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ENERGY-AWARE ADAPTIVE INTEGRATION OF NONLINEAR OSCILLATORY SYSTEMS

Committee of Mathematics and Physics (09MAP)

International Organization on Technical and Physical Problems of Engineering (IOTPE)

Presentation Roadmap

1. Motivation and physical model
2. Energy-aware RK4 acceptance rule
3. Duffing benchmark and numerical comparisons
4. Accuracy, computational cost and sensitivity
5. Practical limitations and conclusions

Why an Energy-Aware Integrator?

Standard adaptive solvers monitor local state error.

Forced and damped oscillators also obey a known power balance.

A step may look numerically acceptable yet poorly reproduce physical energy exchange.

Aim: add an interpretable physics-based check to classical RK4.

Duffing Oscillator — Physical Model

$$\ddot{x} + 2\zeta\omega_0\dot{x} + \omega_0^2 x + \alpha x^3 = F\cos(\Omega t)$$

$\zeta = 0.035$; $\omega_0 = 2\pi$ rad/s; $\alpha = 180$

$F = 8.0$; $\Omega/\omega_0 = 0.92$

Initial state: $x(0) = 0.12$, $v(0) = 0$

Simulation horizon: 20 s

Mechanical Energy and Exchange Law

Stored energy:

$H = \frac{1}{2}v^2 + \frac{1}{2}\omega_0^2 x^2 + \frac{1}{4}\alpha x^4$

Net power:

$P = Fv\cos(\Omega t) - 2\zeta\omega_0 v^2$

Exact physical balance: $dH/dt = P$

The method tests consistency, not conservation.

Normalized Energy-Balance Defect

For one candidate RK4 step of length h:

$\eta_n = | H(y^*) - H(y_n) - (h/2)(P_n + P^*) | / Hs$

Hs is based on a small reference energy and the current and candidate stored energies.

If $\eta_n > \tau E$, reject and retry at a shorter step (when possible).

Transparent Step Acceptance

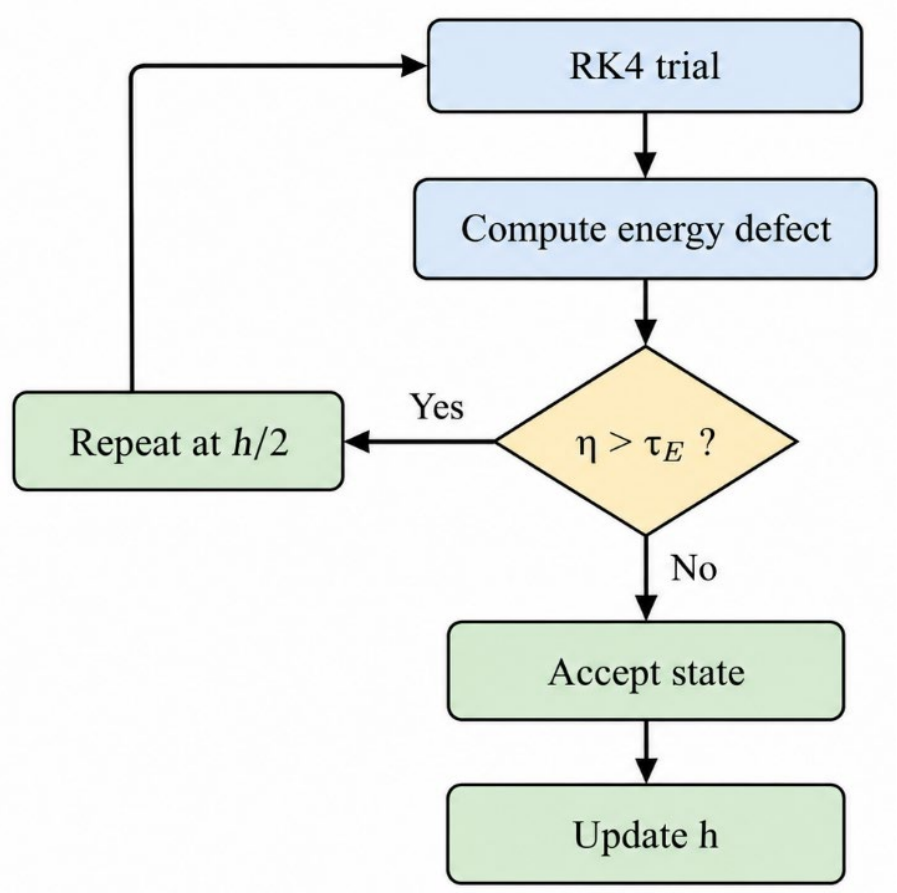
Compute trial RK4 state and energy mismatch.

