The 22nd International Conference on
Technical and Physical Problems of Engineering
ICTPE-2026
31 October 2026
International Organization of IOTPE

OPTIMUM DESIGN OF CABLE CROSS-SECTION IN A LONG-SPAN CABLE-STAYED BRIDGE AS OPTIMUM CONTROL PROBLEM

R.A. Iskanderov, N.S. Mastanzade, N.M. Abdinli

Department of Mechanics, Azerbaijan University of Architecture and Construction, Baku, Azerbaijan

Research objective

Minimum cable material

Determine cable cross-sectional areas while controlling the bridge response.

Optimal-control formulation

Use Pontryagin’s principle to obtain a stationary control and an area expression.

Feasible discrete design

Reduce strand counts iteratively and check cable strength and girder deflection after every cycle.

Bridge configuration and finite element model

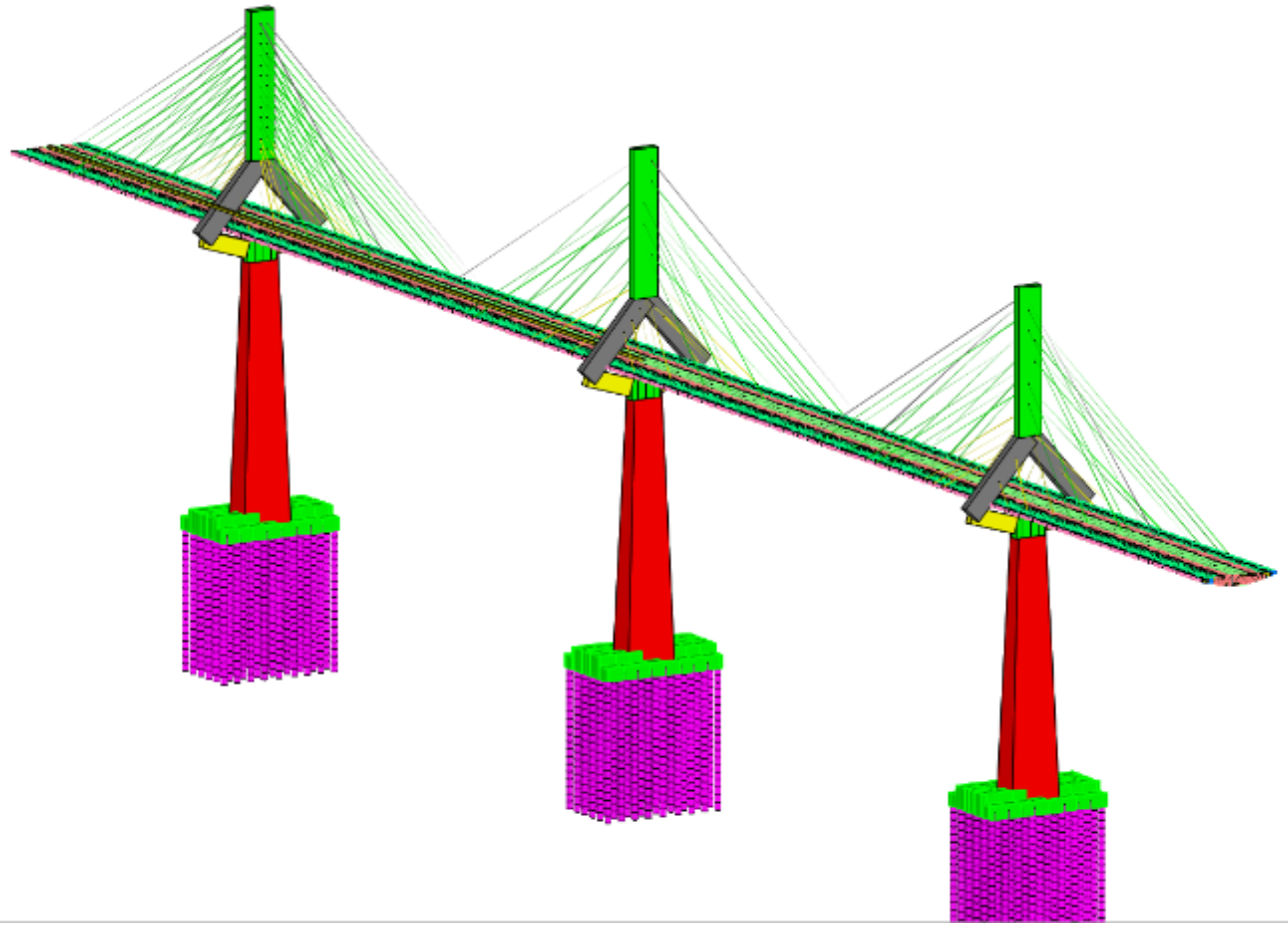


Figure 1. Finite element model from the article

Muganly–Ismayilli–Gabala road
Azerbaijan

Total bridge length: 1,100 m

Central spans: 400 m

Pylon height: 50 m

18 cables in each direction from a pylon

Loads and dynamic model

The analysis considers structural dead weight, traffic loading (A-15) and seismic action.

