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IMPACT OF USING MATHEMATICS LEARNING SOFTWARE ON PUPILS PROBLEM SOLVING AND CONVERTING SEMIOTIC REGISTERS OF NUMERICAL FUNCTIONS

D. Bouanfir [1] M. Abid [2] B. El Wahbi [1]

1. Laboratory of Geometric Analysis and Applications, Faculty of Science, Ibn Tofail University, Kenitra, Morocco bouanfir.douaa@gmail.com, belwahbi@yahoo.fr
2. Laboratory of Scientific Research and Pedagogical Innovation, Regional Center for Education and Training Professions, Madinat Al Irfane, Rabat, Morocco, profabidmed@gmail.com

Abstract- This study investigated the impact of integrating GeoGebra software into the mobilization and consolidation of new knowledge stages in the teaching-learning process of numerical functions studies on second-year Moroccan high school pupils. This research used a quasi-experimental design with two groups (experimental and control) to compare the achievement and difficulties of pupils receiving instruction using GeoGebra software and those adopting the traditional way of problem-solving. The sample consisted of 260 pupils in pre-test and direct post-test and 147 pupils in long-term post-test (LT post-test), divided into two streams: physics chemistry (PC) and mathematics sciences (MS). The obtained data were examined and analyzed using Excel and SPSS software. The results showed that PC and MS pupils' performance to solve numerical function problem was improved in the experimental subgroup (ESG) on the LT post-test ($M_{EPC} = 16.81 > M_{CPC} = 12.84$ and $M_{EMS} = 18.73 > M_{CMS} = 15.64$) compared with the ESG on the direct post-test. In addition, mental calculation difficulties of the calculative, logical and technical types encountered by PC and MS pupils in the ESG of the direct post-test were reduced over time in post-test LT. Moreover, including using of GeoGebra to study numerical functions increased PC and MS pupils' LT performance in the conversion of semiotic representation registers associated to the concept of numerical function.

Keywords: Numerical Function, GeoGebra, Semiotic Representation Register, Register Conversion.

1. INTRODUCTION

Information and communication technologies (ICT) promote the use of a teaching approach based on the implementation of differentiated pedagogy (learner development according to his or her own pace) and autonomy pedagogy (the active investment of the learner in the construction of his or her knowledge) [1]. The use of ICT has been integrated into the education systems of most countries [2].

The use of technology in the teaching process can help produce improved results for pupils [3]. Integrating technology in classroom mathematics teaching and learning has been one of the priorities of the Moroccan curriculum in secondary school [4]. Masri, et al. [3] announced that the best understanding of concepts by students occurs when they have actively participated and motivated by technology. Computers have made some problems and topics more accessible and provided new ways of representing and treating mathematical information to offer choices in content and pedagogy of teaching that didn't exist before [3]. Integrating mathematical software into teaching and learning helps pupils to gain a better understanding of mathematical concepts [5]. Dynamic mathematics software can produce rapid numerical and symbolic calculations, and has been used worldwide, such as MATLAB, Maple, Autograph, Geometer's Sketchpad (GSP) and the graphing calculator [2]. Many mathematics software programs have been considered as computer-assisted learning to help students execute skills based on their active thinking and ability to make choices [6].

At present, schools have to combat pupils' unsatisfactory learning of mathematical concepts to reduce the failure associated with their underperformance. Jad, et al. [7] have concluded that learning difficulties are among the causes of school failure. Pupils found difficulties in understanding mathematical concepts because they couldn't imagine the application of these concepts [3]. To this end, images of pedagogical technology enable pupils to connect their knowledge and experiences with their new information [3]. Examining the applications of pedagogical technology remains a challenge in order to know the importance of these tools in pupils' learning. Many studies have determined effectiveness of teaching and learning mathematics using software. Studies carried out by Magallanes [8] confirmed the significant difference which exists between results obtained with traditional method and those obtained with the Ethnomathematics software, and that students who used the latter method achieved better results.

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Trespalacios and Perez-Quinones [9] demonstrated the existence at post-test of a significant difference (no significant difference at pre-test) after using e-books in university pre-calculus courses. Zengin, et al. [10] concluded that experimental pupils performance score, including integration of GeoGebra software into the teaching and learning of trigonometry for 5 weeks, was better than the control group. In addition, research has shown that the use of technological tools has no advantage on pupils' performance in teaching and learning mathematics [2]. For example, High [11] proved no significant difference in pupils' performance in introductory statistics course using two methods: the traditional method and the use of computers. However, there are open-source mathematics software that can be downloaded and used free of charge in the teaching and learning mathematics [2]. GeoGebra is one of these open-source programs, and was chosen in this study to compare the conventional method of solving a numerical function problem with that using GeoGebra.

2. PROBLEMATIC OF STUDY

Solving problems involving the examination of numerical functions requires the investment of skills in solving many successive activities: calculating domains of definition, calculating limits, calculating derivations, studying variations and drawing graph function. Given that graphical representation requires an imagination of infinite branches, it is this theme that has been chosen to be treated with the open-source software GeoGebra in this study, in order to develop skills for assimilating the graphical appearance of a given numerical function. Consequently, the goal of this study is to examine the effect of integrating GeoGebra software, during the mobilization and consolidation stages of the teaching-learning process, on pupils' performance in solving numerical function study problems, using a didactic analysis framework of semiotic representation registers. This software contains constructive characteristics that make it possible to visualize mathematical concepts in teaching and learning all over the world [12]. Understanding the concept function requires two essential skills: knowledge of semiotic representation registers and the conversion of the different representations produced in one system to another [13].

Our approach aims to evaluate this objective in Moroccan second-year baccalaureate students in physics chemistry (PC), and mathematics sciences (MS). This study is based on the following research question:

- What effect does use GeoGebra software in the mobilization and consolidation of new knowledge stages of the teaching-learning process have on the performance of second-year Moroccan baccalaureate pupils (PC and MS groups) in solving the problem of studying a numerical function and converting the registers of semiotic representation of this concept?

3. CONCEPTUAL ELEMENTS

3.1. GeoGebra Software

GeoGebra software was developed in 2001 by Markus Hohenwarter [14]. It combines a computer algebra system with a dynamic geometry system [12]. It can be installed in Android and Windows [14]. These two systems provide, respectively, visualization and dynamic change capacities [12]. The visualization of algebra and geometry windows enables algebraic representations to be linked with the geometric one [3]. Using GeoGebra software is easier than other software that requires programming skills such as Maple [3]. It helps focus on mathematical concepts while easily constructing tangents, graphs and angles because all instructions are provided in the menu bar [3]. The application of GeoGebra can help students explore mathematical concept in a different and fun learning environment than conventional teaching [3]. GeoGebra software can motivate pupils to teach and learn math's [6]. Moreover, GeoGebra is a pedagogical learning tool that allows students to verify the veracity of their solved mathematical problems [15].

