S{}^4 \tag{12}$$

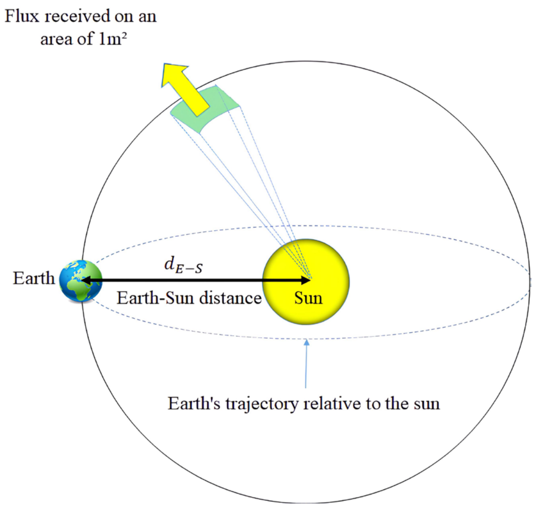

Figure 3. Schema representing the solid angle seen from the earth [24]

The total power radiated by the Sun in space is distributed on the surface S of a sphere centered on the Sun, particularly the sphere of radius equal to the distance Earth-Sun $d_{E\text{-}S}$. We can establish a relationship between the emissivity of the sun and the Total Solar Irradiance received outside the atmosphere I_{OA}, such as:

$$\varepsilon_s = I_{OA} 4\pi d_{E-S}^2 \tag{13}$$

where, I_{OA} is the Total Solar Irradiance per unit area received over an earth-solar distance $d_{E\text{-}S}$.

$$4\pi R_s{}^2 \sigma T_S{}^4 = I_{OA} 4\pi d^2 \tag{14}$$

We deduce the Equation (15):

$$I_{OA} = \frac{R_s{}^2 \sigma T_S{}^4}{d_{E-s}{}^2} \tag{15}$$

The maximum wavelength in the spectrum of the sun corresponds to 501.3 nm [25]. And by applying Wien's law of displacement, we get the temperature of the sun's surface:

$$T_S = \frac{C_{wien}}{\lambda_{max}} = \frac{0.002897\,\text{m.K}}{501.3\times10^{-9}\,\text{m}} = 5778\,\text{K} \tag{16}$$

This result is consistent with the value found in the literature [26]. For $d_{E\text{-}S}$ = $1u\times a$; $1u\times a$=149597870 km; σ=5.67×10^8 Wm^-2K^-2.

The Stefan-Boltzmann constant [23], T_S: Temperature of the sun's surface = 5778 K [25], R_s: The sun radius=6.95823×10^5 km [27], we obtain:

$$I_{OA} = \frac{\left(6.95823\times10^5\right)^2 \times 5.67\times10^{-8} \times 5778^4}{\left(1.49597870\times10^6\right)^2} = 1367\,\text{Wm}^{-2}$$

In order to determine the daily value of the total solar irradiance, we introduced the ellipse parameters as the trajectory of the earth in relation to the sun describes an ellipse. In what follows, we will express I_{OA} as a function of n; the D-day number. Figure 4 presents the trajectory of the earth in relation to the sun, and we notice that the trajectory is elliptical such that the distance varies between two extreme values a minimum distance on January 3 (perihelion) and a maximum distance on July 3 (aphelion) [15, 23].

