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PLACE AND ISSUES OF MODELING IN TEACHING OF MATHEMATICS

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Abstract- In view of the development of the mathematics teaching curriculum towards an approach that aims at using mathematics as a service discipline by integrating the modeling approach, our choice of study is interested in the difficulties of importing this process into secondary education in Morocco. This issue will be addressed through a two-dimensional study. The first dimension involves an examination of the existing mathematical organizations in the institution, focusing on the modeling approach. This dimension is institutional in nature. The second dimension, which pertains to the professional aspect, centers on the teaching practices employed by science core class instructors, specifically their declarative practices. The study is conducted within the theoretical framework of TAD: Anthropological Theory of Didactics, drawing inspiration from the model proposed by BLUM AND LEISS [3]. The convergence of these two dimensions facilitates an understanding of the challenges encountered in implementing this approach.

Keywords: Didactic Modeling, Teaching Practices, Curriculum, Teaching Mathematics.

1. INTRODUCTION AND PROBLEMATIC

Our choice of studying mathematics education through modeling is not arbitrary; it is based on several observations that motivate our thinking. Firstly, mathematics is considered a service discipline. In the current period, there is an increasing tendency to emphasize the instrumental nature of mathematics. In certain international studies, mathematics finds its legitimacy as a service discipline by transferring its concepts and methods to solve external problems posed by the development of knowledge in other sectors of human activity [8, 21]. This function of mathematics necessitates a strong focus on the teaching and learning of the modeling process. Indeed, this process has been considered as an object of study since the early didactic works in France and also in Germany [12].

When confronted with concrete situations from reality, it becomes necessary to translate them into mathematical terms, solve them by applying mathematical knowledge, and finally verify the validity of the solutions in the original context. This approach effectively reflects the process of modeling, as we will discover later on. According to PISA: "If the education system teaches students to apply this type of approach, they will be better equipped to use their mathematical knowledge and skills throughout their lives. They will possess what can be called mathematical literacy" [16]. This literacy is defined by PISA as follows: "mathematical literacy is the capacity of an individual to identify and understand the role that mathematics plays in the world, to make well-founded judgments about mathematical content and practices, and to engage in mathematics-related activities, as a constructive, reflective and active citizen" [16].

Therefore, the PISA study ultimately confirms the importance of teaching and learning the modeling process as a path for students to assimilate this mathematical literacy, which is crucial for shaping them as future critical and constructive citizens. The Moroccan education system comprises a six-year primary cycle, a three-year middle secondary cycle, and a three-year higher secondary cycle. In the Moroccan context, the curriculum for the secondary cycle suggests that mathematical modeling holds a significant place in mathematics education [14]. Upon an initial reading of the general goals of mathematics education in the secondary cycle, program designers demonstrate a willingness to integrate the modeling approach into mathematics. In this study, we focus on the field of elementary algebra in the first year of the higher secondary cycle, known as the common core, particularly in the scientific and technological series. We seek to address the following questions:

- Where can we identify the presence of the modeling approach in mathematics education in the higher secondary cycle (first year)?
- What are the constraints in implementing the modeling approach in Moroccan secondary education?
- What are the explanatory factors behind the teaching practices that hinder the implementation of the modeling process?

These questions will be approached through two dimensions. The first dimension is an analysis of mathematical praxeology's by the institution concerning the modeling approach, which is an institutional dimension.

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The second dimension is a professional dimension based on the declared practices of teachers in the first year of the higher secondary cycle.

2. THEORICAL FRAMWORK

2.1. The Didactic Anthropological Theory (TAD)

This study is framed within the context of the Didactic Anthropological Theory [5], which provides a perspective for describing the constraints that influence teaching practices regarding modeling in first-year secondary education. This approach examines the complex relationships between teachers and institutional and personal requirements.

2.1.1. Institutional Report and Personal Report

The set of interactions that a given person X can have with a given object O, as well as all the ways in which X can relate to O, is denoted as $R(X, O)$. We say that "X knows object O" when we have $R(X, O) \neq \emptyset$, and this is called the personal report of X to O. The institutional report refers to the report of an institution I to an object O, or more precisely, the subjects (Ideals) of institution I in position p in I to this object O, denoted as (p, o). "The institutional report on an object, for a given institutional position, is shaped and reshaped by the set of tasks that must be accomplished, through specific techniques, by the individuals occupying that position. It is through the fulfillment of different tasks that a person is led to carry out throughout their life in various institutions, of which they are subject successively or simultaneously, that their personal report to the considered object emerges" [4].