Reject if $\eta_n > \tau E$ and $h > h_{min}$.

Accept otherwise and advance the simulation clock.

Increase the next step if $\eta_n < \tau E/12$.

Reduce it if $\eta_n > 0.7\tau E$.



Paper Figure 2: decision sequence

Adaptive Step-Size Gate

The multiplier is deliberately discrete: double, keep, or halve the step.

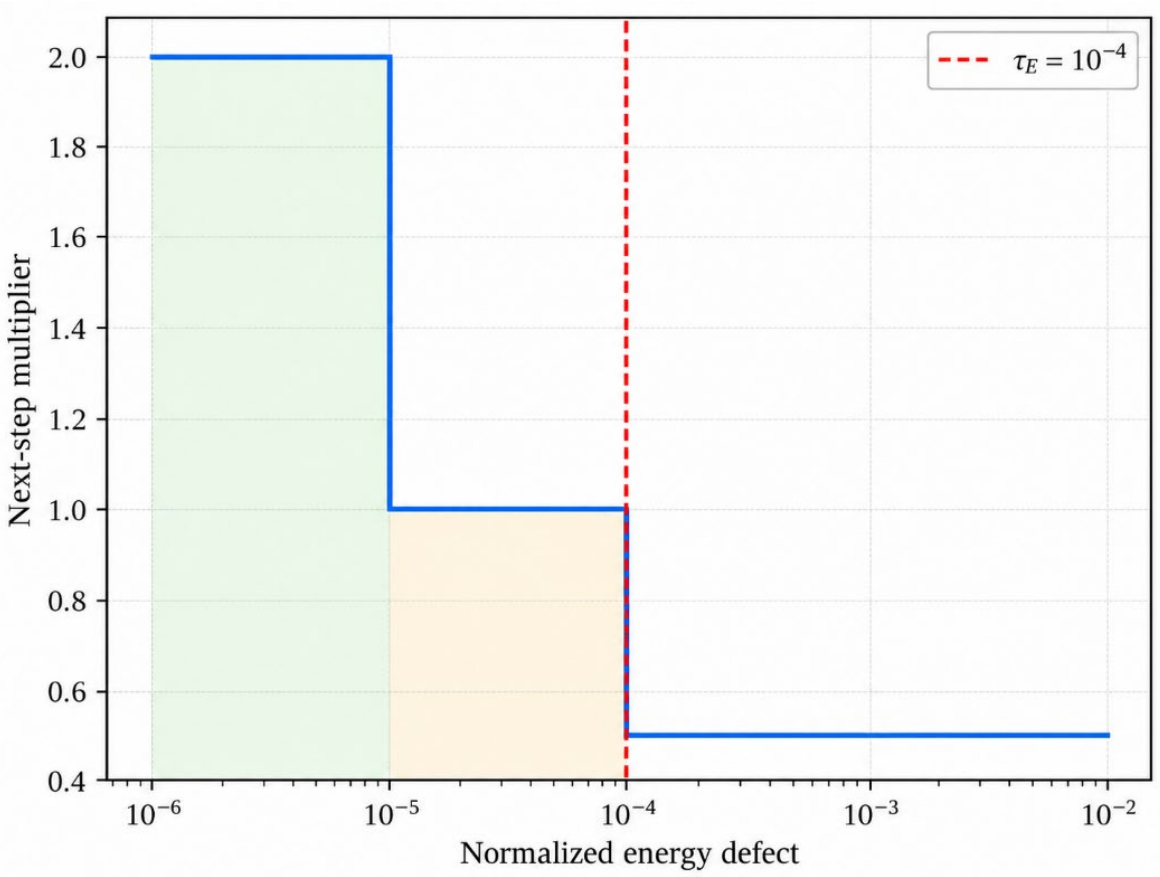
For the nominal experiment:

$\tau E = 10^{-4}$

hmin = 0.0025 s

hmax = 0.04 s

The gate provides interpretable acceptance decisions.



