FREE VIBRATIONS OF A VERTICAL SUPPORT CONSISTING OF THREE ORTHOTROPIC, VISCO-ELASTIC SOIL-CONTACTING CYLINDRICAL PANELS TO BE STIFFENED WITH LONGITUDINAL RIBS

D.S. Ganiyev

Azerbaijan University of Architecture and Construction, Baku, Azerbaijan
qanidilqem@gmail.com

Abstract- It is appropriate to select supporting in the form of formations consisting of soil-filled cylindrical panels when bridges over the mountain rivers. These supporting should be constructed such that one of the control lines would stand against the river flow. Under the action of the river flow there happens a vibrational process in this construction. The present paper was devoted to the study of natural vibrations of such supports.

Keywords: Vibrations, Cylindrical Panels, Supports, River, Soil-Filled.

1. INTRODUCTION

Note that [1] was devoted to one of dynamical strength characteristics, the frequency of natural vibrations of the retaining wall consisting of two soil-contacting, orthotropic cylindrical shells reinforced with discretely distributed annular rods. A problem of natural vibrations of a retaining wall consisting of two orthotropic, viscous-elastic soil-filled cylindrical shells reinforced with discretely distributed longitudinal rods was solved in [2]. In his papers V.Z. Vlasov [3] studied retaining walls and hydrotechnical installations obtained with using three thin-walled granulated medium filled spatial constructions lying on an elastic foundation.

E.K. Agakhanov and A.I. Akayev [4] obtained the solution for a triangular section retaining wall with regard to soil pressure and volume forces of filtration on the vertical side of the wall.

N.M. Sitko offered joint consideration of retaining of wall displacement and deformation of foundation. His results are in [5]. The influence of soil with horizontal surface on vertical plane smooth wall of the retaining wall was considered. N.M. Snitko supposed that in the elastic equilibrium state, in soil there appears a sliding plane and this looks like the Coulomb theory. In [6] L.M. Yemelyanov considered substantiation of stability of deep supports in the spatial system. In calculations, in addition to normal tangential pressures of soils, the reaction on the bottom was taken into account. In majority of cases, the rules for calculating some supports connected with anchors were given.

The problems of connection of concave shells with contour constructions were solved in Kh.R. Seyfullayev’s papers [7-11]. The solutions of differential equations of moment theory of concave shells under arbitrary boundary conditions were tired. Stability and free vibrations of a strueted functionally-graduated cylindrical shell subjected to temperature boundary conditions were analyzed in [12]. Ref. [13] studies statically deformations of retaining walls of spatially-building structures formed by cylindrical shells made of isotropic material.

Retaining walls consisting of three different isotropic materials in a plane strain state were analyzed in [14]. The problem was reduced to the solution of ordinary differential equations and analytic solution was obtained. Ref. [15] was devoted to development of a technique for calculating cylindrical shells made of isotropic material with regard to compression and sliding in a contact surface. Calculations and studies were carried out based on the moment theory of cylindrical shells. Analysis of the executed works shows that during the construction of retaining walls, the stiffened cylindrical shells were not used and the soil reaction was not taken into account.

The present paper is devoted to the study of one the dynamical strength characteristics, the frequency of natural vibrations of a vertical support consisting of three orthotropic, soil-filled cylindrical panels strengthened with discretely distributed longitudinal rods. Using the Hamilton-Ostrogradsky variational principle for finding frequencies of vibrations of a vertical support, a frequency equation was structured, its roots were found and influence of physical and geometrical parameters characterizing the system, were found. Accounting of joint work on the contact line of three cylindrical panels is accepted as contact conditions.