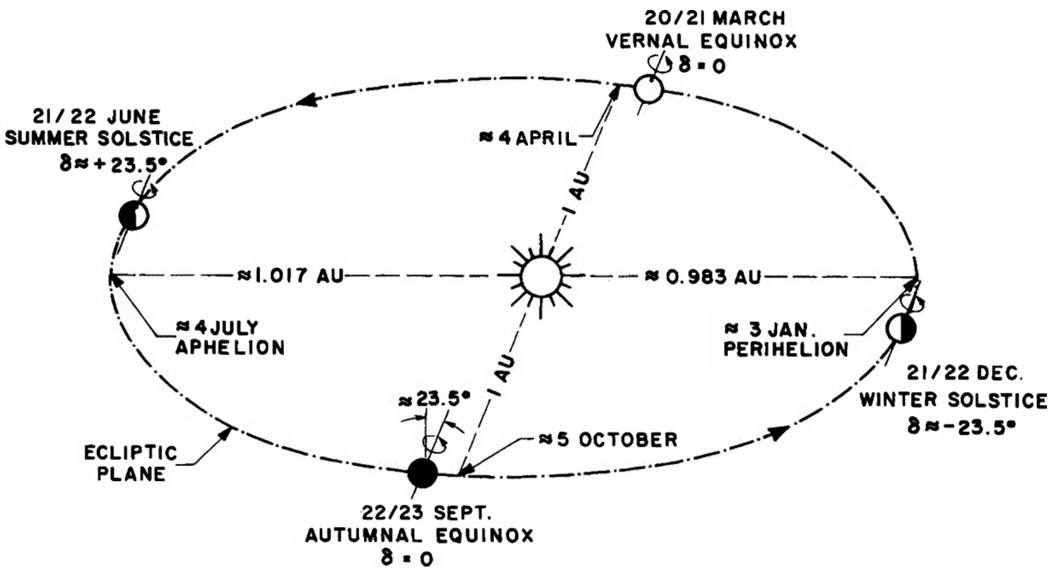

Figure 4. Motion of the earth around the sun [15]

Based on the elliptical coordinates to get the expression of the earth-solar distance:

$$r = d_{E-S} = \frac{a\left(1-e^2\right)}{1+e.cos\theta} \tag{17}$$

And we know that the earth makes one turn around the sun during a period of T=365.25 days so the angular speed is:

$$\omega = \frac{2\pi}{T} \text{ where } \theta = \omega n \tag{18}$$

Thus, replacing the obtained expression of d_{E-S} in Equation (15) we obtain:

$$I_{OA} = \frac{R_s^{\,2}\sigma T_S^{\,4}\left(1+e\,cos\theta\right)^2}{\left(a\left(1-e^2\right)\right)^2} \tag{19}$$

By considering Equation (18), we obtain:

$$I_{OA} = \frac{R_s^{\,2}\sigma T_S^{\,4}\left(1+e.\cos\left(\omega.n\right)\right)^2}{\left(a\left(1-e^2\right)\right)^2} \tag{20}$$

This last Equation (20) represents the variation of the total solar irradiance received outside of the atmosphere as a function of physical parameters.

$a = 1.000001018U.a = 149876762$ km

The semi-major axis of the earth-sun trajectory [28].

$e = \frac{c}{a} = 0.0167086$

The eccentricity of the earth orbit with respect to the sun, where c is the distance between the center of the orbit and the focus corresponding to the sun [28].

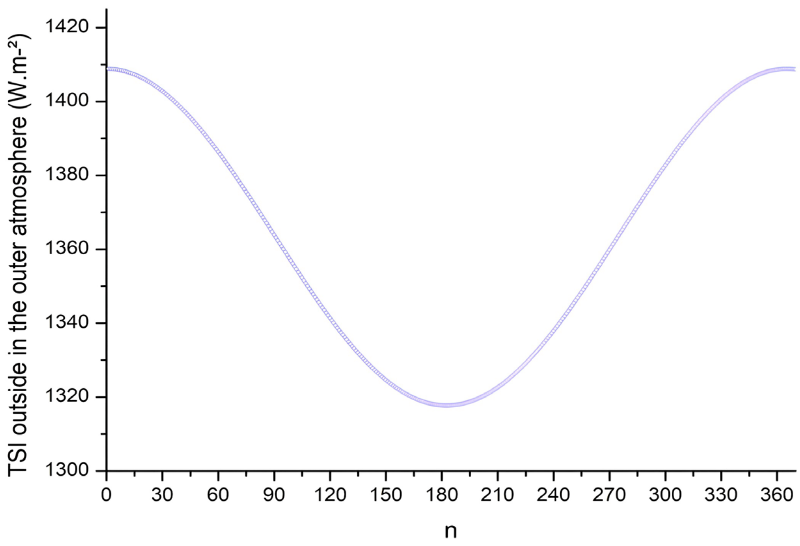

Figure 5. Variations of the theoretical Total Solar Irradiance outside the atmosphere as a function of the D-day number

4. RESULTS AND DISCUSSIONS

In this section, we make a comparison of the different results obtained previously and we use the different mathematical methods to study the reliability of the proposed model compared to the experimental results. From the measurements made by the various satellites from 1978 to 2020, we traced the variation of solar irradiance outside the atmosphere over time, as well as a theoretical model that represents this variation as a function of physical parameters. The plot of Equation (20) allowed us to obtain the graph presented in Figure 5. With the help of our model, we have been able to represent the variations of solar irradiance outside the atmosphere according to a theoretical model, which is based on purely physical laws; this model is well adapted to the satellite measurements and represents a good reliability compared to the experimental results.