Paper Figure 1: step-size multiplier

Numerical Benchmark

Reference: tightly resolved DOP853 (rtol 10^{-12}, atol 10^{-14}).

Fixed RK4: step h = 0.04 s.

Adaptive RK45: state-error control.

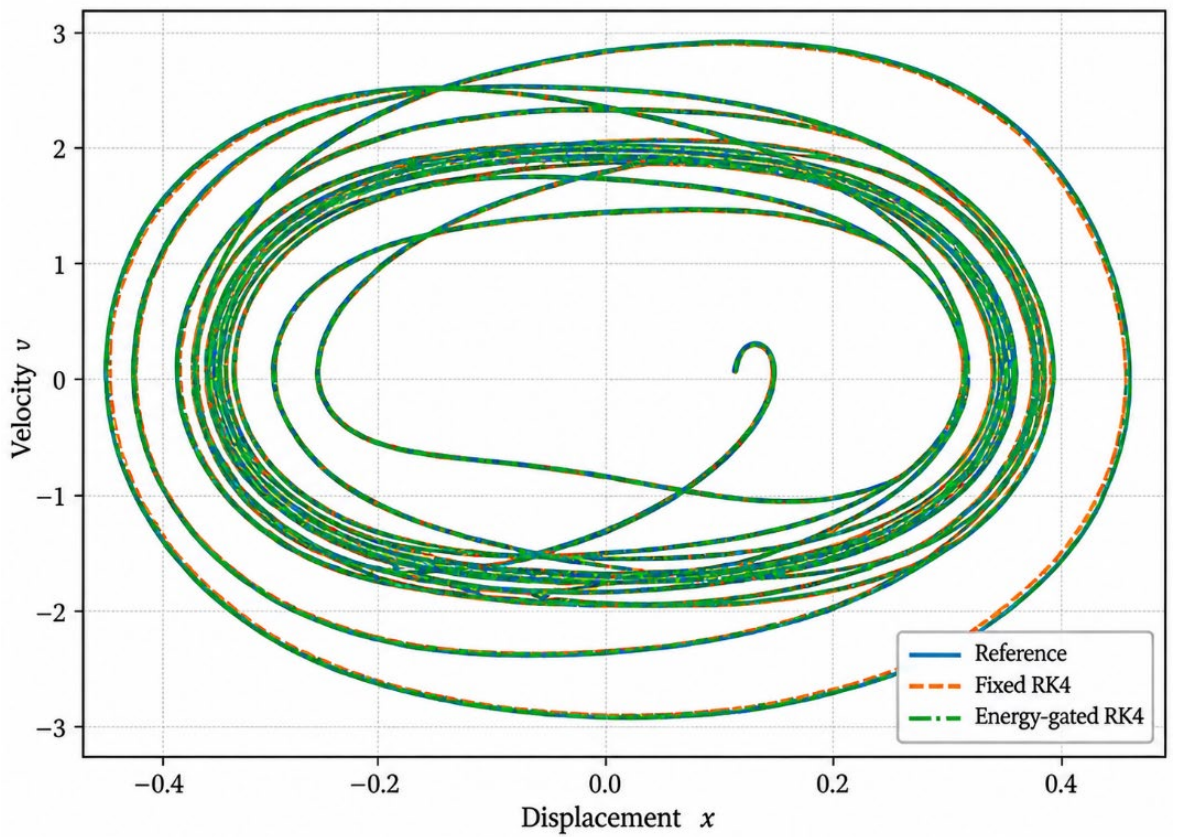
Energy-gated RK4: $\tau E = 10^{-4}$.

Measures: displacement RMSE, maximum error, final-state error, normalized mesh-balance defect and RHS evaluations.

Nonlinear Trajectory Comparison

Fixed-step RK4 captures the overall orbit but gradually accumulates phase displacement.

The energy-gated trajectory stays closer to the DOP853 reference over the 20 s benchmark.