2. PROBLEM STATEMENT

In order to apply the Hamilton-Ostrogradsky variational principle, we write the total energy of the vertical support under investigation. Since the vertical support consists of three shells of cylindrical form with open contour and stiffened elements, their number vary. Furthermore, from the inside the construction is in contact with soil (Figure 1a).
We write potential and kinetic energies of cylindrical shells [15]:

\[ G_i = \frac{b_i R_i}{2} \int \left[ h_{ui} \left( \frac{\partial u_i}{\partial t} \right)^2 + \frac{b_{i2}}{R_i} \frac{\partial u_i}{\partial x_i} + \frac{w_i^2}{R_i^2} \left( h_{ui} + 2h_{2i} + b_{2i} \right) \right] dx_i + b_{66i} \left( \frac{\partial u_i}{\partial \theta_i} \right)^2 + b_{66i} \left( \frac{\partial \theta_i}{\partial \theta_i} \right)^2 \]

\[ + 2 \left( h_{2i} + b_{2i2} \right) \left( \frac{\partial u_i}{\partial x_i} \right)^2 + 2b_{2i} \left( \frac{\partial u_i}{\partial x_i} \right)^2 + \frac{w_i^2}{R_i^2} \left( \frac{\partial \theta_i}{\partial \theta_i} \right)^2 \]

\[ = -2 \left( h_{2i} + b_{2i2} \right) \frac{w_i}{R_i^2} \left( \frac{\partial \theta_i}{\partial \theta_i} \right) + \frac{b_{2i} w_i}{R_i^2} \left( \frac{\partial \theta_i}{\partial \theta_i} \right) + \frac{w_i^2}{R_i^2} \left( \frac{\partial \theta_i}{\partial \theta_i} \right) \] 

\[ + b_{66i} \left( \frac{\partial u_i}{\partial \theta_i} \right)^2 + b_{66i} \left( \frac{\partial \theta_i}{\partial \theta_i} \right)^2 \]

\[ + 2 \left( h_{2i} + b_{2i2} \right) \left( \frac{\partial u_i}{\partial x_i} \right)^2 + 2b_{2i} \left( \frac{\partial u_i}{\partial x_i} \right)^2 + \frac{w_i^2}{R_i^2} \left( \frac{\partial \theta_i}{\partial \theta_i} \right)^2 \]

\[ = -2 \left( h_{2i} + b_{2i2} \right) \frac{w_i}{R_i^2} \left( \frac{\partial \theta_i}{\partial \theta_i} \right) + \frac{b_{2i} w_i}{R_i^2} \left( \frac{\partial \theta_i}{\partial \theta_i} \right) + \frac{w_i^2}{R_i^2} \left( \frac{\partial \theta_i}{\partial \theta_i} \right) \]

\[ + b_{66i} \left( \frac{\partial u_i}{\partial \theta_i} \right)^2 + b_{66i} \left( \frac{\partial \theta_i}{\partial \theta_i} \right)^2 \]

\[ + 2 \left( h_{2i} + b_{2i2} \right) \left( \frac{\partial u_i}{\partial x_i} \right)^2 + 2b_{2i} \left( \frac{\partial u_i}{\partial x_i} \right)^2 + \frac{w_i^2}{R_i^2} \left( \frac{\partial \theta_i}{\partial \theta_i} \right)^2 \] (1)

where, \( i = 1 \) corresponds to the first cylindrical shell, \( i = 2 \) to the second cylindrical shell, \( i = 3 \) to the third cylindrical shell constituting vertical riser (Figure 1); \( u_i, \theta_i, w_i \) are displacements of ribs, \( R_i, h_i \) are the radii and thickness of cylindrical shells, \( h_{ui}, h_{2i}, h_{66i} \) are the main module of elasticity of orthotropic material, \( E_{ii}, E_{2i} \) are module of elasticity in the direction of coordinate axis \( x_i \) and \( \theta_i \), respectively, \( v_{ui}, v_{2i} \) is a Poisson ratio.

Figure 1. Cylindrical shell constituting vertical riser
rod to the surface of the cylindrical shell, \( \rho_{ji} \) is density of the material of the \( j \)th rods, \( \varphi_{ji}, \theta_{qi} \) are turning and twist angles of the cross-section of the \( j \)th rod and are expressed by the shell displacement in the following form:

\[
\varphi_{qi} \left( \theta_j \right) = \varphi_i \left( x_{ji}, \theta_j \right) = -\frac{\partial w_i}{\partial x} \big|_{x=x_i}
\]

by the shell displacement in the following form:

As a result, we get the total energy of the system in the form:

\[
\prod = \sum_{i=1}^{3} \left( G_i + K_j + H_i + A_i \right).
\]  

(5)