$$M\,\ddot{u} + C\,\dot{u} + K\,u = f_{eff}(t) + B\,w(t)$$

$$f_{eff}(t) = -M\,\ddot{u}_{gr}(t)$$

u: displacement M: mass C: damping K: stiffness
w(t): control input B: control coefficient

Kinematic roller supports form part of the seismic-response concept.

State variables and optimization criterion

$$x_1 = u \qquad x_2 = \dot{u}$$

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -(K/M)x_1 - (C/M)x_2 + f_{eff}/M + (B/M)w$$

$$I = \int [\alpha x_1^2 + \beta x_2^2 + \gamma w^2]\, dt \qquad \text{(minimize)}$$

Bryson's rule

$$\alpha = 1/x_{1,max}^2 \qquad \beta = 1/x_{2,max}^2 \qquad \gamma = 1/w_{max}^2$$

Reported weights: $\alpha = 1.589$, $\beta = 14.56$, $\gamma = 128.9$

Hamiltonian and stationary control

$$H = \alpha x_1^2 + \beta x_2^2 + \gamma w^2 + p_1 x_2 + p_2[-(K/M)x_1 - (C/M)x_2 + f_{eff}/M + (B/M)w]$$

State equations: $\dot{x} = \partial H/\partial p$

Adjoint equations: $\dot{p} = -\partial H/\partial x$

$$\partial H/\partial w = 2\gamma w + (B/M)p_2 = 0$$

$$\mathbf{w^* = -Bp_2 / (2\gamma M)}$$

Unconstrained control expression

Cable cross-section as the design variable

$$w = A \quad \text{and} \quad k(w) = Ew/L$$

The cross-sectional area enters both the stiffness term and the control penalty.

$$2\gamma w - p_2 E x_1/(Lm) + p_2 b/M = 0$$

$$\mathbf{w_{opt} = [p_2/(2\gamma)]\ [Ex_1/(Lm) - b/M]}$$

A feasible cable area must also satisfy strength, deflection and available strand sizes.

Reported numerical solution

$$p_1 = -2(s_{11}x_1 + s_{12}x_2)$$
$$p_2 = -2(s_{12}x_1 + s_{22}x_2)$$

Parameter	Reported value
K/m	4.168
C/m	0.20416
s_{11}	124.8211
s_{12}	0.1907
s_{22}	29.934