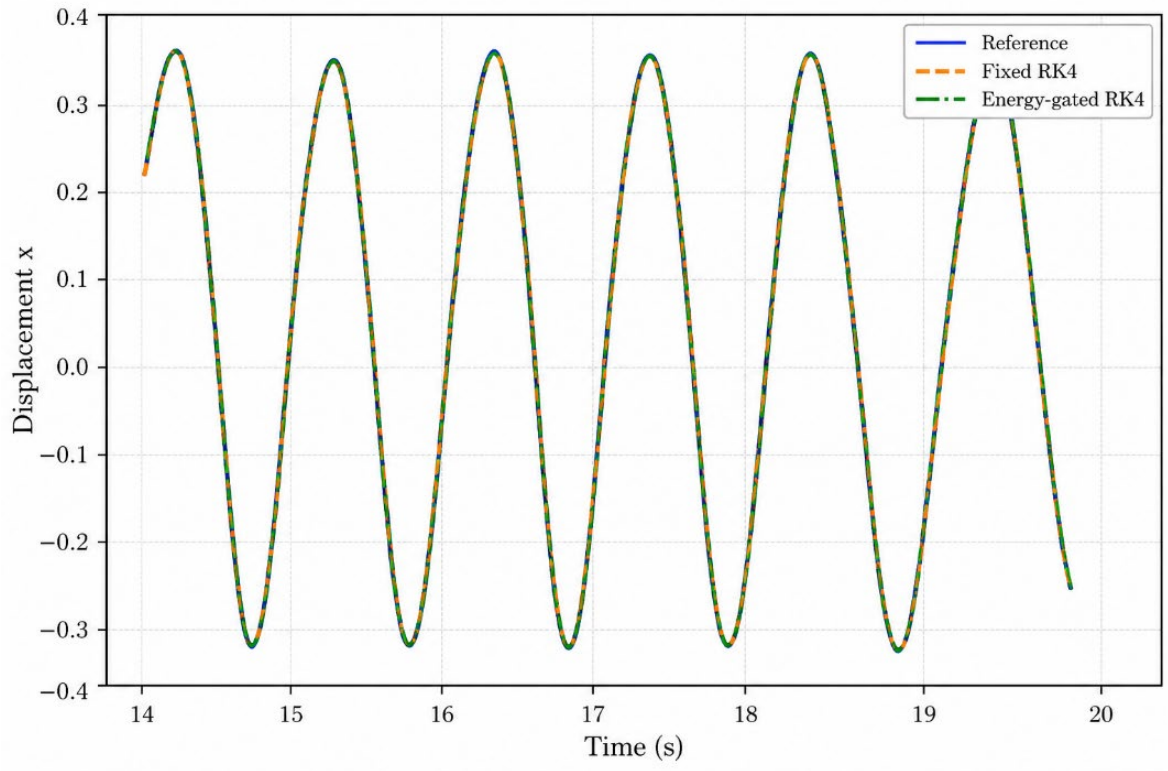


Paper Figure 3: nonlinear phase portrait

Final Six Seconds

The coarse RK4 trajectory shows a small timing shift and amplitude difference.

Energy-gated RK4 remains close to the reference during the final oscillations.



Paper Figure 4: displacement comparison

Adaptive Step Behavior

At $\tau E = 10^{-4}$:

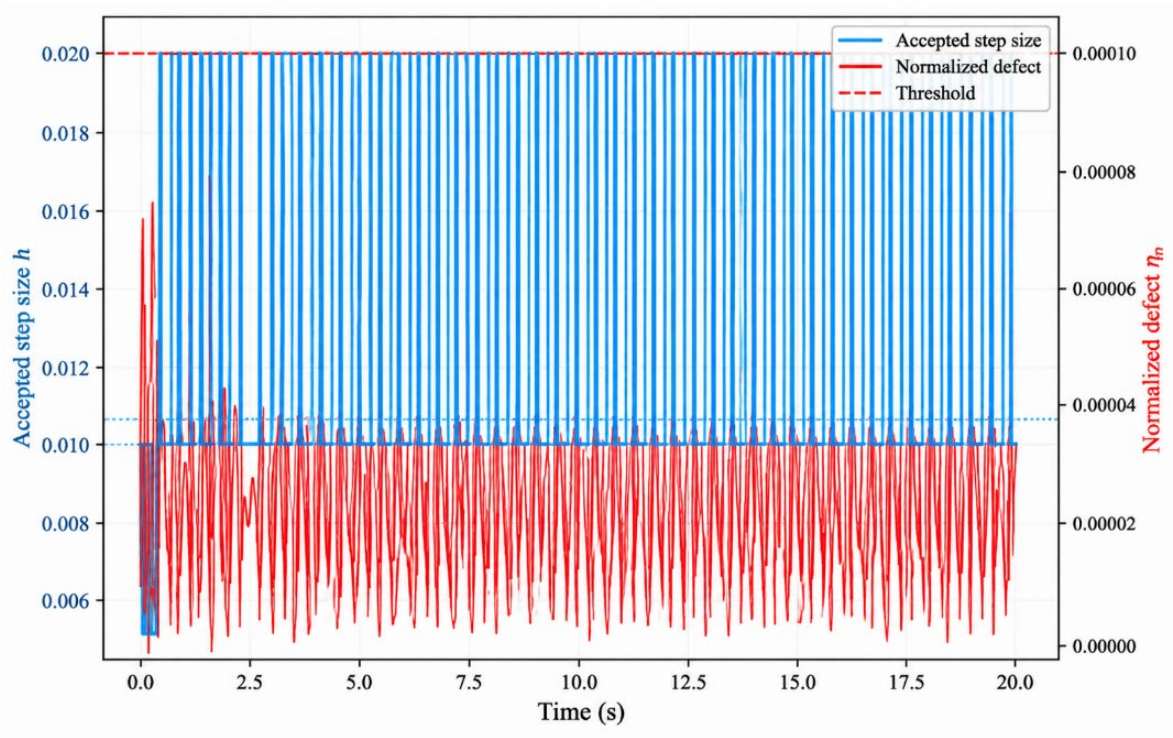
1,863 accepted steps

49 rejected trials

Mean accepted step: 0.010735 s

7,648 right-hand-side evaluations

Accepted step sizes vary even under periodic forcing.



Paper Figure 5: step sizes and energy defect

Accuracy and Computational Cost

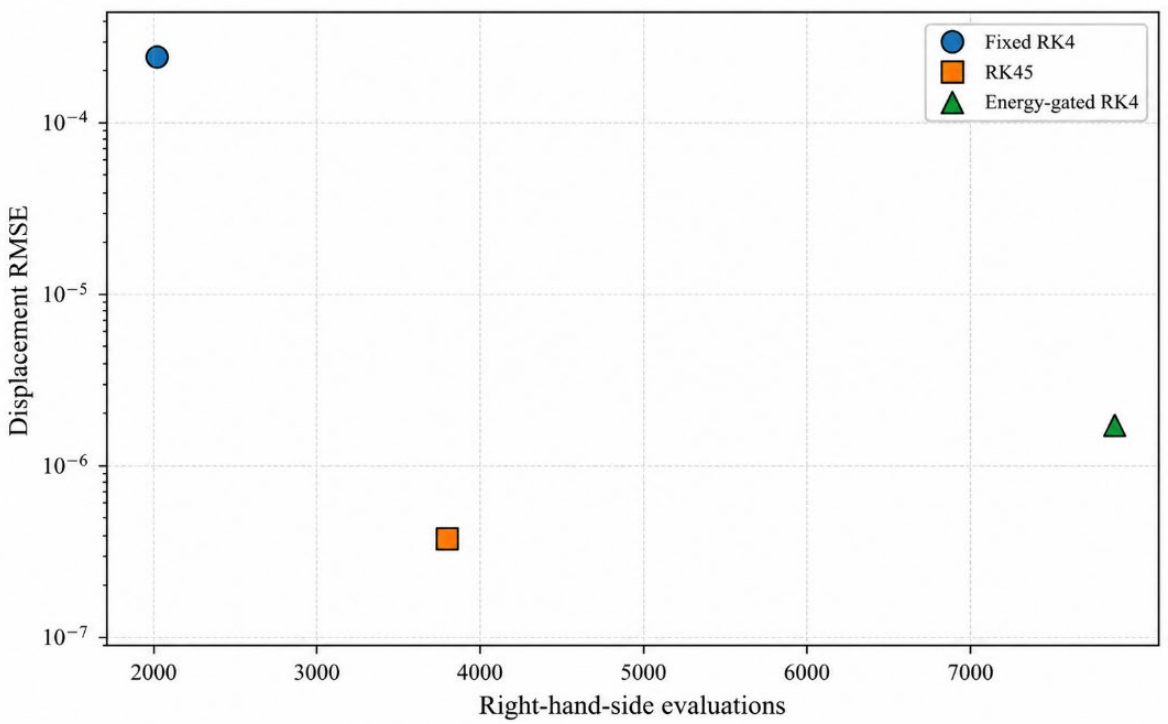
Displacement RMSE / RHS evaluations:

Fixed RK4: 1.88×10^{-4} / 2,004

Adaptive RK45: 5.71×10^{-7} / 3,878

Energy-gated RK4: 1.66×10^{-6} / 7,648

Energy-gated RK4 reduces RMSE ≈113× relative to coarse fixed RK4, but RK45 is more economical for state error alone.