We can therefore write that the value of the total solar irradiance outside the atmosphere varies according to the formula presented in Equation (20) defined earlier.

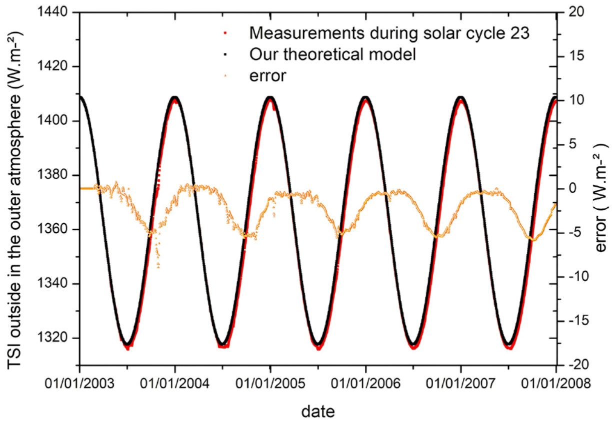

Figure 6. Comparison between the measurements during solar cycle 23 and the theoretical values

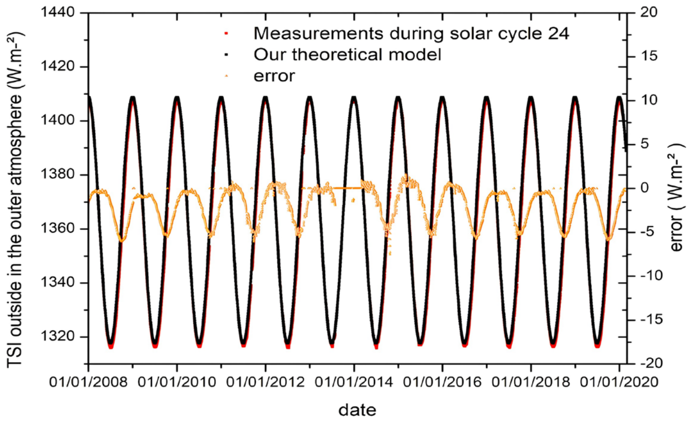

Figure 7. Comparison between the measurements during solar cycle 24 and the theoretical values

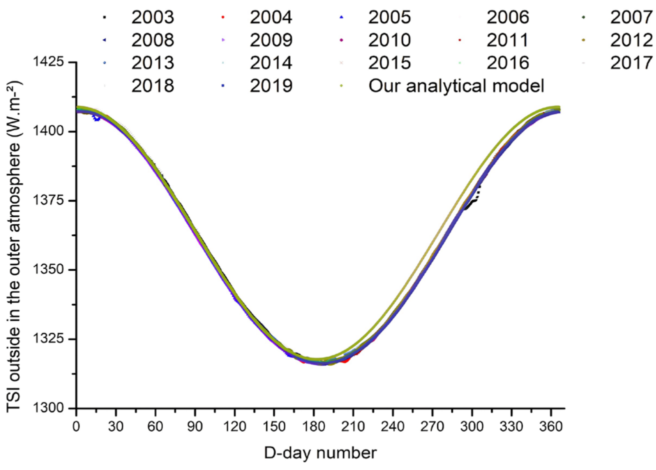

Figure 8. Comparison between the annual measurement's values and the theoretical values according to D-day number

Table 1. Analysis of the correlation between the proposed theoretical model and the empirical Spencer model as a function of the total solar irradiance outside the atmosphere

Year of study	Pearson correlation coefficient	
	Our analytical model	Spencer's model
2003	0.998	0.997
2004	0.998	0.998
2005	0.999	0.998
2006	0.998	0.998
2007	0.998	0.998
2008	0.998	0.998
2009	0.999	0.999
2010	0.999	0.999
2011	0.998	0.998
2012	0.998	0.996
2013	1.000	0.998
2014	0.998	0.998
2015	0.998	0.998
2016	0.998	0.998
2017	0.999	0.998
2018	0.998	0.999
2019	0.998	0.998

The graphs in black in Figures 6 and 7 represent the evolution of the irradiance outside the terrestrial atmosphere calculated theoretically so the graphs in red represent the measurements values during the solar cycles 23 and 24. The graph in green of Figure 8 represents also the theoretical model that we proposed for the variation of the daily solar irradiance in outside atmosphere and the other graphs correspond to the measurements values, we notice that the theoretical graph is in perfect agreement with the satellite measurements.