Reported adjoint values: $p_1 = -222.834$, $p_2 = -55.778$

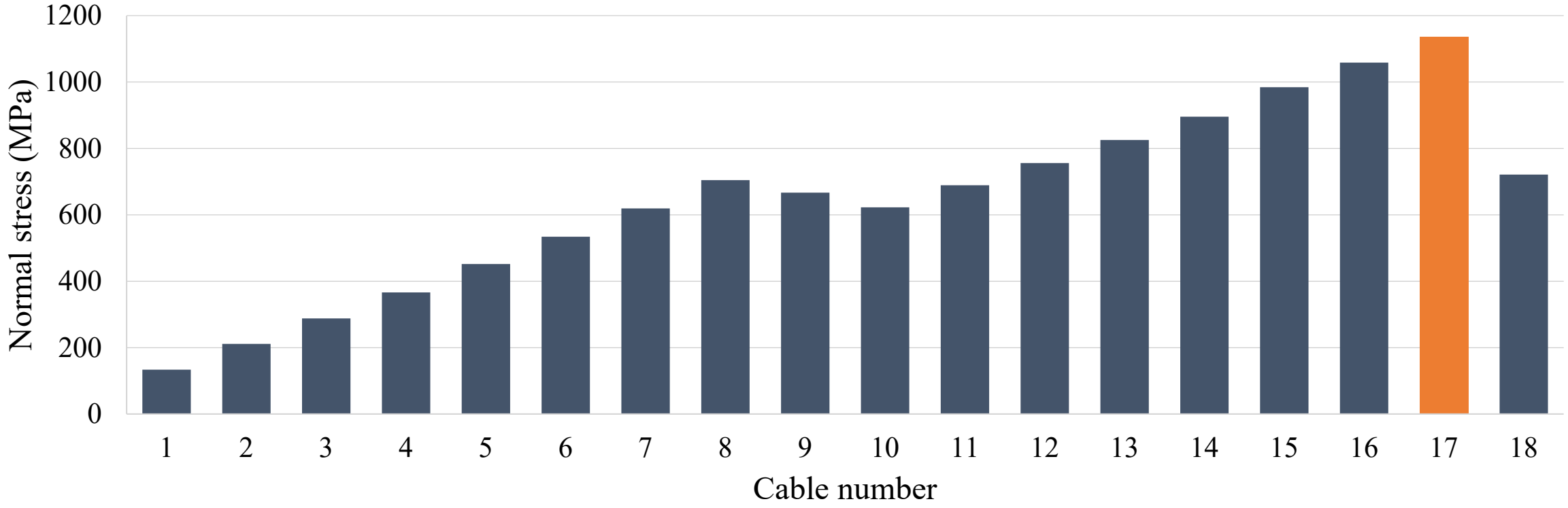
Baseline cable areas

Cable numbers	Strands per cable	Area per cable (mm²)	Group area (mm²)
1–8	31	4,340	34,720
9–17	37	5,180	46,620
18	48	6,720	6,720
Total	18 cables		88,060

Total cross-sectional area = 88,060 mm² = 0.08806 m²

Group subtotals calculated from Table 1

Baseline cable stress distribution



Cable 17 has the highest reported stress: 1,136.5 MPa

Source: Table 1, all 18 cables

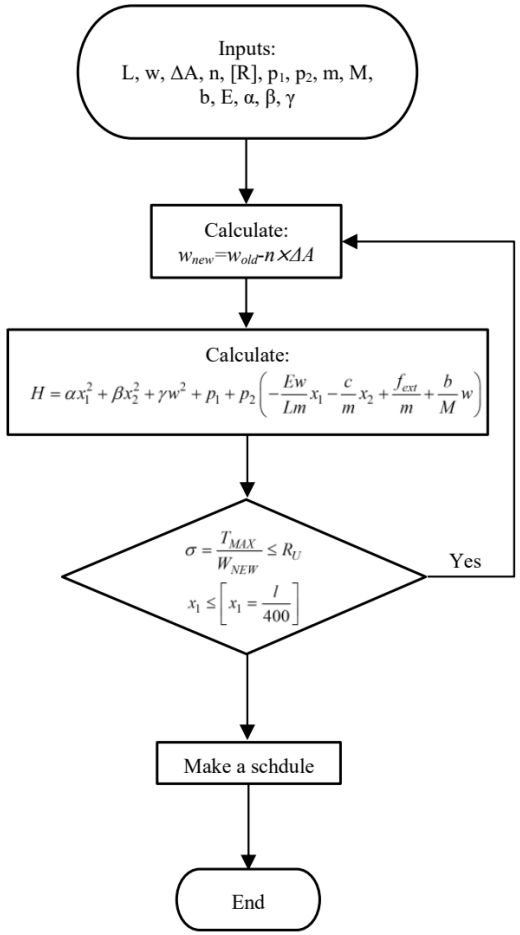


Figure 2 from the article

Iterative strand-removal procedure

$$w_{new} = w_{old} - n\Delta A$$

1. Start from the baseline cable areas.
2. Remove strands symmetrically.
3. Recalculate forces and deformation.
4. Test strength and deflection limits.
5. Retain the last feasible configuration.

Strand area needs reconciliation: the text uses 182.32 mm², while Table 1 implies 140 mm² per strand.