Paper Figure 6: error versus computational cost

Physical-Balance Comparison

Normalized mesh-balance defect:

Fixed RK4: 2.02×10^{-3}

Energy-gated RK4: 1.96×10^{-4}

This is approximately a 10.3-fold decrease.

The RK45 postprocessed balance metric depends on its output mesh; it is not evidence of RK45 solver failure.

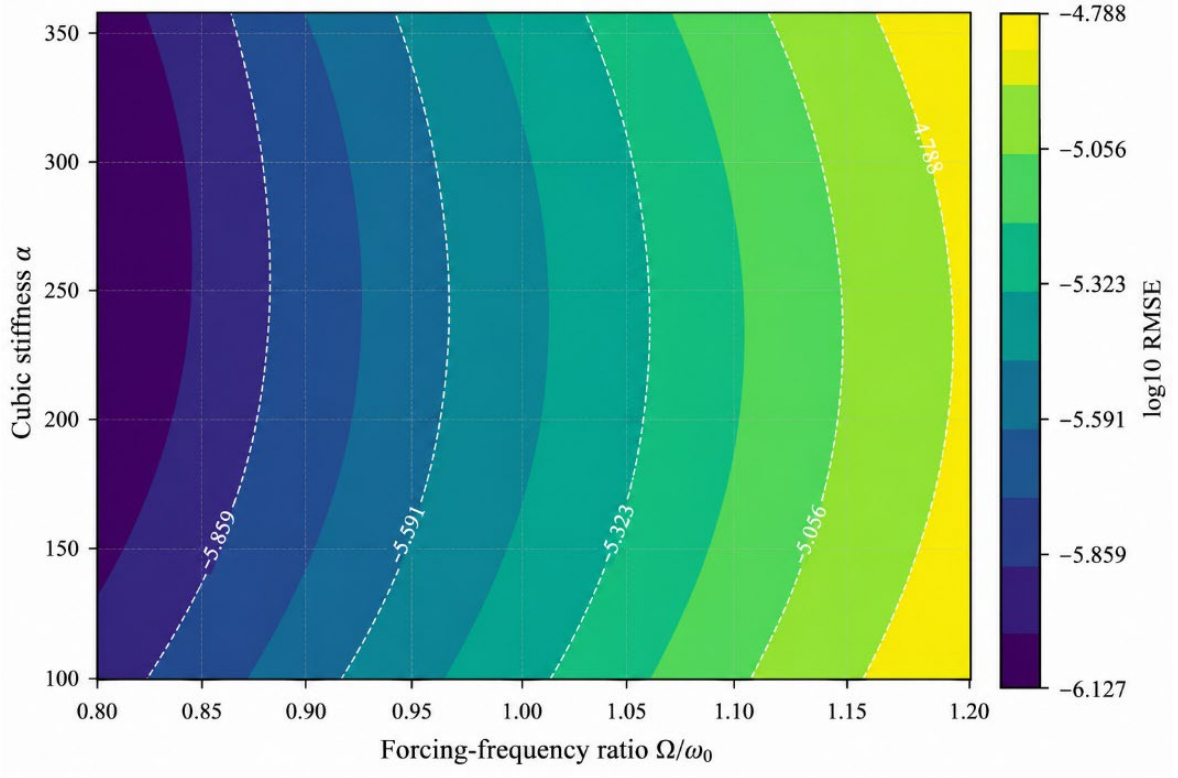
Sensitivity to Model Parameters

Cubic stiffness α: 80–360

Frequency ratio Ω/ω₀: 0.80–1.20

The forcing-frequency ratio more strongly affects displacement error in the tested range.

Increasing α does not cause uniformly worsening accuracy.