3.2. Semiotic Register

Understanding mathematical concepts requires the exploitation of semiotic representation registers [16]. These registers are the treatment rules and points of view treated during the examination of a mathematical problem. Mathematical objects are necessarily manipulated through registers of semiotic representations (algebraic representations, graphical representations, figurative representations, natural language register) [17]. Semiotic representations serve not only to activate the cognitive process of thinking, but also to develop mental representations, to perform different cognitive functions and to produce knowledge [18]. They represent productions composed of using signs that belong to a representation system, whose operation and significance follow precise constraints [19]. These representations explain and work mathematical objects, which are considered to be abstractions of thought [18].

To understand a mathematical object, we need to work with many registers of representation [20]. Working with all these registers is done with the help of conversion between them, which is the act of providing a new register that is seen as a guide to treatments carried out in another register [16]. Marc [21] announces that the passage between registers is an essential element for the understanding of a mathematical object, knowing that each register is characterized by its specific treatments and properties. He also adds that the passage between two registers is done according to conversion rules without making any transfer of properties from the origin register to the arrival register. The understanding of a mathematical concept and the progression of learning are based on conversion between semiotic representation registers [16]. In this problem-solving study of numerical functions, the treatment carried out is based on conversion between four registers of semiotic representation presented in the following semiotic chain

[21]: algebraic symbolic register (ASR), Table of variation register (TVR), Natural language register (NLR) and Graphic register (GR). In a mathematical concept, when there are n representations, there are n-1 treatments and conversions in the associated semiotic representation register chain and n! treatments and conversions between the representations in total [22].

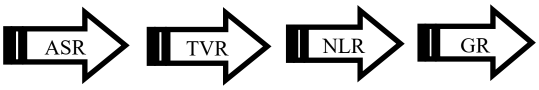

Figure 1. Semiotic chain associated with a function numeric [21]

3.3. Mental Activity

Performing a mental activity involves a high degree of knowledge, in order to manipulate different problem situations. The performance of knowing semiotic representations is one of that knowledge that allows the individual's mental representations to be made visible to others (Duval, 2006). To study a numerical function, it is necessary to have prior knowledge of the different typologies of mental activity linked to this concept: calculative activity, logical activity (developing the quality of reasoning), technical activity (creating the variation table and graphical representation of the function). In this study, we aim to test these mental activities in the context of transforming initial data from a representation in a register of the concept function, in order to obtain terminal data in the same register.

3.4. Numeric Function

The term function is of primary importance in mathematical thinking, associated with the study of different analytic expressions and dependent on geometric quantities linked with the shape of curves [23]. Lagrange declared in 1806 that functions are operations performed on known quantities in order to obtain the values of other unknown quantities, according to him, a function represents a combination of operations [19]. The concept of function evolved with the evolution of three mental images: geometry (expression of curves), algebra (expression of formulas) and function's logical definition (corresponding to a machine's input-output mental image) [24]. Concept of function is important in the learning process, since its study enables students to make connections between its multiple representations: numerical representations, graphical representations, symbolic representations (equations), verbal representations [25]. The numerical function is used to describe different situations using geometry and Cartesian representation [23].

3.5. Moroccan Pedagogical Engineering for Numerical Function Course in the 2nd Year of the Baccalaureate

The Moroccan curriculum recommends a specific amount of time for teaching and learning numerical functions in the second year of the Moroccan baccalaureate PC and MS streams. The main parts of the numerical function chapter are: continuity, derivability and function study, primitive function, logarithmic and exponential functions. The present study focuses on derivability and function study (10 hours for PC and 12 hours for MS), which involves teaching and learning the study of numerical functions.

4. MATERIALS AND METHODS

4.1. Research Conception

To collect and analyze the data from this study, we adopted a control and experimental plan and a quantitative and analytical methodology in order to raise the difference in the realization of the objectives of the teaching and learning the numerical function study with and without integration the GeoGebra software, in Moroccan 2nd year baccalaureate pupils. This conception makes it possible to compare the degree of mastery of problem solving and conversion between semiotic representation registers linked to function concepts.

4.2. Target Population

The study pupils taking part in pre-test and direct post-test were 260 Moroccan learners (172 pupils in the PC group and 88 pupils in the MS group) in the 2nd baccalaureate year from the city of Casablanca during the 2022-2023 school year. In the PC group, there were two subgroups: 86 in the control subgroup (noted CSG) and 86 in the experimental subgroup. In the MS group, there were two subgroups as well: 44 in the CSG and 44 in the ESG, Table 1. While 147 Moroccan pupils from the study's sample (69 pupils from the CSG and 78 pupils from the experimental subgroup) participated in post-test LT, during the same 2022-2023 school year as Table 1.

Table 1. Distribution of groups in pre-test, direct post-test & LT post-test

	Control			Experimental		
	Pre-test	Direct post-test	LT post-test	Pre-test	Direct post-test	LT post-test
PC group	86	86	33	86	86	39
MS group	44	44	36	44	44	39

4.3. Instrument of Research

Data were collected using three tests to evaluate the realization of numerical functions study (pre-test, direct post-test and LT post-test). The pre-test included four exercises that examined pupils' knowledge of the justification and precision of definition domain, the calculation of limits, the study of function continuity and the study of function derivability. The direct post-test included two problems in the conception of numerical function study (Table 2): The first problem for the PC group concerning the realization of the study of the following complex numerical function $f(x)=x\sqrt{1+x^2}-x^2$ and the second one for the MS group whose aim is the realization of the examination of the given complex numerical function $k(x)=\sqrt[3]{x^2}+\sqrt[3]{x}-x$. The LT post-test included a problem of realization of the numerical function study $g(x)=1+\dfrac{x}{\sqrt{1+x^2}}$ for both PC and MS groups (Table 3).