This institutional report constitutes the essential system of conditions and constraints under which it takes shape and evolves into a second report, namely the personal report of teachers regarding algebraic modeling.

2.1.2. Praxiological Organization or Praxeology

Modeling mathematical activity through the lens of the Anthropological Theory of Didactics (ATD) [5] is done by breaking down this activity into four main elements: task, technique, technology, and theory. According to this model, institutional practices can be analyzed by decomposing the system into tasks (t) that belong to different types of tasks (T) [6]. Each task (t) is performed using a specific technique, which is justified by a technology corresponding to a rational discourse explaining this technique. Ultimately, each technology is based on the foundations of a theory. In the context of this study, we use this modeling to analyze mathematical organizations that emerge in the field of algebraic modeling. Our goal is to highlight aspects that are often overlooked or hidden in the existing dynamics between these different mathematical organizations, whether they are punctual, local, regional, or global. Additionally, we will analyze the didactic praxeology's that accompany the implementation of these mathematical organizations by identifying the hierarchy of levels of didactic co-determination [7] that correspond to the levels of mathematical determinations explicitly outlined in this study.

2.2. Levels of Didactic Co-Determination

A hierarchy of levels of didactic codetermination [7] helps to specify the different scales of praxeological organizations and position them in relation to each other. These levels of didactic codetermination can be divided into two categories: higher levels of codetermination, such as pedagogy, school, society, and civilization, and lower sub-levels or levels of codetermination, such as discipline, domain, sector, theme, and subject of study. These sub-levels can be related to levels of mathematical determination.

- Discipline (LV1): Mathematics is considered a combination of different fields of study.
- Domain of study (LV2): Algebra is a global organization that encompasses multiple regional organizations.
- Sector (LV3) refers to regional organization, which in turn is a combination of local organizations based on the same theory.
- Themes of study (LV4) correspond to local organization. These themes are grouped into regional organizations designated as study sectors.
- Subjects of study (LV5) generally refer to a type of task T, they are specific questions and organizations. They are grouped in a more or less clear manner within local organizations that represent the themes of study.

These sub-levels correspond to the different divisions that structure the curriculum and the mathematical organizations taught according to the institution in question. The following table illustrates this structuring in the Moroccan context.

Table 1. Levels of didactic codetermination related to modeling [2]

Civilization	Level-3	Teacher professional development and student profile development
Society	Level-2	Interdisciplinary approach and socio-constructivism
School	Level -1	Qualifying secondary cycle
Pedagogy	Level 0	Tutor teacher and active student
Discipline	Level 1	Mathematics
Field of study	Level 2	Algebra (global praxeology)
Sectors of study	Level 3	Modelling (regional praxeology's)
Topics of study	Level 4	Numerical, algebraic and functional modeling (local praxeology)
Study topics	Level 5	Setting in equations, modeling by algebraic expressions algebraic expressions, modeling by functional relations (punctual praxeology's)

Several studies on modeling focus on the theme level, which allows for the construction of specific mathematical praxeology's for particular problems. Thus, modeling is seen as a means of introducing a theme or mathematical concepts. Once this introduction is done, the modeled system no longer serves a purpose. Other works operate at the discipline level, aiming at of the development multidisciplinary skills without direct reference to mathematical content. This approach aligns with the notion of an integrated curriculum, where mathematical content is formulated in terms of competencies, including those related to modeling. In the perspective of "Realistic Mathematics Education" (RME) [10] the aim is to position the modeling problem at the sector level. This approach is based on the understanding that mathematics is the result of human activity [11], and students acquire formal mathematical knowledge by utilizing their informal knowledge with the assistance of the teacher. This transformation of concrete informal knowledge into formal knowledge occurs through familiar contextual problems for students, which can be found across all themes and subjects related to modeling.