The external forces \( q_{s}, q_{s}, q_{z} \) of the shell acting on cylindrical shell and contained in Equation (2) will be taken in the form:

\[
q_{s} = q_{s} = 0
\]

\[
q_{z} = p_{k} w_{1} + k_{1} \left( \frac{\partial^2 w_{1}}{\partial x^2} + \frac{\partial^2 w_{1}}{\partial v^2} \right) \left( \int_{0}^{1} \Gamma (t - \tau) w_{1}(\tau) d \tau \right)
\]

\[
q_{z} = p_{2} w_{2} - k_{2} \left( \frac{\partial^2 w_{2}}{\partial x^2} + \frac{\partial^2 w_{2}}{\partial v^2} \right) \left( \int_{0}^{1} \Gamma (t - \tau) w_{2}(\tau) d \tau \right)
\]

(6)

where, \( p_{1}, k_{1,2} \) are rigidity factors of soil under compression and sliding, respectively, \( \Gamma (t) = A e^{-\gamma t} \) is a relaxation core, \( A, \gamma \) are constants. We add contact and boundary conditions to Equations (2) and (5). Assume that the cylindrical shell is elastically connected, i.e. in the contact (along the line \( AB, CD, EF \)) the following conditions are fulfilled.

\[
w_{1}(x) \big|_{\theta_{1} = \theta_{0}} = w_{2}(x) \big|_{\theta_{2} = \theta_{0}} = 0
\]

\[
\partial w_{1}(x) \big|_{\theta_{1} = \theta_{0}} = \partial w_{2}(x) \big|_{\theta_{2} = \theta_{0}} = 0
\]

\[
u_{1}(x) \big|_{\theta_{1} = \theta_{0}} = u_{2}(x) \big|_{\theta_{2} = \theta_{0}} = 0
\]

\[
\frac{\partial w_{1}(x)}{\partial x} \big|_{\theta_{1} = \theta_{0}} = \frac{\partial w_{2}(x)}{\partial x} \big|_{\theta_{2} = \theta_{0}} = 0
\]

\[
w_{2}(x) \big|_{\theta_{2} = \theta_{2}} = w_{3}(x) \big|_{\theta_{3} = \theta_{2}} = 0
\]

\[
\partial w_{2}(x) \big|_{\theta_{2} = \theta_{2}} = \partial w_{3}(x) \big|_{\theta_{3} = \theta_{2}} = 0
\]

\[
u_{2}(x) \big|_{\theta_{2} = \theta_{2}} = u_{3}(x) \big|_{\theta_{3} = \theta_{2}} = 0
\]

\[
\frac{\partial w_{2}(x)}{\partial x} \big|_{\theta_{2} = \theta_{2}} = \frac{\partial w_{3}(x)}{\partial x} \big|_{\theta_{3} = \theta_{2}} = 0
\]

(7)

It is accepted to assume that cylindrical shells are reliably fixed on ideal diaphragms on the lines \( x = a, x = a \), this time boundary conditions are expressed as follows:

\[
\partial_{\theta} = 0, \quad w_{1} = 0, \quad T_{11} = 0, \quad M_{11} = 0
\]

(8)

where, \( T_{11}, M_{11} \) are the forces and moments acting on cross-sections of cylindrical shells (Figure 1).

Using the Ostrogradsky-Hamilton stationarity condition determining the vibrations of vertical risers, created by the connection of cylindrical panels, we can get the total energy equation

\[
\partial W = 0
\]

(9)

where, \( W = \int_{0}^{1} \prod dt \) is Hamliton’s action. In the equality \( \partial W = 0 \) we execute the variation operations and taking into account that independent variables \( \partial w_{1}, \partial w_{2}, \partial w_{3} \) are arbitrary for finding frequencies of free vibrations of vertical risers obtained by connection of cylindrical shells dynamically contacting with soil, we get a frequency equations Thus the solution of the problem of vibrations of vertical risers obtained by connection of cylindrical shells dynamically interacting with soil is reduced to joint integration of total energy of the structure (5) under contact conditions (7) and boundary conditions (8).