Table 2. Conversions and treatments activities of each question of the proposed direct post-test for PC and MS groups

item	Groups	Treatments and conversions activities
Q_1 and Q_2	PC and MS	Concept of definition domain and limit of a function: using calculator and logical mentals treatments and the algebraic notation to justify definition domain of f / k and to calculate the limit of f / k at the bounds of D_f /D_k in order to express algebraic symbolic register "ASR"
Q_3	PC and MS	Concept of variation table: using technical mental treatments and the necessary conditions of variation table to express the table of variation register "TVR"
Q_4	PC and MS	Concept of branch infinite: using calculator and logical mentals treatments and the conditions adopted to function f to justify the branch infinite in order to express the symbolic algebraic register "ASR" and the final answer with the natural language register "NLR".
Q_5	PC	Using the previous answers of questions and knowing the notion of tangent (the tangent equation is given in the statement) and reciprocal function concepts to drawing the curves (C_f) and ($C_{f\text{-}1}$) with technical mental treatments in order to express the graphical register "GR".
Q_5	MS	Concept of derivability of function: using calculator and logical mental treatments and the algebraic notation to justify the derivability of function k and to interpret geometrically the result (the function admits a vertical half-tangent) in order to express the algebraic symbolic register "ASR" and the natural language register "NLR".
Q_6	MS	Using the previous answers of questions and knowing restriction and reciprocal functions concepts to drawing the curves (C_k), (C_g) and $(C_{g^{-1}})$ with technical mental treatments in order to express the graphical register "GR"

Table 3. Conversions and treatments activities of each question of the proposed LT post-test for PC and MS groups

item	Treatments and conversions activities
Q_1 and Q_2	Concept of definition domain and limit of a function: using calculatory and logical mental treatments and the algebraic notation to justify definition domain of g and to calculate the limit of g at the bounds of Dg in order to express the algebraic symbolic register "ASR".
Q_3	Concept of variation table: using technical mental treatments and the necessary conditions of variation table to express the table of variation register "TVR"
Q_4	Concept of branch infinite: using calculator and logical mentals treatments and the conditions adopted to function g to justify branch infinite in order to express symbolic algebraic register "ASR" and final answer with natural language register "NLR"
Q_5	Using the previous answers of questions and knowing the notion of tangent (the tangent equation is given in the statement) and inflexion point and center of symmetry and reciprocal function concepts to drawing the curves (C_g) and $(C_{g^{-1}})$ with technical mental treatments in order to express the graphical register "GR"

The three tests took the form of a supervised test lasting 20 minutes for the pre-test and 60 minutes for both post-tests. Each question in the three tests corresponds to the performance of a mental activity. Results were collected using a good scale with Cronbach's alpha between 0.6 and 0.7. The criteria for fulfilling the learning objectives and competencies in the examination of numerical functions in the context of this study require that pupils sample obtain a minimum average score of 67% (2/3) on the test items to prove their mastery of this crucial mathematical skill. Test items with scores below 67% are a sign of potential learning difficulties for the pupils.

4.4. Test Procedure

The pre-test was carried out before teaching the numerical function study, and direct post-testing was done after teaching and correcting exercises in class. The experimental study includes a middle part (between pre-test and direct post-test) that focuses on the teaching-learning process with regard to the mobilization and consolidation of knowledge by way of the completion of exercises and problems, in which the GeoGebra software is introduced and explained and exercises involving the study of numerical functions are solved. Only pupils in the experimental PC and MS groups were allowed to utilize GeoGebra in these exercises' activities. The number of pupils in the initial sample (direct post-test) was reduced because their teachers declined to participate in the LT post-test study because the pupils were preoccupied with the final exam and there wasn't enough time to complete the program. The LT post-test was conducted five months after the direct post-test. It should be noted that the tests were given to four mathematics teachers with long teaching experience to check their reliability, and each test is awarded a 20-point mark.

4.5. The GeoGebra Software Intervention Process

The 6 steps of the study were carried out as follows: 1) the pre-test phase, 2) the course teaching phase, 3) the GeoGebra pupil initiation phase, 4) the GeoGebra integration phase in the training exercises section, 5) the direct post-test phase, 6) the LT post-test phase. Phases 5 and 6 of the direct and LT post-tests were run for 60 minutes for each post-test and each group. Pupils were grouped into control and experimental subgroups. The treatment of ESG followed all of these steps in contrast to the CSG, who's the absence of two phases (2 and 3) while retaining phase 4 without integration of GeoGebra.

In phase number 4 of the experimental subgroup, pupils were taught in class how to solve problems involving the study of numerical functions using GeoGebra software. The intervention with the pupils took place in the following steps:

1) give the pupils a moment to read and understand the statement and do the calculations by hand in an individual way.

2) give the pupils the opportunity to check their answers with GeoGebra. Then invite pupils to apply GeoGebra to other numerical functions in the school textbook. The usefulness of using the GeoGebra checker for each question in the study of the typical function $F(x)=\ln(\frac{x}{2-x})$ solved in class is as follows:

Q_1) Determine D_F with justification "after typing the function $F(x)$ in the input field of the algebra window, the definition domain (0, 2) appears in the graph".

Q_2) Calculate the limits of F at the bounds of D_F "the limits of the function $F(x)$ are observed from the graph and calculated in the algebra window using the following notation:

$a = LimitAbove(F,0) = -\infty$ and

($b = LimitBelow(F,2) = +\infty$).

Q_3) Draw the table of variation of F and

Q_4) Studying the infinite branches "the variation of the function $F(x)$ and the infinite branches can be seen from the graph of the function".

Q_5) Draw in the same frame the curves (C_F) and $(C_{F^{-1}})$ "after applying all the data in the algebra window, the graphical representation appears in GeoGebra".

4.6. Procedure for Data Analysis

Descriptive and inferential statistics were applied to the obtained data to be analyzed. The Kolmogorov-Smirnov and Shapiro-Wilk tests were used to establish the test type and to check the test results for normality in the data distribution.

4.7. Ethical Research Considerations

Authorization for this research was obtained prior to the start of the study from the provincial education direction, Morocco. The names of the schools, teachers and pupils are not mentioned in the research report to guarantee their confidentiality.

5. RESULTS

5.1. Pupils' Performance on the Pre-Test

5.1.1. Descriptive Analysis of PC and MS Group According to Pre-Test

In the PC group, the pre-test mean was equal to $M_{CPC} = 10.13$ in the control PC group and $M_{EPC} = 10.65$ in the experimental PC group, with a minimum score of $5/20$ and a maximum score of $18/20$ in both subgroups. Whereas in the MS group, the pre-test mean is equal to $M_{CMS} = 16.72$ in the control *MS* group and $M_{EMS} = 15.72$ in the experimental *MS* group, with a minimum score of $7/20$ and a maximum score of $20/20$ in both sub-groups.