Strength and deformation constraints

Cable strength

$$\sigma_i = T_{max,i} / A_i \leq R_u$$

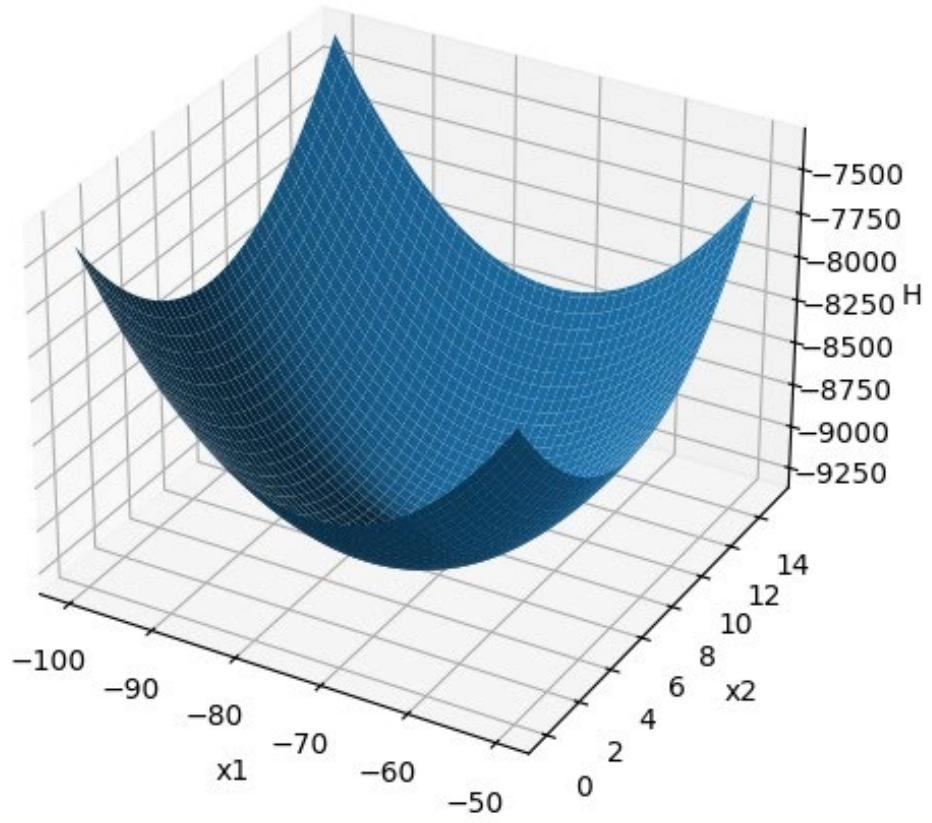
Use the maximum combined static and dynamic cable force.

Girder deformation

$$x_{1,max} \leq [x_1] = L/400$$

Stop reducing strand counts when either constraint fails.
Maintain symmetry during strand removal.

Hamiltonian interpretation



Original Figure 3, with axes labelled x_1 and x_2

For fixed state and adjoint variables

H(w) contains a positive quadratic term γw^2.

When $\gamma > 0$, the stationary point minimizes H along w.

The optimum changes with the current state and p_2.

Conclusions

Analytical sizing framework

The formulation relates cable area to structural response and adjoint variables.

3–4 cycles reported

The paper reports convergence of the iterative cross-section procedure.

15–20% girder-weight reduction reported

The paper associates this reduction with cable number and arrangement optimization.

The article does not provide a final optimized cable schedule or a before-and-after weight calculation.

Appendix: cable data 1–9

Cable	Strands × diameter (in)	Area (mm²)	Angle (°)	Length (m)	Force (kN)	Stress (MPa)	Unit weight (N/m)
1	31×0.6	4,340	31.988	11.7903	580.3	133.7	340.69
2	31×0.6	4,340	31.092	17.5165	917.2	211.3	340.69
3	31×0.6	4,340	30.638	23.2449	1,250	288	340.69
4	31×0.6	4,340	30.366	28.974	1,589.5	366.2	340.69
5	31×0.6	4,340	30.179	34.7029	1,961.4	452	340.69
6	31×0.6	4,340	30.047	40.4339	2,318.6	534.2	340.69
7	31×0.6	4,340	29.948	46.164	2,688.3	619.4	340.69
8	31×0.6	4,340	29.871	51.894	3,057.5	704.5	340.69
9	37×0.6	5,180	29.809	57.6245	3,454.4	666.9	340.69

Source: Table 1. Decimal notation and unit capitalization normalized.

The 22nd International Conference on Technical and Physical Problems of Engineering (ICTPE-2026)

Appendix: cable data 10–18

Cable	Strands × diameter (in)	Area (mm²)	Angle (°)	Length (m)	Force (kN)	Stress (MPa)	Unit weight (N/m)
10	37×0.6	5,180	29.758	63.3549	3,225.7	622.7	340.69
11	37×0.6	5,180	29.716	69.0854	3,569	689	340.69
12	37×0.6	5,180	29.680	74.8158	3,916.5	756	340.69
13	37×0.6	5,180	29.649	80.5462	4,276.6	825.6	340.69
14	37×0.6	5,180	29.623	86.2757	4,639.2	895.6	340.69
15	37×0.6	5,180	29.599	92.0072	5,099	984.4	340.69
16	37×0.6	5,180	29.579	97.7377	5,483	1,058.5	340.69
17	37×0.6	5,180	29.560	103.4682	5,887	1,136.5	340.69
18	48×0.6	6,720	29.544	109.1988	4,847	721.3	340.69

Source: Table 1. Decimal notation and unit capitalization normalized.