So, we can say that our model is well adequate to the *TSI* measurements: good correlation with the measured values, as well as the fluctuation margin for the mean, standard deviation, minimum value and maximum value. To determine the reliability of our model, we used statistical indicators and studied correlation using the Pearson correlation coefficient.

We obtained the results presented in Table 2 we can easily see that our proposed theoretical model presents a good correlation with respect to the empirical model proposed by Spencer. Concerning the statistical indicators mentioned before, we summarize the results on the following graphs by comparing the real measurements with the analytical model proposed by our team and with the empirical one proposed by Spencer.

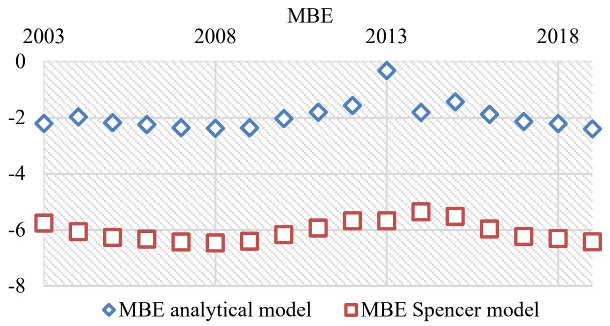

Figure 9. Evolution of MBE statistical indicator for our analytical model (blue) and the empirical Spencer model (red) as a function of the real measurements obtained by the satellites

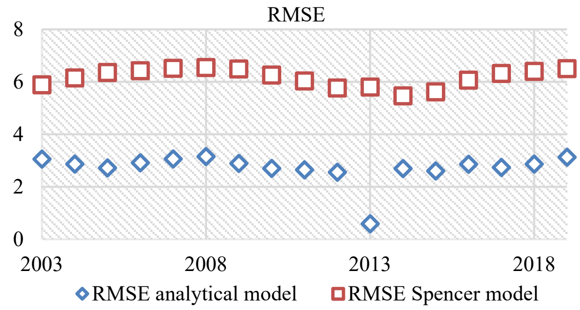

Figure 10. Evolution of statistical indicator RMSE for the two models: our analytical model (blue) and the empirical Spencer model (red) according to the real measurements obtained by the satellites

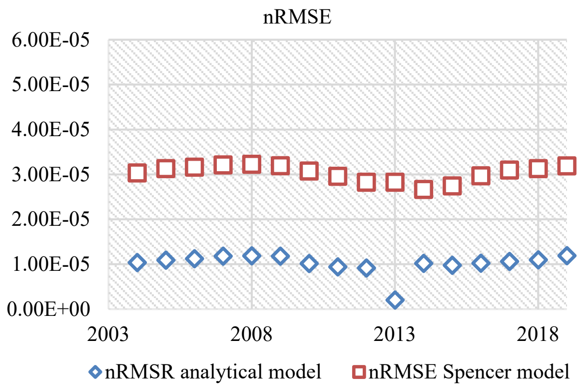

Figure 11. Evolution of statistical indicator nRMSE for the two models: our analytical model (blue) and the empirical Spencer model (red) according to the real measurements obtained by the satellites

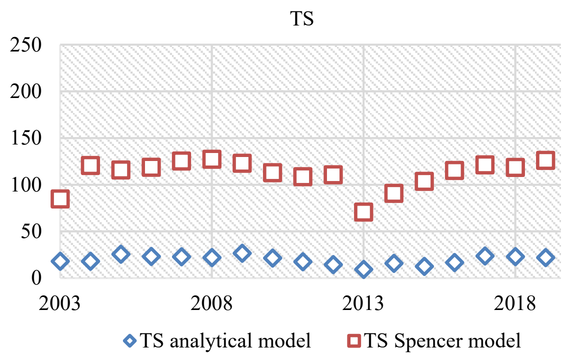

Figure 12. Evolution of TS statistical indicator for our analytical model (blue) and the empirical Spencer model (red) as a function of the real measurements obtained by the satellites