2.3. Ecological Analysis

The ecological approach in didactics [1, 15] emphasizes that the introduction of an object into the educational role of the object within the system of objects with which it interacts. To specifically study system cannot be considered natural, as it disrupts the ecological balance of the system by destroying certain interrelationships between existing objects and creating new ones. The study of ecological conditions, such as levels of didactic determination [7], helps to understand that each object of knowledge cannot exist in isolation but is related to other objects of knowledge. From this perspective, the law of "everything structured" is developed, asserting that the viability of an object of knowledge depends on other objects without which it has no reason to exist [15]. The ecological analysis of an object of knowledge is based on two key notions: habitat, which refers to the contexts in which the object evolves and its conceptual environment, and niche, which describes the modeling in algebra, the tools of the ecological approach in didactics allow for the analysis of "text of knowledge" to identify the institutional relationship with the object of knowledge from the teacher's point of view and to characterize the position occupied by modeling in algebra.

The ecological analysis of the teaching text explores how the universe of the taught knowledge is institutionally delimited. It involves analyzing the interrelationships between objects, sub-objects, and super-objects present in the teaching text, highlighting their habitats and ecological niches, as well as trophic levels, etc. At this level, the problem of identifying institutional objects and analyzing institutional and official relationships with an object *O*, as well as their evolution, can be posed [1]. Thus, to address the questions of what the teacher teaches and how they teach it, a mathematical praxeology allows for modeling the answer to the first question, while a didactic praxeology allows for modeling the answer to the second question. In our view, the ecological approach enables the characterization of didactic praxeology's developed by the institution by structuring the highlighted levels of didactic codetermination. This allows for defining two institutional relationships, one from the teacher's perspective and the other from the student's perspective (topo genesis) [1].

These approaches allow us to question the existence of modeling as an object of teaching. By combining the notions of habitat and niche with the levels of didactic codetermination, we are able to describe the institutional relationship developed with algebraic modeling more precisely. However, this approach is often perceived as a pedagogical modality rather than an explicit teaching objective. Therefore, we approach it through the levels of codetermination to which it is linked in the Moroccan educational system, while also questioning the functions fulfilled by this problem-solving activity in official texts. We also utilize the concept of the semiotic representation system to characterize the mathematical organizations developed in the textbook regarding modeling [9]. This approach involves converting the data from the initial problem into the symbolic representation system (algebraic, numerical, or graphical). Once these theoretical elements are established, we present the references of our epistemological framework for this research.

3. EPISTEMOLOGICAL FRAMEWORK

3.1. The Modeling Process

In order to study the difficulties teachers, face in implementing modeling practice in their classrooms, a literature review on this process in mathematics education helped us to distinguish some aspects of this approach. As already mentioned, from our literature review on modeling, we noticed how international studies like PISA (2003) had as objectives the formation of a critical citizen, and also to have "mathematical literacy" in the individual [16].

The modeling process remains an important part of this culture, and we have presented several reasons for the implementation of this approach in teaching. The process of modeling is seen in the perspectives as a broader process than that of mathematization, which is a part of the modeling cycle [22]. And as Blum points out, the term modeling does not refer only to the passage from a situation to a mathematical model (as may have been the case), but to the "whole process of structuring, mathematizing, working mathematically, and interpreting/validating" [16].

It is at the level of actual practice that the modeling culture can be built through the role that the teacher must play so that students can engage in the modeling process. At this level, all these researches affirm that there are difficulties for teachers to apply this type of approach in their classes. The RME current refers to two types of mathematization, a horizontal mathematization in which mathematical tools are mobilized and used to structure and resolve a situation (from the real world to the world of representations, symbols) and a vertical mathematization which is played out at a purely mathematical level (circumscribed to the world of symbols alone) [10].

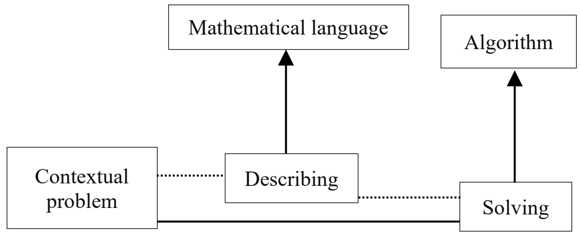


Figure 1. Horizontal and vertical mathematization [19]

The work of Blum and Leiss: Among the modeling cycles proposed in secondary education is that of Blum and Leiss, this type of cycle already considers a stage that can be called intermediate between the real situation and the mathematical model, in the English-speaking works this stage is called "real model" [10]. Blum and Leiss describe the modeling process in six stages as shown in Figure 2.