5.1.2. Homogeneity of Variances Test between CSG and ESG According to Pre-Test for PC Groups

According to Kolmogorov-Smirnov, the p-value of the pretest score for the control and experimental subgroups is equal to 0.076 and 0.082 respectively which are greater than 0.05, so the available data were normally distributed. According to Shapiro-Wilk, the p-values of the pre-test score for the control and experimental subgroups are equal to 0.038 and 0.011 respectively which are less than 0.05, so the available data were not normally distributed. According to Table 4, the distribution of the pre-test score for both subgroups (control and experimental) is quasi-normal, as the *Z* Skewness and *Z* Kurtosis values fall within the range -3.29 and +3.29.

Table 4. Skewness symmetry index and Kurtosis flattening index

	Pre-test score for control PC group	Pre-test score for experimental PC group
Skewness	0.326	0.228
Std. Error of Skewness	0.260	0.260
Z Skewness	1.253	0.876
Kurtosis	–0.460	–0.670
Std. Error of Kurtosis	0.514	0.514
Z Kurtosis	–0.894	–1.303

Table 5 demonstrates that, for the two subgroups of the PC group, the significant value of Levene's test of pre-test score variance homogeneity is equal to 0.556, which is greater than 0.05, indicating that the variance of PC pupils in the CSG is equal to the variance of PC pupils in the experimental one. The significant value of the student t-test is equal to 0.308 which is greater than 0.05, so there is no significant distinction between the mean pre-test score of the two PC subgroups (control and experimental).

Table 5. Student's t-test for pre-test score of control and experimental for PC group

	Levene's Test for Equality of Variances		*t*-test for Equality of Means						
	F	Sig.	*t*	*df*	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
								Lower	Upper
Equal variances assumed	0.348	0.556	–1.023	170	0.308	–0.51163	0.50032	–1.49927	0.47602

5.1.3. Homogeneity of Variances Test between CSG and ESG According to Pre-Test for MS Groups

The available pre-test score data from the two subgroups (control and experimental) of the MS group were not normally distributed, as the Kolmogorov-Smirnov p-value and the Shapiro-Wilk p-value are less than 0.001, which is less than 0.05. The Mann-Whitney p-value equals 0.406 which is higher than 0.05. There was therefore no significant distinction between the pre-test of pupils in the two subgroups (control and experimental) of the MS group. So, the variance of MS pupils in the CSG is equal to the variance of MS pupils in the ESG (Table 6).

Table 6. Mann-Whitney test for MS group (pre-test)[a]

Mann-Whitney *U*	870.000
Wilcoxon *W*	1860.000
Z	–0.831
Asymp. Sig. (2-tailed)	0.406
a. Grouping Variable: Subgroups (control and experimental)	

5.2. Pupils' Performance on the Direct Post-Test

5.2.1. Descriptive Analysis of PC Group

Table 7 summarizes all the descriptive data collected from 172 students in the PC group concerning problem solving in numerical function study with the traditional method and that integrating GeoGebra software in the training exercises and in the direct post-test. The mean score for the PC group in the CSG was $M_{CPC} = 10.51$ with a standard deviation of 3.76, and a mean score of $M_{EPC} = 10.62$ in the ESG with a standard deviation of 3.14. The lowest scores for the control and experimental subgroups were $5.75/20$ and $7/20$ respectively, while the highest scores for the control and experimental subgroups were $16/20$ and 17/20, respectively.

According to criteria for fulfilling the learning competencies in the study, the results of the PC group given in Table 7 show those pupils in the CSG have difficulties in drawing graphical representation of functions (Q_5). Whereas pupils in the ESG have difficulties in: calculating limits (Q_2), treatment of the variation table (Q_3), studying infinite branches (Q_4) and graphical representation of functions (Q_5).

Table 7. descriptive statistics based on item score and direct post-test score of PC group

Subgroups	N	Questions	Scale	Mean score/Item	Criteria average score (%)
Control	86	Q_1	1/1	1.00	100
		Q_2	3/3	2.93	97
		Q_3	2/2	1.98	99
		Q_4	4/4	2.791	70
		Q_5	10/10	1.814	18
Experimental	86	Q_1	1/1	0.872	87
		Q_2	3/3	1.744	58
		Q_3	2/2	1.163	58
		Q_4	4/4	1.541	39
		Q_5	10/10	5.448	55

5.1.1.

5.2.2. Comparison of Direct Post-Test Scores between Experimental and Control PC Subgroup

The available data were not normally distributed in the direct post-test of the CSG and the ESG of the PC group. Indeed, for both subgroups of the PC group, the Kolmogorov-Smirnov p-value and the Shapiro-Wilk p-value are less than 0.001 which is less than 0.05. The Mann-Whitney p-value equals 0.344 which is higher than 0.05. there was any significant difference between the direct post-test (use of GeoGebra in this test) of pupils in the two subgroups (control and experimental) of the PC group (Table 8).

Table 8. Mann-Whitney test for PC group (direct post-test)[a]

Mann-Whitney U	3391.000
Wilcoxon W	7132.000
Z	–0.947
Asymp. Sig. (2-tailed)	0.344
a. Grouping Variable: Subgroups (control and experimental)	

5.2.3. Descriptive Analysis of MS Group

Table 9 summarizes all the descriptive data collected from 88 pupils in the MS group concerning numerical function study problem solving with the traditional method and that integrating GeoGebra software only in the training exercises and not in the direct post-test. The mean score in the CSG was $M_{CMS} = 13.02$ with a standard deviation of 5.76, and a mean score of $M_{EMS} = 13.84$ in the ESG with a standard deviation of 5.80. The lowest scores for the control and experimental subgroups were $5/20$ and $7/20$ respectively, while the highest score for both subgroups was 20/20. According to criteria for fulfilling the learning competencies in the study, the results of the MS group, presented in Table 9 show that pupils in the CSG have difficulties in: studying infinite branches (Q_4) and drawing graphical representation of function (Q_6). Whereas pupils in the ESG have difficulties in: treatment of variation table (Q_3) and drawing graphical representation of function (Q_6).

Table 9. descriptive statistics based on item score and direct post-test score of MS group

Subgroups	N	Questions	Scale	Mean score/Item	Criteria average score (%)
Control	44	Q1	1/1	1.00	100
		Q2	3/3	2.73	91
		Q3	3/3	2.182	73
		Q4	3/3	1.73	58
		Q5	3/3	2.114	70
		Q6	7/7	3.182	45
Experimental	44	Q1	1/1	1.00	100
		Q2	3/3	2.41	80
		Q3	3/3	1.91	64
		Q4	3/3	2.841	95
		Q5	3/3	2.451	82
		Q6	7/7	3.182	45

5.2.4. Comparison of Direct Post-Test Scores between Experimental and Control MS Subgroup

The available data were not normally distributed in the direct post-test of the CSG and ESG of the MS group. Indeed, for both subgroups of the MS group, the Kolmogorov-Smirnov p-value and the Shapiro-Wilk p-value are less than 0.004 which is less than 0.05. The Mann-Whitney p-value equals 0. 182 which is higher than 0.05. So, there is not a significant distinction between the direct post-test (no use of GeoGebra in this test) of pupils in the two subgroups (control and experimental) of the MS group (Table 10).