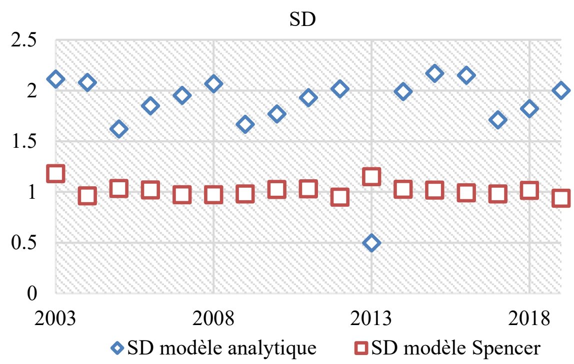

Figure 13. Evolution of SD statistical indicator for our analytical model (blue) and the empirical Spencer model (red) as a function of the real measurements obtained by the satellites

In Table 3, we present a statistical comparison of the measured values with the obtained results from our theoretical model and Spencer's empirical model. The exploitation of Table 3 shows that we have obtained a good reliability of our model according to the measurements. According to the study of the performance of the two models using statistical indicators we notice that our theoretical model has an average of MBE of -1.9609612, RMSE of 2.70656936, nRMSE of 0.00992198, TS of 19.6364853 and a SD of 1.848266.

Table 2. Statistical study of the measurements and the results obtained by our theoretical model and the Spencer empirical model

Year of study	N	Solar irradiance outside the atmosphere			standard deviation
		Minimum $W.m^{-2}$	Maximum $W.m^{-2}$	Average $W.m^{-2}$	
2003	301	1315.92	1408.02	1353.39	29.4409
2004	366	1316.59	1407.78	1361.21	32.3614
2005	365	1315.87	1407.75	1360.88	32.1889
2006	365	1316.10	1407.51	1360.81	32.1548
2007	359	1316.09	1407.40	1361.24	32.2244
2008	366	1316.01	1407.43	1360.80	32.2836
2009	354	1316.22	1407.21	1359.48	31.8003
2010	351	1316.38	1407.23	1360.18	32.2366
2011	355	1316.55	1407.92	1361.43	32.3580
2012	342	1315.98	1408.06	1359.16	32.0436
2013	205	1316.38	1407.90	1358.22	33.4130
2014	302	1315.91	1407.99	1353.51	29.4476
2015	365	1317.05	1408.52	1361.62	32.2788
2016	366	1316.57	1408.11	1361.30	32.3119
2017	363	1316.08	1407.45	1361.10	32.1834
2018	363	1316.34	1407.40	1361.03	32.1236
2019	338	1316.29	1407.28	1363.06	32.2737
Our Theoretical Model	366	1317.73	1408.83	1363.18	32.2856
Spencer's empirical model	366	1315.33	1408.53	1360.95	33.1545

However, for the empirical model proposed by Spencer has an average of MBE of -6.063963, RMSE of 6.14886359, nRMSE of 0.02499112, TS of 111.389603 and a SD of 1.01478864. Similarly for the Pearson correlation coefficient, we obtained a mean of 0.9984 with our theoretical model, on the other hand we found a mean of 0.9980 with the empirical model proposed by Spencer. In the following table, we present the different values given in the literature [4], as well as the value obtained by our research team.

Our scientific contribution gives a value in accordance with what has been found by scientific researchers in the field. and the model proposed by our team is in agreement with the literature and the satellite measurements. As a final result. after this scientific contribution. we can express the variation of solar irradiance outside the atmosphere in the form of Equation (20) presented below. which represents our main result.

Table 3. *TSI* values reported in the literature [4] and our contribution

Authors	year	*TSI* value ($W.m^{-2}$)
P Moon	1940	1323
Aldrich and Abbot	1948	1325.8
Aldrich and Hoover	1952	1349.5
Hatami, et al.	2020	1357
Gueymard, et al.	2013	1361.1
Simpson	1928	1362.8
Our contribution	2023	1363.18
Fellak, et al.	2017	1365
Labs and Neckel	1968	1365.6
Ameur, et al.	2020	1366
Jiang, et al.	2021	1367
Guldentops	2016	1368
Paulescu	2003	1369
Matsumoto, et al.	2014	1370
Hoyt	2015	1373
Allen	1950	1374.7
Allen	1955	1374.7
Nicolet	1951	1380
Allen	1958	1380.2
Gast	1965	1390
Johnson	1954	1395.6
Stair and Johnston	1956	1430.5
Schuepp	1949	1367.7 to 1416.5

The exploitation of Figure 14 clearly shows that our theoretical model presents a good reliability compared to the empirical model; theoretical model presents a smaller RMSE than the empirical one. as well as the correlation coefficient of our analytical model which is larger than the Spencer's model.