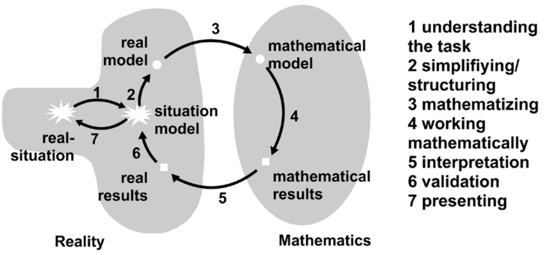


Figure 2. Modeling scheme from BLUM and LEISS (2005) [3]

In this same category, we can classify the work of Henry who introduced the stage of "pseudo-concrete model" which is an intermediate model between reality and the mathematical model, this model is considered as a first level of abstraction of what is called "real situation": "the passage from the extra-mathematical domain to the "pseudo-concrete" domain is done by reducing the concrete system to aspects of this system relevant in relation to the questions to be answered through mathematical modeling, while remaining in a non-strictly mathematical language" (...) this pseudo-concrete model can be more or less close to the mathematical model to be built or conversely to the situation to be modeled" [13].

4. METHODOLOGY

In order to analyze institutional practices at the beginning of high school, we rely on the official instructions and the textbook. The official guidelines are studied from the educational programs. For each level of education, there are pedagogical guidelines that are considered as a main reference to be followed by teachers alongside the official textbooks (textbooks certified by the Ministry). These official documents reflect the content of the knowledge to be taught and provide information on the curricular perspective adopted by the institution. We combine this work with studies in the didactics of algebra that are based on semiotic and anthropological approaches. In our study, we intersect with work in algebra didactics that falls under both semiotic and anthropological approaches. These two complementary approaches allow us to better understand teachers' personal and institutional relationships with algebra through modeling. The work based on the anthropological approach proposes epistemological models for teaching algebra. They help us to examine the didactic and trans positive phenomena that determine the place and function of algebra in the school curriculum. On the other hand, the semiotic approach emphasizes the semiotic stakes of the modeling approach, in particular the difficulties linked to the conversion between different registers of representation.

Indeed, we have been inspired by the work of RUIZ-MUNZON and AL [17] for the identification of mathematical praxeology's related to modeling, in this research, the authors present a teaching model that suggests that the emergence of algebra occurs through a progressive process of algebraization of the calculation programs. This teaching model is structured around the following elements:

- First stage: in this stage the kind of problems are those that require the use of algebraic expressions from what called by Chevallard "the programs of calculation" (A pc is defined by a sequence of operation of numbers that can be carried out in order to find finally a numerical result concert). This pc can be distinguished by quantities to operate called (arguments of the PC) and in order to solve an arithmetic problem; in the school habit the method is to reduce this PC to sequences of PC with two arguments and one operation. The result of one is used as an argument of the next. The technique used in solving this type of problem is that of analysis-synthesis [17].
- Second step: The type of problem that consists of algebraic techniques of equational calculus which gives the use of "=" a meaning of conditional equality between two PCs. Equations.
- Third step: the algebraic formulas that appear when the distinction between unknowns and parameters disappears.

In order to answer the questions related to our problem, we consider the regional OM which is the algebraic modeling in our case and we distinguish four local mathematical organizations OML.

- OML1: the modeling of situations by numerical expressions
- OML2: the modeling of situations by algebraic expressions
- OML3: the modeling of situations by equations
- OML4: the modeling of situations by functions

These four Local Mathematical Organizations are closely linked, because the use of certain types of tasks of a local LMO implies the use of other types of tasks of the local LMOs or of the others. For example, the OML 1 tasks "model the problem by an equation" may refer to an equivalence of computational programs by translating each computational program, where one of the programs may then refer to OML 2 using formulas that subsequently allow them to be equated.

We identify the lower levels of didactic co-determination within the algebraic domain and the links that maintain them. Next, we characterize the OMLs developed around modeling. Our study is illustrated by elements of prior analysis highlighting the dominant contexts (extra-mathematical, geometrical), the registers of representation most solicited, the types of tasks suggested, as well as the explicit or implicit elements of the technological-theoretical block related to the established techniques.