Table 10. Mann-Whitney test for MS group (direct post-test)[a]

Mann-Whitney U	810.000
Wilcoxon W	1800.000
Z	–1.336
Asymp. Sig. (2-tailed)	0.182
a. Grouping Variable: Subgroups (control and experimental)	

5.3. Pupils' Performance on the LT Post-Test

5.3.1. Descriptive Analysis of PC Group

Table 11 shows the descriptive data collected from 72 students in the PC group, distributed between pupils in the CSG who solved the numerical function study problem using the traditional method and those in the experimental one who solved the problem using GeoGebra software only in the training exercises and not in post-test LT. The average score for the CSG was $M_{CMS} = 12.84$ with a standard deviation of 6.32, while the mean score for the ESG was $M_{EPC} = 16.81$ with a standard deviation of 2.84. The lowest scores for the control and experimental subgroups were 5/20 and 10.25/20, respectively, while the highest scores for the control and experimental subgroups were 20/20. According to criteria for fulfilling the learning competencies in the study, the results of the PC group, presented in Table 11 show that pupils in the CSG have difficulties in: treatment of variation tables (Q_3) and drawing graphical representation of functions (Q_5). Pupils in the ESG had difficulties only in drawing graphical representation of the function (Q_5).

Table 11. descriptive statistics based on item score and LT post-test score of PC group

Subgroups	N	Questions	Scale	Mean score/Item	Criteria average score (%)
Control	33	Q_1	1/1	1.00	100
		Q_2	4/4	3.273	82
		Q_3	3.25/3.25	2.07	64
		Q_4	4/4	3.273	82
		Q_5	7.75/7.75	3.23	42
Experimental	39	Q_1	1/1	1.00	100
		Q_2	4/4	4.00	100
		Q_3	3.25/3.25	3.25	100
		Q_4	4/4	3.95	99
		Q_5	7.75/7.75	4.62	62

5.3.2. Comparison of LT Post-Test Scores between Experimental and Control PC Subgroup

The available data were not normally distributed in the LT post-test of the CSG and the ESG of the PC group, since for both subgroups the Kolmogorov-Smirnov p-value and the Shapiro-Wilk p-value are less than 0.001, which is less than 0.05. The Mann-Whitney p-value equals 0.011, which is less than 0.05, so there is a significant difference in the LT post-test results of pupils in the two subgroups of the PC group. As the mean rank $MR_{EPC} = 42.15 \succ MR_{CPC} = 29.82$, we conclude that the performance of the PC group in LT post-test average score of the ESG is superior to that of the control one (Tables 12 and 13).

Table 12. Ranks of Mann-Whitney test for PC group (LT post-test)

	N	Mean Rank	Sum of Ranks
LT post-test score for CSG	33	29.82	984.00
LT post-test score for ESG	39	42.15	1644.00

Table 13. Mann-Whitney test for PC group (LT post-test)[a]

Mann-Whitney U	423.000
Wilcoxon W	984.000
Z	–2.549
Asymp. Sig. (2-tailed)	0.011
a. Grouping Variable: Subgroups (control and experimental)	

5.3.3. Descriptive Analysis of MS Group

Table 14 shows the descriptive data collected from 75 MS group pupils, distributed between those in the CSG and those in the experimental one. The mean score for the CSG was M_{CMS}=15.64 with a standard deviation of 3.86, while the average score for the ESG was $M_{EMS} = 18.73$ with a standard deviation of 2.21. The lowest scores for the control and experimental subgroups were 10.25/20 and 12.25/20, respectively, while the highest scores for the control and experimental subgroups were 20/20. According to criteria for fulfilling the learning competencies in the study, the results of the MS group, presented in Table 14 show that pupils in the CSG have difficulty in drawing graphical representation of function (Q_5) and pupils in the ESG have any difficulties.

Table 14. descriptive statistics based on item score and LT post-test score of MS group

Subgroups	N	Questions	Scale	Mean score/Item	Criteria average score (%)
Control	36	Q_1	1/1	1.0000	100
		Q_2	4/4	4.0000	100
		Q_3	3.25/3.25	3.2500	100
		Q_4	4/4	3.8333	100
		Q_5	7.75/7.75	3.5625	45
Experimental	39	Q_1	1/1	1.0000	100
		Q_2	4/4	4.0000	100
		Q_3	3.25/3.25	3.2500	100
		Q_4	4/4	3.9487	98
		Q_5	7.75/7.75	6.7105	86

5.3.4. Comparison of LT Post-Test Scores between Experimental and Control MS Subgroup

The available data were not normally distributed in the LT post-test of the CSG and the ESG of the MS group, since for both subgroups the Kolmogorov-Smirnov p-value and the Shapiro-Wilk p-value are less than 0.001, which is less than 0.05. The Mann-Whitney p-value of less than 0.001 is less than 0.05, there is therefore a significant distinction in the LT post-test results of pupils in the two subgroups (control and experimental) of the MS group. As the mean rank $MR_{EMS} = 46.00 \succ MR_{CMS} = 29.33$, we conclude that the performance of the MS group in LT post-test mean score of ESG is superior to that of the CSG (Tables 15 and 16).

Table 15. Ranks of Mann-Whitney test for MS group (LT post-test)

	N	Mean Rank	Sum of Ranks
LT post-test score for CSG	36	29.33	1056.00
LT post-test score for ESG	39	46.00	1794.00

Table 16. Mann-Whitney test for MS group (LT post-test)[a]

Mann-Whitney U	390.000
Wilcoxon W	1056.000
Z	–3.688
Asymp. Sig. (2-tailed)	0.000
a. Grouping Variable: Subgroups (control and experimental)	

5.4. Comparison between Direct Post-Test and LT Post-Test According to Semiotic Registers of ESG of PC and MS Groups

Figures 2 and 3 compare the percentage realization of semiotic representation registers related to numerical function in the experimental subgroups of the two groups PC and MS between the direct post-test and the LT post-test. Figure 2 indicates that the performance of PC pupils varies considerably in the conversion of semiotic representation registers from ASR to TVR (38%) and from TVR to NLR (43%) and from NLR to GR (9%) in favor of the ESG in the long post-test. However, Figure 3 indicates that the performance of MS pupils varies considerably in conversion of semiotic representation registers from ASR to TVR (36%) and from TVR to NLR (5%) and from NLR to GR (42%) in favor of the ESG in the LT post-test.