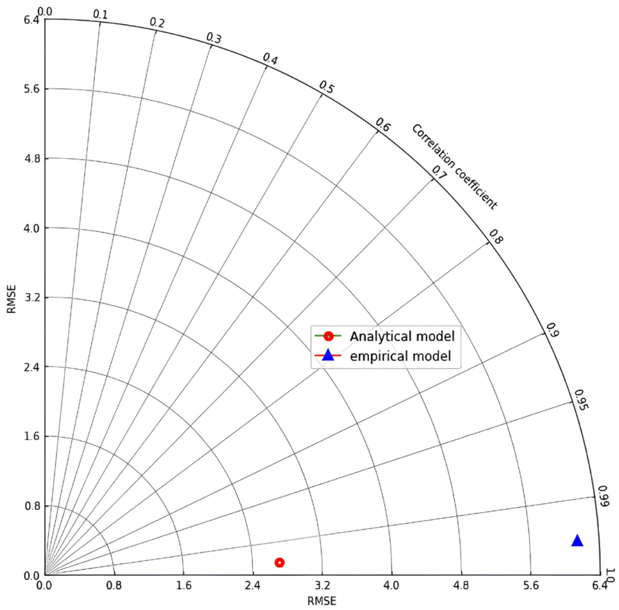

Figure 14. Taylor diagram for both models: analytical and empirical

5. CONCLUSION

The exact value of the total solar irradiance (solar constant) represents a challenge for scientists. Because of the difficulty of measurement on earth. the physicists chose to make it out of atmosphere and the first mission was launched in 1978. For the purpose of knowing with great precision or certainty the value of total irradiance outside the atmosphere. this radiation or solar energy affects many of the phenomena that have the greatest influence on the earth's global climate. Without considering short-term and long-term variations. it is impossible to obtain clear and significant information. These measurements show that the value of the solar constant (at the distance 1 a.u.) varies between 1362 and 1374 $W.m^{-2}$. this fluctuation is due to the variability of solar activity. On the other hand. the value of the total solar irradiance outside the atmosphere varies between two extreme values 1315 and 1408 $W.m^{-2}$.

This fluctuation is due to the variation of the earth-solar distance. The problem here is to find a theoretical model that answers the question of how the value of the total solar irradiance outside the earth's atmosphere varies on a daily basis. That was why our starting point is to elaborate. in one hand. a theoretical model based on the physical laws. constants and parameters dependent on the earth's trajectory in relation to the sun. And on the other hand. to carry out a statistical study and make a comparison of our proposed model with those proposed in literature.

So, we have developed a theoretical model. governed by Equation (20). able to describe clearly the daily variations of the solar irradiance out of the atmosphere which gives results in good reliability with the satellite measurements. and remain in best agreement with other models given in literature. By using statistical indicators and correlation coefficients. we show that our model is well correlated with satellite measurements and more accurate than the Spencer's model. Our model will undoubtedly serve as a tangible reference in the field of meteorological. climatic and energetic modelling.

NOMENCLATURES

1. Acronyms

MBE	Mean Bias Error
RMSE	Root Mean Square Error
nRMSE	Normalized Root Mean Square Error
TS	Test Statistic
SD	Standard Deviation

2. Symbols / Parameters

ρ: Pearson's correlation coefficient
n: D-day number
I_{OA}: Total Solar Irradiance outside atmosphere
I_{OAmean}: The average value of the *TSI*
e_{xi}: Model error
N: Total number of the measurements in the dataset
ε: Energy emitted by a black body per unit of time and unit of area
σ: Stefan-Boltzmann constant
T_S: Temperature of the sun's surface
ε_S: Total emissivity of the sun
S: Surface of the sun
$d_{E\text{-}S}$: Earth-Sun distance
C_{wien}: Wien's constant
$\lambda_{\max}$: Maximum wavelength
R_S: Sun radius
TSI: Total Solar Irradiance
ω: Angular speed
a: Semi-major axis of the earth-sun trajectory
e: Eccentricity of the earth orbit with respect to the sun