We confront this analysis with the study of teachers' declared practices regarding modeling by using a questionnaire. The questionnaire includes questions aimed at gathering information about the meaning teachers attribute to modeling in mathematics, the functions they associate with this process, as well as the possible obstacles encountered in implementing this approach in their classrooms.

5. INSTITUTIONAL PRACTICES

5.1. Role of Mathematical Modeling in Secondary Education

The latest reform in secondary mathematics education, implemented since November 2007, places great importance on the resolution of mathematical and non-mathematical problems. The new textbooks contain activities that involve modeling practices alongside the construction of new algebraic knowledge. This new didactic approach aligns with the organization developed around problem-solving. The pedagogical guidelines emphasize the importance of developing disciplinary skills, including the ability of students to develop various functions for algebraic modeling.

In the general goals of mathematics education, particularly in the second and third goals, "developing students' problem-solving abilities and mathematical communication," it is indicated that "the ability of students to model situations should be developed" [14]. Developing students' problem-solving abilities includes:

Developing their ability to formulate problems based on mathematical or familiar/unfamiliar situations and express them using mathematical models. In these guidelines, we find that there are other different functions of modeling that contribute to the development of students' skills in mathematical knowledge, mathematical communication, mathematical reasoning, as well as cognitive and semiotic levels. It also addresses the integrity of students in everyday life.

5.1.1. Regarding Mathematical Knowledge and Communication

At this level, the focus is on developing students' ability to communicate mathematically and develop their skills in one or more domains [14]. Developing students' ability in modeling situations or presenting a proof, explaining a strategy, or solving a problem using oral and written expression, as well as the use of drawings, graphs, or algebraic methods.

5.1.2. Regarding Mathematical Reasoning

At this level, the focus is on developing students' ability to use deductive and inductive reasoning, as well as critical thinking, as we can see in this translated excerpt [14].

- Developing their ability to engage in mathematical discovery from suitable models.
- Developing their ability to understand and apply inductive reasoning.
- Developing their ability to understand and apply deductive reasoning.
- Acquiring the ability to use various methods of proof.
- Developing their ability to understand and apply reasoning methods.
- Developing their ability to formulate conjectures, produce proofs, and provide evidence.
- Acquiring the relevance of reflection and the ability to make judgments.
- Developing their ability to ensure the validity of their ideas.
- Developing their ability to provide examples and counterexamples.
- Developing their ability to appreciate the power of reasoning as a fundamental part of mathematics [14].

5.1.3. Cognitive Aspect and Disciplinary Aspect

On the cognitive level, it involves using mathematics to understand phenomena in other disciplines and to integrate with the real world. On the disciplinary level, it involves developing an interdisciplinary approach by drawing upon mathematical knowledge.

- Acquiring essential knowledge and skills in different branches of mathematics.
- Acquiring sufficient mathematical knowledge and skills to pursue future studies or to integrate into the workforce.
- Acquiring mathematical knowledge and skills to understand and grasp the content of other academic units, especially those in the scientific and technological fields.
- Acquiring essential skills to utilize new technologies [14].

In summary, this initial review of the curriculum has highlighted a lack of explicit explanation regarding the various identified aspects and a lack of concrete examples to illustrate these guidelines. This deficiency in terms of clarity and practical examples may hinder teachers' understanding and adoption of this approach.

5.2. Role of Algebraic Modeling in Curriculum of the First Year of Secondary Education

Modeling is present in various domains of activity within this level (numerical, algebraic, geometric, statistical, and analytical). Additionally, the curricula are formulated in terms of competencies, which we consider from an anthropological perspective as elements of the practical-technical block and knowledge to be taught, which can be seen as constituents of the technologic-theoretical block. This provides information about the types of tasks and expected techniques. We have chosen to classify these competencies into the different domains of activity in both the curriculum and the chosen textbook (Najah in mathematics), which is the most commonly used textbook by teachers.

Table 2. Competencies required for the required skills for modeling in the first year of the Moroccan secondary qualifying level [14]

Sectors of activities	Expected capabilities	Pedagogical recommendations
Numerical Activities	C1: Use parity and prime factor decomposition and PGCD and PPCM to solve simple problems involving the natural numbers.	