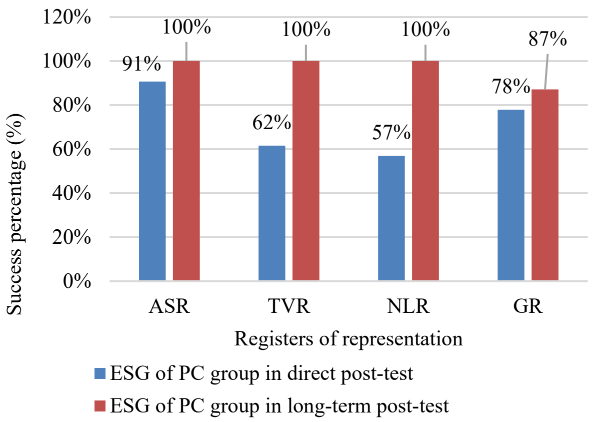

Figure 2. Comparison between direct and LT post-test of ESG of PC group according to successful conversion rate between semiotic registers

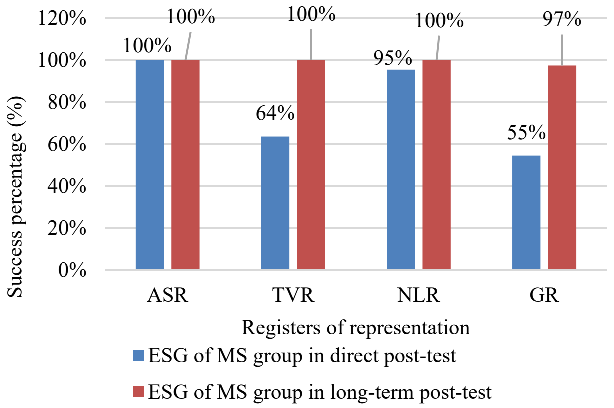

Figure 3. Comparison between direct and LT post-test of ESG of MS group according to successful conversion rate between semiotic registers

6. DISCUSSION

The current study examines the effects, of GeoGebra software integration on the mobilization and consolidation process of the teaching and learning phase, of studying a numerical function in second-year baccalaureate pupils (aged 18) in the Moroccan context. To achieve this, the research design used a quasi-experimental study with two groups-experimental and control - where the experimental group received an intervention that followed the method of integrating GeoGebra software in problem solving, whereas the control group adopted the traditional problem-solving strategy of numerical functions.

In the direct post-test, treatment is performed with two complex numerical functions: $f(x)=x\sqrt{1+x^2}-x^2$ for the PC group and $k(x)=\sqrt[3]{x^2}+\sqrt[3]{x}-x$ for the MS group. The findings of this direct post-test analysis indicate that there isn't any significant distinction in mean score between experimental and control subgroups for both PC and MS groups: for the PC group ($M_{EPC}=10.62$ and $M_{CPC}=10.51$) and for MS ($M_{EMS}=13.84$ and $M_{CMS}=13.02$). These findings are consistent with Puteh and Rohaidah's findings from 2002 [26], which proved that teaching and learning certain mathematical concepts cannot be adequately accomplished by evaluating the effectiveness of integrating GeoGebra in a short period of time. Pupils' mathematical problem-solving skills using GeoGebra have improved over time [27]. Masri, et al. [3] suggested that the study be carried out over a longer time (one or two years) in order to provide better findings.

In the LT post-test, the goal of the treatment is to resolve the issue of treating the same complex numerical function $g(x)=1+\frac{x}{\sqrt{1+x^2}}$, in order to evaluate and determine the role that the integration of GeoGebra software has played in resolving the numerical function study issue between the two types of groups, PC and MS. In the PC and MS groups, the mean score between the experimental and control subgroups differs significantly, according to an analysis of the results of this LT post-test. While this difference favors the ESG in the PC group with a mean score of $M_{EPC}=16.81>M_{CPC}=12.84$, it favors the ESG in the MS group with a mean score of $M_{EMS}=18.73>M_{CMS}=15.64$. These findings demonstrate that pupils in the experimental group have an additional dynamic tool for using GeoGebra software that enables them to more fully understand mathematical concepts and problem-solving techniques in a playful, interactive and visual manner. This is supported by a study by Murni and al. [28] who found that students who used GeoGebra-assisted learning had better problem-solving skills than those who used the traditional learning method. Additionally, GeoGebra helped pupils do better in math, according to Bakar and colleagues [2]. According to Saha et al. [29], GeoGebra-assisted teaching in conjunction with traditional classroom teaching is more effective than traditional teaching alone.

The results from the post-tests (direct and LT) were examined based on the average score criterion, which enables us to conclude that a group of pupils have difficulty mastering the learning objectives if they received an average score less than 67%. Table 17 displays the difficulties and their types. A technical type of mental arithmetic difficulty was present for the PC group's pupils in the CSG of both post-tests (direct and LT), but with a different number of items (one item in the control group of the direct post-test "Q5" and two items in the control group of the LT post-test "Q3 and Q5"). While pupils in the ESG of the two post-tests (direct and LT) encountered difficulties with mental calculation of the calculative and logical types in the direct post-test and difficulties with mental calculation of the technical types in the LT post-test but with a different number of items (four items in the direct post-test, "Q2, Q3, Q4, and Q5" and one item in the LT post-test, "Q5").

For the MS group, pupils in the CSG of the two post-tests (direct and LT) had difficulties with mental calculations of the calculating, logical, and technical types in the post-test direct and difficulties with mental calculations of the technical type in the LT post-test but with a different number of items (two items in the post-test direct "Q4, Q6" and one item in the LT post-test

"Q5"). While the pupils in the ESG of the direct post-test faced math difficulties of the technical type "Q3 and Q6," they weren't confronted with mathematics difficulties in the LT post-test. We can conclude that the number of difficulties found in the LT post-test by pupils in the experimental sub-groups (PC and MS) was significantly lower than that identified in the direct post. These results can be explained by the fact that the pupils were engaged with the end-of-year final exam, so they worked, in a regular and intensive way, the maximum number of exercises and practice problems using the GeoGebra help tool, unlike the pupils in the control sub-groups (PC and MS), who worked in the traditional way, without dynamic support such as GeoGebra, to improve their skills in problem-solving skills related to numerical functions.