3. PROBLEM SOLUTION

We look for displacement of the points of cylindrical shells in the form:

\[
u_{i} = u_{0i} \sin \chi \xi \left( \cos \varphi_{i} t + \sin \varphi_{i} t \right) \sin \omega_{i} t
\]

\[
\partial_{\theta} = \varphi_{0i} \left( \cos \varphi_{i} t + \sin \varphi_{i} t \right) \sin \omega_{i} t
\]

\[
w_{i} = w_{0i} \left( \cos \varphi_{i} t + \sin \varphi_{i} t \right) \sin \omega_{i} t
\]

(10)

where, \( u_{0i}, \varphi_{0i}, \omega_{i} \) are unknown constants, \( \xi = \frac{x_{i}}{a}, \xi, \eta \) are wave numbers of cylindrical shell in direction of generatrix and circular direction, \( 0 \leq \theta_{1} \leq \theta_{1}, 0 \leq \theta_{2} \leq \theta_{2}, 0 \leq \theta_{3} \leq \theta_{3}, \omega_{i} = \sqrt{\left( 1 - \frac{\omega_{i}^{2}}{E_{11}} \right) \mu R_{i}^{2} a^{2}}
\]

Using solutions (10) from the contact condition (7) we express the coefficients \( u_{02}, \varphi_{02}, \omega_{02} \) and \( u_{03}, \varphi_{03}, \omega_{03} \) by the constants \( u_{01}, \varphi_{01}, \omega_{01} \): \( u_{02} = u_{01} \left( \cos \varphi_{01} t + \sin \varphi_{01} t \right) \), \( \varphi_{02} = \varphi_{01} \left( \cos \varphi_{01} t + \sin \varphi_{01} t \right) \), \( \omega_{02} = \omega_{01} \left( \cos \varphi_{01} t + \sin \varphi_{01} t \right) \), \( u_{03} = u_{01} \left( \cos \varphi_{01} t + \sin \varphi_{01} t \right) \left( \cos \varphi_{02} t + \sin \varphi_{02} t \right) \left( \cos \varphi_{03} t + \sin \varphi_{03} t \right) \), \( \varphi_{03} = \varphi_{01} \left( \cos \varphi_{01} t + \sin \varphi_{01} t \right) \left( \cos \varphi_{02} t + \sin \varphi_{02} t \right) \left( \cos \varphi_{03} t + \sin \varphi_{03} t \right) \), \( \omega_{03} = \omega_{01} \left( \cos \varphi_{01} t + \sin \varphi_{01} t \right) \left( \cos \varphi_{02} t + \sin \varphi_{02} t \right) \left( \cos \varphi_{03} t + \sin \varphi_{03} t \right) \)

Note that the following conditions should be fulfilled

\[
\left( \cos \varphi_{01} t + \sin \varphi_{01} t \right) \left( \cos \varphi_{02} t + \sin \varphi_{02} t \right) \times \left( \cos \varphi_{03} t + \sin \varphi_{03} t \right) = 1
\]
Substituting solution (10) in (5), taking into account the expression of the constant \( u_{02}, \vartheta_{02}, \vartheta_{01} \) and \( u_{00}, \vartheta_{00}, \vartheta_{01} \) through the constant \( u_{01}, \vartheta_{01}, \vartheta_{01} \) the total energy (5) we get a second order polynomial with respect to \( u_{01}, \vartheta_{01}, \vartheta_{01} \):

\[
\Pi = \varphi_1 u_{01}^2 + \varphi_2 \vartheta_{01}^2 + \varphi_3 u_{01} \vartheta_{01} + \varphi_4 u_{01} \vartheta_{01} + \varphi_5 u_{01} \vartheta_{01} + \varphi_6 \vartheta_{01} \vartheta_{01} = 0
\]

The expressions for the coefficients \( \varphi_1, \varphi_2, \varphi_3, \varphi_4, \varphi_5, \varphi_6 \) are bulky and we do not give them here.