Algebraic Activities	C2: "Graphically represent the solutions of inequalities or systems of inequalities of the first degree with two unknowns, and use this representation in the rejoining of the plane and in the solution of simple linear programming problems "	- Problems from everyday life or from other subjects should be proposed in order to accustom students to mathematize situations and to solve them. - The graphical representation of the solutions of a first degree inequation with two unknowns will be used to solve some simple linear programming problems.
Statistical activities	C3: Read and interpret statistical graphs; C4: Interpret positional and dispersion parameters	- We will use examples from everyday life or from other disciplines (History - Geography, Biology, Chemistry...), authentic situations, in order to initiate students to collect statistical data, organize them in tables and represent them graphically - The calculation of statistical parameters and their interpretation will be done in order to answer questions related to the study of phenomena and to make deductions
Analytical activities	C5: Express, using the concept of function, situations from everyday life or from other disciplines	-To approach the notion of function and its graphical representation, one can use, as far as possible, software that allows the construction of curves of functions, one can also make this approach from well-chosen situations of geometry, physics, economy or everyday life; - Students should be trained to mathematize situations and solve various problems when studying the extremums of a function;
Geometric Activities	C6: Use the scalar product in solving geometric problems; C7: Use the properties of space geometry to solve real-life problems	The role of this tool in the determination of geometric locations in the plane, in the calculation of lengths, areas and angle measurements will be emphasized;

5.3. Role of Modeling in the Chosen Textbook

By addressing the activities in which we observed the presence of modeling, we were able to identify the OML (Local Mathematical Organizations) associated with this approach through a task analysis, as well as the techniques proposed to the students in the textbook. In order to carry out this analysis, we developed a coding system to facilitate the work, which takes as Equation (1):

$$T_i - N_P - N_S \quad (1)$$

where, T_i is typology of the situation involving the relevant task (preparatory activity; solved exercises; end-of-chapter exercises and problems), N_P is page number in the textbook, and N_S is number of the learning situation containing the relevant task.

We present the results of the quantitative analysis conducted to identify these local mathematical organizations.

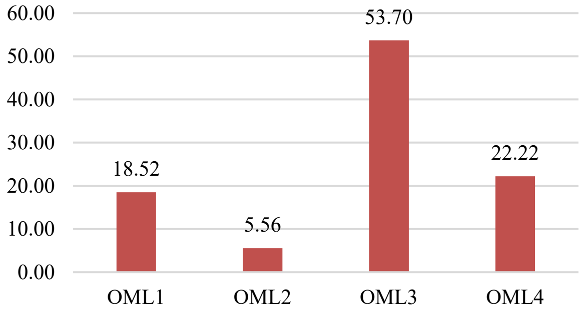


Figure 3. Results of percentage of situations according to local mathematical organizations

5.3.1. A Reorganization of the Lower Levels of Didactic Codevelopment

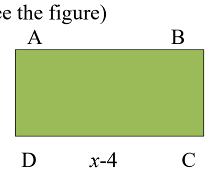
Algebraic notation is used frequently and consistently, in many situations, we observe interconnectivity between algebraic and graphical representations that emerges in the various tasks proposed. The concepts of quadratic polynomial functions, rational functions, linear and affine functions as well as systems of equations with two unknowns, are addressed. Additionally, the chapters are organized in a new manner, associating the domains of equations and functions. In general, algebraic modeling through graphs is highly prevalent in this new organization of studies, as demonstrated by the examples extracted from the textbook (Figures 4 and 5).

A company manufactures two types of belts, A and B. Each A belt is sold with a profit of 2 dirhams, and each B belt is sold with a profit of 1.5 dirhams.

The manufacturing of a belt A takes 2 hours, and the Manufacturing of a belt B takes 1 hour, while the company has only 1000 hours of work per day. There is enough leather to produce 800 belts A and B per day, and there are 400 buckles for belts A and 700 buckles for belts B. Create a daily production plan to maximize the company's profit.

Figure 4. Concrete problem1 taken from the textbook [18]

Let ABCD be a rectangle (see the figure)



1) Determine the possible values of x
2) Calculate the area $A(x)$ of the rectangle ABCD as a function of x
3) Graphically represent the function $A(x)$ over the interval [0, 4]
4) Determine the value of x for which the area of rectangle ABCD is maximum

Figure 5. Concrete problem2 taken from the textbook [18]

5.3.2. The Habitats and Niches Assigned to Modeling

The ecological analysis allowed us to categorize the types of tasks that involve a modeling approach through the contextualized problems proposed in the official textbook, based on the niches and habitats associated with the levels of didactic codevelopment highlighted by our study.