This is approved by Iranzo and Fortuny [30] who indicate that GeoGebra enables pupils to diagnose their learning difficulties in order to find problem-solving pathways.

Furthermore, analysis of the direct post-test and the LT post-test indicates that the ability of both the PC and MS groups to convert between the four registers (the 3 conversions in Figure [4]) was in favor of the experimental subgroup: for the PC group (38% difference in conversion from ASR to TVR, 43% difference in conversion from TVR to NLR, 9% difference in conversion from NLR to GR) and for the MS group (36% difference in conversion from ASR to TVR, 5% difference in conversion from TVR to NLR, 42% difference in conversion from NLR to GR).

Table 17. PC and MS pupils' difficulties in solving the direct and LT post-tests

Tests	Groups	Subgroups	Difficulties	Types of difficulties
Direct post-test	PC	Control	Draw graphical function (Q_5)	Technical activities.
		Experimental	Calculate limit (Q_2)	Calculative and logical activities.
			Treat variation table (Q_3)	Technical activities.
			Examine the infinite branch (Q_4)	Calculative and logical activities.
			Draw graphical function (Q_5)	Technical activities.
	MS	Control	Examine the infinite branch (Q_4)	Calculative and logical activities.
			Draw graphical function (Q_6)	Technical activities
		Experimental	Treat variation table (Q_3)	Technical activities
			Draw graphical function (Q_6)	Technical activities
LT post-test	PC	Control	Treat variation table (Q_3)	Technical activities
			Draw graphical function (Q_5)	Technical activities
		Experimental	Draw graphical function (Q_5)	Technical activities
	MS	Control	Draw graphical function (Q_5)	Technical activities
		Experimental	No difficulties	-

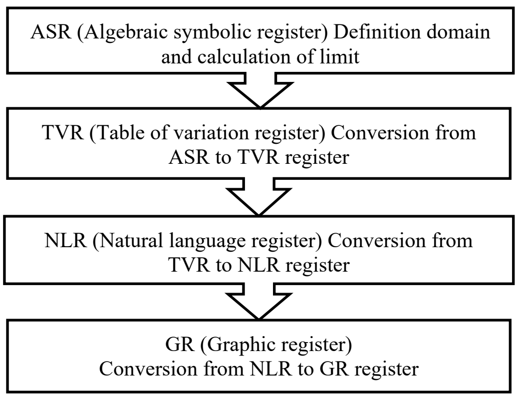

Figure 4. Semiotic representations registers tested in the LT post-test

Even if the complete treatment of the problem does not involve the realization of semiotic representation registers, the resolution of numerical function study problems with GeoGebra allowed us to obtain an evolution of register conversion in the LT post-test for the PC and MS groups. This evolution can be explained by the fact that GeoGebra has combated (in the long term) mental arithmetic difficulties of a calculative and logical type, as well as technical ones: justification of definition domain, limit calculation, drawing the variation table, treatment of infinite branches, graphical representation of the function.

7. CONCLUSION

The study's findings, regarding the integration of GeoGebra software during the mobilization and consolidation phases of the teaching-learning process, showed that pupils in the experimental groups (PC and MS), of the second year of the Moroccan baccalaureate, demonstrated a significant and lasting improvement in their mental arithmetic skills, in the LT post-test. Additionally, there was notable progress in their ability to transform and interpret semiotic representations associated with the concept of numerical functions.

REFERENCES

[1] N. Ameziane, T. Hassouni, K.A. Bentaleb, A. Chahlaoui, "Evaluation of Integration of new ICTS in Teaching-Learning of School System", International Journal on Technical and Physical Problems of Engineering (IJTPE), Issue 54, Vol. 15, No 1, pp. 127-134, March 2023.

[2] K.A. Bakar, A.F.M. Ayub, R.A. Tarmizi, "Exploring the Effectiveness of Using GeoGebra and E-Transformation in Teaching and Learning Mathematics", Advanced Educational Technologies, January 2010.

[3] R. Masri, T.S. Hiong, N.M. Tajudin, Z. Zamzamir, "The Effects of Using GeoGebra Teaching Strategy in Malaysian Secondary Schools: A Case Study from Sibu, Sarawak", Malaysian Journal of Society and Space, Issue 12, No. 7, pp. 13-25, 2016.