REFERENCES

[1] S.S. Alrwashdeh, "Comprehensive Analysis and Evaluation of the Impact of Variable Receiver Absorbance on Energy Production in Parabolic Trough Systems", International Journal on Technical and Physical Problems of Engineering (IJTPE), Issue 56, Vol. 15, No. 3, pp. 212-216, September 2023.
[2] M. Benaichi, A. Karkri, A. Chetouani, J.E. Salhi, R. Zarrouk, S. Talbi, A. Ed Daoui, "3D Numerical Modeling and Simulation of Effects of Non-Uniform Irradiation on Performance of Photovoltaic", International Journal on Technical and Physical Problems of Engineering (IJTPE), Issue 56, Vol. 15, No. 3, pp. 225-233, September 2023.
[3] G. Kockarts, "Physical Aeronomy and Atmospheric Chemistry", De Boeck, Brussels, Belgium, 2000.
[4] J. Almorox, C. Voyant, N. Bailek, A. Kuriqi, J.A. Arnaldo, "Total Solar Irradiance's Effect on the Performance of Empirical Models for Estimating Global Solar Radiation: An Empirical-Based Review", Energy, Vol. 236, pp. 1-10, 2021.
[5] C.A. Gueymard, "A Reevaluation of the Solar Constant Based on a 42-Year Total Solar Irradiance Time Series and a Reconciliation of Spaceborne Observations", Sol. Energy., Vol. 168, pp. 2-9, 2018.
[6] M.P. Thekaekara, R. Kruger, C.H. Duncan, "Solar Irradiance Measurements from a Research Aircraft", Appl. Opt., Vol. 8, No. 8, pp. 1713-1732, 1969.
[7] P. Foukal, C. Frohlich, H. Spruit, T.M.L. Wigley, "Variations in Solar Luminosity and their Effect on the Earth's Climate", Nature, Vol. 443. No. 7108, pp. 161-166, 2006.
[8] C. Pouillet, "Memory on Solar Heat: on the Radiating and Absorbing Powers of Atmospheric Air and on the Temperature of Space", Reports of the Academy of Sciences Sessions, Vol. 7, pp. 24-28, 1838.
[9] J.A. Eddy, "Founding the Astrophysical Observatory: The Langley Years", JHA., Vol. XXI, pp. 111-120, 1990.
[10] C.G. Abbot, "The Solar Constant of Radiation", Am. Philos. Soc., Vol. 50, No. 199, pp. 235-245, 1911.

[11] V.D. Hoyt. "The Smithsonian Astrophysical Observatory Solar Constant Program", Rev. Geophys. Sp. Phys., Vol. 17, No. 3, pp. 427-458, 1979.
[12] D.V. Hoyt, K.H. Schatten, "The Role of the Sun in Climate Change", Oxford, Oxford University Press, pp. 1-279, New York, USA, 1997.
[13] S. Dewitte, D. Crommelynck, S. Mekaoui, "Measurement and Uncertainty of the Long-Term Total Solar Irradiance Trend", Sol. Phys., No. June, pp. 1-10, 2004.
[14] Laboratory for Atmospheric and Space Physics, "Total Solar Irradiance Data", Total Solar Irradiance Data, December 2021, https://lasp.colorado.edu/home/sorce/data/tsi-data/.
[15] M. Iqbal, "An Introduction to Solar Radiation", Academic Press Canada, New York, USA, 1983.
[16] J.A. Duffie, W.A. Beckman, "Solar Engineering of Thermal Processes", Wiley, New York, USA, 2013.
[17] S.A.M. Maleki, H. Hizam, C. Gomes, "Estimation of Hourly, Daily and Monthly Global Solar radiation on Inclined Surfaces: Models Re-Visited", Energies, Vol. 10, No. 1, pp. 1-28, 2017.
[18] C. Hajjaj, et al., "Evaluation. Comparison and Experimental Validation of Different PV Power Prediction Models under Semi-Arid Climate", Energy Convers. Manag., Vol. 173, No. August, pp. 476-488, 2018.
[19] G. Pi, S. Perelman, J.P. Guerschman, "How to Evaluate Models: Observed vs. Predicted or Predicted vs. Observed?", Ecol. Model., Vol. 6. pp. 316-322, 2008.
[20] D. Tschopp, A.R. Jensen, J. Dragsted, P. Ohnewein, S. Furbo, "Measurement and Modeling of Diffuse Irradiance Masking on Tilted Planes for Solar Engineering Applications", Sol. Energy., Vol. 231, No. December, pp. 365-378, 2022.
[21] D. Granados Lopez, A. Suarez Garcia, M. Diez Mediavilla, C. Alonso trist, "Feature Selection for CIE Standard Sky Classification", Sol. Energy., Vol. 218, No. March, pp. 95-107, 2021.
[22] I. Tarhi, T. Hassouni, E.M. Al Ibrahmi, D. Lamri, C. El Mahjoub, "Mathematical Modeling in Physics and Conceptions of Learners: Force and Differential Equation", International Journal on Technical and Physical Problems of Engineering (IJTPE), Issue 56, Vol. 15, No. 3, pp. 1-8, September 2023.
[23] J. Bernard, "Solar Energy Calculations and Optimization", Ellipses, Paris, France, 2004.
[24] K. Gibbs, "Solar Energy", Solar Energy, November 2022, www.schoolphysics.co.uk/age16-19/Astrophysics/text/Solar_energy/index.html.
[25] L. Ma, J. Nie, J. Yang, "Two forms of Wien's Displacement Law", Latin-American J. Phys. Educ., Vol. 3, No. 3, pp. 566-568, 2009.
[26] Canadian Space Agency, "The Sun", The Sun, January 2021, www.asc-csa.gc.ca/fra/astronomie/systeme-solaire/soleil.asp.
[27] M. Meftah, et al., "Solar Radius Determined from Picard/Sodism Observations and Extremely Weak Wavelength Dependence in the Visible and the Near-Infrared", Astron. Astrophys., Vol. 616, No. A64, pp. 1-10, 2018.
[28] J. Simon, P. Bretagnon, J. Chapront, M. Chapront Touze, "Numerical Expressions for Precession Formulae and Mean Elements for the Moon and the Planets", Astron. Astrophys., (Berlin. Print), Vol. 282, No. 2, pp. 663-683, 1994.