Paper Figure 7: sensitivity sweep

Tightening the Energy Threshold

$\tau E \rightarrow$ RMSE / RHS evaluations:

$10^{-4} \rightarrow 1.66\times10^{-6}$ / 7,648

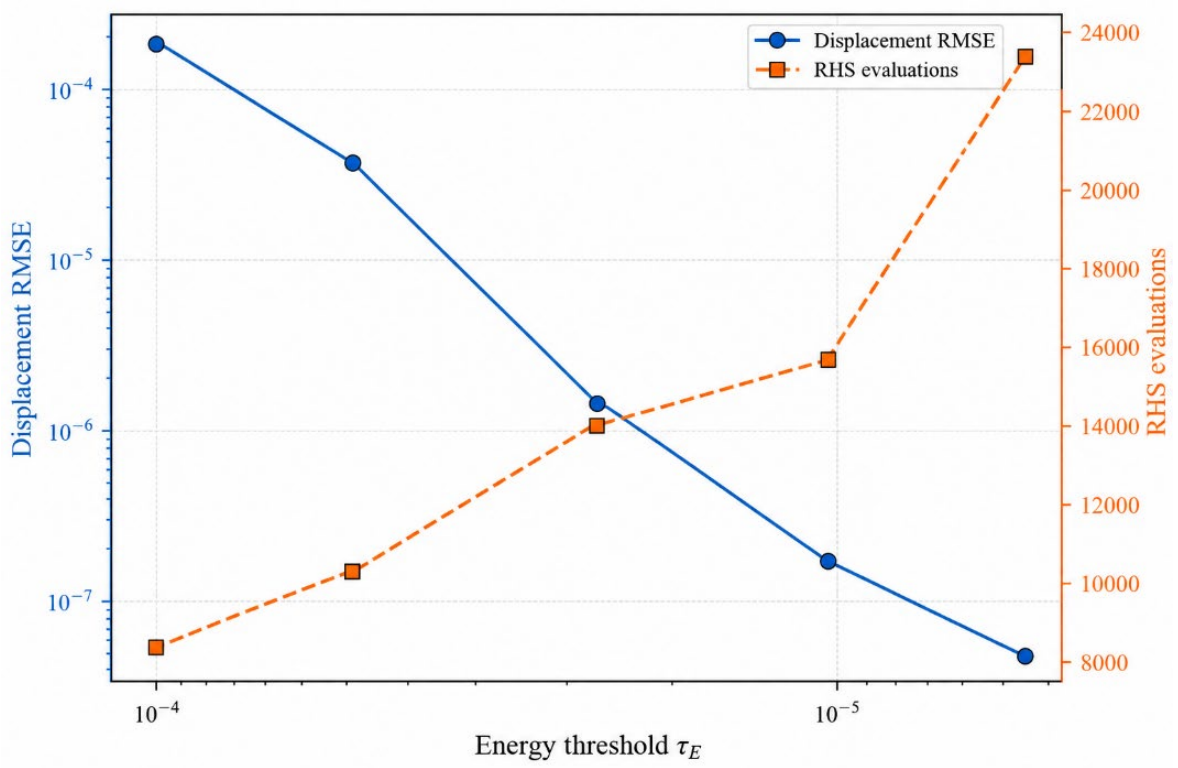
$5\times10^{-5} \rightarrow 7.62\times10^{-7}$ / 10,320

$2\times10^{-5} \rightarrow 1.86\times10^{-7}$ / 14,092

$10^{-5} \rightarrow 9.94\times10^{-8}$ / 15,416

$5\times10^{-6} \rightarrow 5.17\times10^{-8}$ / 23,436

The benefit comes with increasing work.



Paper Figure 8: accuracy–cost trade-off

Engineering Uses and Limitations

Possible use: vibration, RLC circuits, thermal systems, electromechanical models, and digital twins.

Limitations: two-state, nonstiff benchmark; trapezoidal power quadrature; no independent global state-error guarantee.

Future direction: combine conventional state-error control with separate physical-balance gates.

Conclusions

Energy balance adds a transparent physical consistency test to standard RK4.

On this benchmark, energy-gated RK4 improves coarse fixed-step trajectory accuracy and mesh energy balance.

RK45 remains more efficient when displacement error is the sole objective.

The proposed physical gate should complement—not replace—conventional local-error estimation.