[4] Ministry of National Education, "Official Instructions and Programs for Teaching Mathematics in High SCHOOLS 2007", Morocco.
[5] K.A. Bakar, A.F.M. Ayub, R.A. Tarmizi, "Utilization of Computer Technology in Learning Transformation", International Journal of Education and Information Technologies, Issue 2, Vol. 4, pp. 91-98, 2010.
[6] K.A. Bakar, A.F.M. Ayub, R.A. Tarmizi, W.S. Luan, "Exploring Secondary School Students' Motivation Using Technologies in Teaching and Learning Mathematics", Procedia Social and Behavioral Sciences, Issue 2, Vol 2, No. 2, pp. 4650-4654, January 2010.
[7] N. Jad, K. Raouf, K. Elkababi, M. Radid, "Teaching Practices of Scientific Awakening Related to Management of Representations of Primary School Learners: Inspectors Viewpoints", International Journal on Technical and Physical Problems of Engineering (IJTPE), Issue 50, Vol. 14, No. 1, pp. 50-56, March 2022.
[8] A.M. Magallanes, "Comparison of Student Test Score in a Coordinate Plane Unit Using Traditional Classroom Techniques Versus Traditional Techniques Coupled with an Ethnomathematics Software at Torch Middle School", Thesis Master, National University, 2003.
[9] J.H. Trespalacios, M.A. Perez Quinones, "Teaching Functions to Pre-Calculus Students: An Electronic Workbook", The 29th ASEE/IEEE Frontiers in Education Conference, pp. 12d519-12d524, 1999.
[10] Y. Zengin, H. Furkan, T. Kutluka, "The Effect of Dynamic Mathematics Software GeoGebra on Student Achievement in Teaching of Trigonometry", Procedia - Social and Behavioral Sciences, Vol. 31, pp. 183-187, January 2012.
[11] R.V. High, "Some Variables in Relation to Students' Choice of Statistics Classes: Traditional Versus Computer-Supported Instruction", Educational Resources Information Center (ERIC), p. 13, 1998.
[12] Y.W.A. Lu, "English and Taiwaneses Upper Secondary Teachers' Approaches to the Use of GeoGebra", Acta Scientiae, Vol. 10, No. 2, pp. 38-56, 2008.
[13] R. Duval, "Register of Semiotic Representation and Cognitive Functioning of Thought", Annals of Didactics and Cognitive Sciences, Vol. 5, pp. 37-65, 1993.
[14] A. Septian, S. Inayah, R.F. Suwarman, R. Nugraha, "GeoGebra-Assisted Problem Based Learning to Improve Mathematical Problem-Solving Ability", Advances in Social Science Education and Humanities Research, Vol. 467, pp. 67-71, 2020.
[15] I.U. Machromah, M.E.R. Purnomo, C.K. Sari, "Learning Calculus with GeoGebra at College", Journal of Physics: Conference Series, Vol. 1180, No. 1, p. 012008, 2019.
[16] R. Duval, "A Cognitive Analysis of Problems of Comprehension in a Learning of Mathematics", Journal Educational Studies in Mathematics, Vol. 61, No. 1, pp. 103-131, 2006.
[17] R. Duval, R. Sutherland, J. Mason, "Geometrical Pictures: Kinds of Representation and Specific Processings", Exploiting Mental Imagery with Computers in Mathematics Education, pp. 142-157, 1995.
[18] R. Duval, "Representation, Vision and Visualization: Cognitive Functions in Mathematical Thinking. Basic Issues for Learning", The Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education, p. 25, 1999.
[19] C.G.R. Avendano, "Teaching and Learning of Numerical Functions in a Context of Modeling and Mathematical Work", Paris City University, Paris, France, 2020.
[20] R. Duval, "Register of Semiotic Representation and Cognitive Functioning of Thought", Annals of Didactics and Cognitive Sciences, Vol. 5, pp. 37-65, 1993.
[21] R. Marc, "Frame Changes in Mathematical Practice and Regine Douady's Frame Game", Lille Didirem University of Science and Technology, Paris, France, April 2019.
[22] D.L. McGee, "An Operational Approach to Conceptual Understanding Using Semiotic Theory", Interdisciplinary Approaches to Semiotics, A.L.V. Azcarate, (Ed.), InTech, 2017.
[23] J.P. Ponte, "The History of the Concept of Function and Some Educational Implications", The Mathematics, Vol. 3, No. 2, p. 9, 1992.
[24] I. Kleiner, "Evolution of the Function Concept: A Brief Survey", The College Mathematics Journal, Issue 4, Vol. 20, No. 4, pp. 282-300, September 1989.
[25] I. Waisman, M. Leikin, S. Shaul, R. Leikin, "Brain Activity Associated with Translation between Graphical and Symbolic Representations of Functions in Generally Gifted and Excelling in Mathematics Adolescents", International Journal of Science and Mathematics Education, Vol. 12, No. 3, pp. 669-696, 2014.
[26] M. Puteh, D. Rohaidah, "Geometer's SketchPad: Its Use in the Learning of Additional Mathematics", National Conference on Mathematics Education, pp. 270-281, Tanjong Malim, Malaysia, 2002.
[27] Z. Kohen, M. Amram, M. Dagan, T. Miranda, "Self-Efficacy and Problem-Solving Skills in Mathematics: The Effect of Instruction-Based Dynamic Versus Static Visualization", Interactive Learning Environments, Issue 4, Vol. 30, No. 4, pp. 759-778, April 2022.
[28] V. Murni, S. Sariyasa, I.M. Ardana, "GeoGebra Assist Discovery Learning Model for Problem Solving Ability and Attitude toward Mathematics", Journal of Physics: Conference Series, Vol. 895, No. 1, p. 012049, 2017.
[29] R.A. Saha, A.F.M. Ayub, R.A. Tarmizi, "The Effects of GeoGebra on Mathematics Achievement: Enlightening Coordinate Geometry Learning", Procedia - Social and Behavioral Sciences, Vol. 8, pp. 686-693, 2010.
[30] N. Iranzo, J.M. Fortuny, "Influence of GeoGebra on Problem Solving Strategies", Brill, Vol. 6, pp. 91-103, January 2011.

BIOGRAPHIES

Name: **Douaa**
Surname: **Bouanfir**
Birthday: 11.12.1992
Birthplace: Casablanca, Morocco
Bachelor: Science and Technology in Mathematics and Computer Science, Department of Mathematics, Faculty of Technical Sciences, Hassan II University, Casablanca, Morocco, 2016
Master: Teaching and Training Professions in Mathematics, Department of Mathematics, Faculty of Science, Ibn Tofail University, Kenitra, Morocco, 2019
Doctorate: Student, Didactics, Educational Sciences, Teaching and Training Professions in Mathematics and Experimental Sciences, Department of Mathematics, Faculty of Science, Ibn Tofail University, Kenitra, Morocco, Since 2019
Research Interests: Teaching Difficulties and Digital Function Learning
Scientific Publications: 1 Paper

Name: **Mohammed**
Surname: **Abid**
Birthday: 07.10.1969
Birthplace: Sidi Bennour, Morocco
Bachelor: Chemistry, Department of Chemistry, Faculty of Sciences, Ibn Tofail University, Kenitra, Morocco, 1995
Master: Chemistry, Department of Chemistry, Faculty of Sciences, Ibn Tofail University, Kenitra, Morocco, 1997
Doctorate: Applied Chemistry, Department of Chemistry, Faculty of Sciences, Ibn Tofail University, Kenitra, Morocco, 2000
The Last Scientific Position: Prof., Regional Center for Education and Training Professions, Rabat, Morocco, Since 2011 - Assoc. Prof., Faculty of Sciences, Ibn Tofail University, Kenitra, Morocco, Since 2015
Research Interests: Science Didactic, Chemistry of Glasses
Scientific Publications: 28 Papers, 1 Thesis

Name: **Bouazza**
Surname: **El Wahbi**
Birthday: 01.01.1967
Birthplace: Rabat, Morocco
Bachelor: Mathematics, Department of Mathematics, Faculty of Science, Mohamed V University, Rabat, Morocco, 1989
Master: Mathematics, Department of Mathematics, Faculty of Science, Mohamed V University, Rabat, Morocco, 1993
Doctorate: Department of Mathematics, Faculty of Science, University of Mohamed V, Rabat, Morocco, 1998
The Last Scientific Position: Prof., Mathematics, Ibn Tofail University, Kenitra, Morocco, Since 2001
Research Interests: Mathematical Analysis and Application, Didactics of Mathematics
Scientific Publications: 30 Papers, 20 Communications
Scientific Memberships: Member of Laboratory of Analysis, Geometry and applications (LAGA)