Table 3. Niches and habits associated with modeling in the first year of Moroccan secondary education

Habitats	Niches
Arithmetic-numerical	Solving real-world problems. Solving problems involving proportionality.
Algebraic	Solving problems that require modeling using equations, inequalities, and systems of equations. Solving problems that lead to functional relationships. Solving problems that can be modeled by functions (linear, affine, polynomial, and rational functions).
Geometric	Solving geometry problems that lead to common geometric figures.
Analytical/graphical framework	Solving situations that require a graph. Solving a system of inequalities that models a problem graphically and interpreting the results.

5.3.3. Role of Modeling in New Organization of Study

In the textbook, there are three sections that involve modeling work based on the status of the problems posed in the official textbook. These sections refer to introductory activities, solved exercises and problems, and end-of-chapter exercises. The introductory activities are sometimes accompanied by a phase of institutionalizing concepts or procedures, followed by a section dedicated to exercises. We conducted an analysis of these situations to identify the functions attributed to the modeling approach.

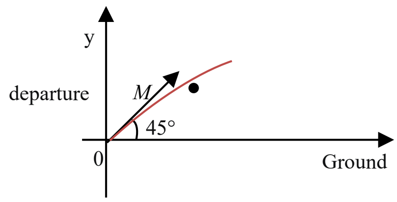
Table 4. Functions attributed to modeling based on problem characteristics [2]

	Preparatory activities	Solved exercises	Exercises and problems
Functions assigned to mathematical modeling	Function of knowledge construction	Function of consolidation and continuous learning	Application and mobilization of resources
Problem characteristics	Property introductions; discovery of solution algorithms or new notions and concepts	Improving skills (in general) (algebraic expressions, functional relationships, equations processing)	Immediate application of learning. Integration of learning and the utilization of knowledge in all its aspects. Practice of different forms of reasoning.

In the activities provided, the model is either introduced beforehand or partially constructed, and it is the student's task to operate on this model to solve it. In other words, the initial steps of horizontal mathematization are often provided by the statement itself: identifying unknown or variable, using letters to represent them, and sometimes providing indications of algebraic relationships to establish (equations or functional relations), as shown in these excerpts from the manual.

Algebraic-functional modeling, which aims to develop aspects related to functions, also implicitly connects the initial domain of quantities (the physical system) with the mathematical model (mathematical functions). However, this modeling is often embedded within the problem statement itself, either through formulas that students can use or by breaking down the task into subtasks with specific instructions. This gives the impression that these problems focus more on applying acquired knowledge rather than constructing or independently manipulating a pre-existing model as illustrated by the example in Figure 6.

From a ground position, a solid M is launched into the air at an angle of 45° and an initial velocity v_0 = 30 km/h, as shown in the figure.



It is known that the equation of M's trajectory is given by the relation.
$y = -0.144x^2 + x$ (x and y in meters).
1) Determine the two numbers a and b such that:
$y = -0.144(x - a)^2 + b$.
2) In an orthogonal coordinate system, which we will choose, plot the trajectory of M.
3) At which point will the solid M land? What is the maximum height that the solid M can reach before falling?

Figure 6. Concrete problem3 taken from the textbook [18]

Two car rental companies offer the following rates: The first company offers a fixed fee of 550 dirhams plus 6.30 dirhams for each kilometer traveled. The second company offers a fixed fee of 600 dirhams plus 6.10 dirhams for each kilometer traveled. Determine the distance from which the rate of the second company is more advantageous.

Figure 7. Concrete problem4 taken from the textbook [18]

5.4. Summary of the Analysis

The analyses conducted on the program and the chosen manual for the first year of the Moroccan secondary qualifying cycle highlight institutional relationships with algebraic objects. These relationships seem to evolve in a context that provides opportunities for internal and external mathematical applications through problem-solving and problem situation resolution. These situations encompass a variety of contexts and a diversification of experiential domains, following a constructivist and interdisciplinary approach to learning. In this approach, considerable importance is given to flexibility between different semiotic languages and mathematical frameworks. Local mathematical organizations through equations are extensively addressed in contextualized problems, accounting for approximately 54%, while functions or functional relationships come in second place in terms of frequency, at approximately 22% (Figure 3).