If we vary the expression \( \Pi \) along the constants \( u_{01}, \vartheta_{01}, \vartheta_{01} \) and equate to zero the coefficients of independent variables we get the following system of homogeneous algebraic equations

\[
\begin{align*}
2 \varphi_1 u_{01} + 2 \varphi_4 \vartheta_{01} + \varphi_5 u_{01} &= 0 \\
\varphi_4 u_{01} + 2 \varphi_2 \vartheta_{01} + \varphi_6 u_{01} &= 0 \\
\varphi_5 u_{01} + 2 \varphi_6 \vartheta_{01} &= 0
\end{align*}
\]

(11)

Since the system (1) is a homogeneous system of linear algebraic equations, a necessary and sufficient condition for its nonzero solution is the equality of its main determinant to zero. As a result, we get the following frequency equation

\[
\begin{vmatrix}
2 \varphi_1 & \varphi_4 & \varphi_5 \\
\varphi_4 & 2 \varphi_2 & \varphi_6 \\
\varphi_5 & \varphi_6 & 2 \varphi_3
\end{vmatrix} = 0
\]

(12)

We write equation (12) in the form:

\[
4 \varphi_1 \varphi_4 \varphi_6 \varphi_{22} \varphi_{33} + \varphi_4 \varphi_5 \varphi_6 \varphi_{33} - \varphi_3 \varphi_5 \varphi_6 \varphi_{22} - \varphi_3 \varphi_5 \varphi_6 \varphi_{22} - \varphi_2 \varphi_5 \varphi_6 \varphi_{11} - \varphi_2 \varphi_5 \varphi_6 \varphi_{11} = 0
\]

(13)

4. NUMERICAL RESULTS

Equation (13) was calculated by the numerical method. The parameters contained in the solution of the problem were taken as:

\[
\begin{align*}
p_1 &= p_2 = 7 \times 10^8 \text{H/m}^2, k_{\mu} = 11 \times 10^6 \frac{\text{H}}{\text{m}^2}, \frac{a}{R_1} = 3 \\
v_1 &= v_2 = 0.35, R_1 = 160 \text{mm}, h_1 = 0.45 \text{mm} \\
b_{11} &= 18.3 \text{QPa}, b_{12} = 2.77 \text{QPa}, b_{22} = 25.2 \text{QPa} \\
b_{66} &= 3.5 \text{QPa}, \rho_1 = \rho_2 = 1850 \text{kg/m}^3 \\
E_{\mu} &= 6.67 \times 10^9 \text{H/m}^2, \chi = 1, n = 8, h_\mu = 1.39 \text{mm} \\
I_{kp\mu} &= 0.48 \text{mm}^4, I_{sl\mu} = 19.9 \text{mm}^4, F_\mu = 0.45 \text{mm}^2
\end{align*}
\]

The results of calculations were given in Figure 2 in the form of dependence of frequency parameter \( \omega_1 \) on \( \vartheta_{01} \) in Figure 3 on the ratio \( a/R_1 \) in Figure 4 on the amount of lateral rods on the surface of the first cylinder.

As can be seen from Figure 2, when the angle increases, the value of the frequency parameter also increases. As the length of cylindrical shells increases, as can be seen from Figure 3, the value of frequency parameter decreases. The value of frequency parameter increases due to increase of orthotropic properties of the cylindrical shell. As can be seen from Figure 4, frequencies of natural vibrations of retaining walls increase with increasing the amount of rods.

REFERENCES


BIOGRAPHY

Dilgam Seyfeddin Ganiyev was born in Goyler, Shamaki, Azerbaijan, in 1981. He graduated from Faculty of Transportation, Azerbaijan University of Architecture and Construction, Baku, Azerbaijan in 2002. In the same year, he received his M.Sc. degree from the same university. During 2004-2007, he got his Ph.D. education at the same university. In 2004, he defended his Ph.D. thesis, earning his Ph.D. degree in Technical Sciences. In 2005, he started his career and worked as a leading engineer in a number of projects of Azerbaijan. He also worked as a bridge engineer in Akin Project company. He was awarded with “Tereqi” medal in 2018. He was elected as a member of International Academy of Transport in 2017. He was awarded with the Jubilee Medal of Azerbaijan "100 Years of Azerbaijan Automobile Roads (1918-2018)" in 2018. Currently, he works as a Chief Engineer in Institute of Azerbaijan State Agency of Automobile Roads, Baku, Azerbaijan. He is also a lecturer in Azerbaijan University of Architecture and Construction.