BIOGRAPHIES

Name: **Mohammed**
Surname: **El Malki**
Birthday: 05.10.1991
Birthplace: Settat, Morrocco
Bachelor: Renewable Energies, Faculty of Sciences, University of Chouaib Doukkali, El Jadida, Morocco, 2016
Master: Materials, Radiation Energy and Environment, Faculty of Sciences, University of Chouaib Doukkali, El Jadida Morocco, 2018
Doctorate: Student, University of Chouaib Doukkali, El Jadida, Morocco, Since 2018
The Last Scientific Position: Teacher, Physics and Chemistry, High School, Morocco
Research Interests: Materials of Physics, Theoretical, Physics Experimental Physics
Scientific Publications: 1 Paper

Name: **Charaf**
Surname: **Hajjaj**
Birthday: 02.10.1990
Birthplace: Bennour, Morocco
Bachelor: Physics in Electronics, Department of Physics, Faculty of Sciences, University of Chouaib Doukkali, El Jadida, Morocco, 2012
Master: Telecommunications Networks and Industrial Electronics, Faculty of Sciences, University of Chouaib Doukkali, El Jadida, Morocco, 2014
Doctorate: Implementation and Characterization of a Hybrid PVT Device - Modeling and Prediction of PV Yield in a Semi-Arid Climate, Research Institute for Solar Energy and New Energies, University of Chouaib Doukkali, El Jadida, Morocco, 2019
The Last Scientific Position: Assoc. Prof., Higher School of Technology of Essaouira, Cadi Ayyad University, Marrakesh, Morocco, Since 2019
Research Interests: Energy Management, Energy Efficiency, Energy Modeling, Sustainable Energy
Scientific Publications: 29 Papers

Name: **Hamid**
Surname: **Nebdi**
Birthday: 01.10.1968
Birthplace: El Jadida, Morocco
Bachelor: Physics/Electronics, Department of Physics, Faculty of Sciences, University of Chouaib Doukkali, El Jadida, Morocco, 1993
Master: Radiation-Matter Physics, Faculty of Science Ben M'Sik, Hassan II University, Casablanca, Morocco, 1995
Doctorate: Atomic Physics, University Catholic of Louvain, Ottignies-Louvain-la-Neuve, Belgium, 2000
The Last Scientific Position: Prof., Department of Physics, Faculty of Sciences, University of Chouaib Doukkali, El Jadida, Morocco, Since 2011
Research Interests: Space Weather, Atomic and Laser Physics, Education, Innovation in Science, Technology and Modeling
Scientific Publications: 76 Papers, 5 Projects, 7 Theses
Scientific Memberships: SPANET, GIRGEA, AERDDS