There is enhanced interaction between algebraic language and graphical language. The establishment of a modeling practice based on the relationships between functions and equations appears to promote a better understanding of the concepts involved (unknown/variable, static aspect/variation, variable/parameter, functional dependence) and facilitate the adoption of a functional mindset, transitioning from a conception of equations in terms of known and unknown quantities to a conception in terms of dependent and independent variables. Despite the importance given to modeling practice in the official programs, the proposed situations, which are based on concrete contexts and largely guide the process of horizontal mathematization, seem to fall short of achieving this objective. Indeed, most of these situations are concrete and do not sufficiently highlight the modeling work or foster the development of the skills required by this approach.

Furthermore, the predominance of horizontal mathematization in most activities raises questions about the relevance of these practices. Is it simply a pedagogical teaching modality? Is it related to a didactic organization that accompanies other mathematical domains? Can we not consider the use of pre-established models paradoxical, given that the programs aim to develop skills within a constructivist approach?

6. TEACHING PRACTICES

In education in general, teachers are the primary implementers of improvements in line with institutional practices. In this study, we aim to understand the challenges teachers face in adjusting their teaching practices to institutional incentives that reflect the curriculum. We attempt to address the difficulties encountered in integrating the modeling approach into mathematics education in the classroom. To seek explanatory reasons for the following questions mentioned in the methodology section:

What constraints exist in implementing the modeling approach in Moroccan secondary education?

What explanatory arguments can be made regarding teachers' practices that hinder the development of the modeling approach?

We test the following research hypotheses that guide our research work:

- Hypothesis 1: Horizontal modeling is perceived by teachers as a means to connect mathematics with reality (everyday life) through the problems suggested by the institution, replacing the traditional approach of direct application of mathematical knowledge.
- Hypothesis 2: The implicit presence of modeling, rather than explicit, in institutional practices is an obstacle that influences teachers' engagement with this mathematical approach.
- Hypothesis 3: For certain situations in the textbook, institutional challenges related to mathematical practices around algebraic modeling are not always clear to teachers.

6.1. Experiments

Our experiment is based on a survey conducted among teachers, consisting of questions that provide insights into various aspects of our study based on their reported practices. The sample consists of 54 mathematics teachers from secondary schools in different regions of Morocco. The questionnaire was shared online through teacher groups on social networks and the Moroccan Mathematics Association (Agadir), which distributed the questionnaire among teachers. In all the questions, we provided multiple-choice options to be checked, while allowing teachers the freedom to express themselves using "other" in case they wanted to add their remarks and suggestions.

However, for the following two questions, teachers are required to answer with "yes" or "no":

Do you consider it important to develop modeling skills in students at the beginning of secondary education?

Do you encounter obstacles in implementing this type of approach in your classes?

➢ This survey considers three dimensions:

- The epistemological dimension, which pertains to the concept of a model and the definition attributed to modeling.
- The functional dimension, which relates to the role assigned to this approach in mathematical practices.
- The organizational dimension, which addresses the difficulties, conditions, and constraints that teachers face during the teaching and learning of modeling.

6.2. Analysis of Experimental Results

From an epistemological perspective, this viewpoint was approached by obtaining responses to the following questions:

➢ In your opinion, what definition can be given to mathematical modeling? What are the characteristics of this approach, in your opinion?

Table 5. Contains the obtained responses, along with their percentages

Significant expressions of mathematical modeling	Percentage of responses
The percentage of responses for the skill of transitioning from a concrete situation to a mathematical model	40.7 %
A tool to establish a connection between mathematics and reality	46.3%
The ability to translate a real-life situation into mathematical notation	37 %
The ability to identify the mathematical tools needed to solve a given situation	38.9%
Developing and testing a model	14.8%

The results indicate that the most commonly associated expressions with modeling are those that consider it as a skill to transition from a concrete situation to a mathematical model (40.7%), or as a tool to establish a connection between mathematics and reality (46.3%). Nevertheless, the least frequently mentioned viewpoint pertains to the idea that modeling encompasses the development and testing of a model, accounting for only 14.8% of the declared responses.

From a functional perspective: Before asking this second question, we started by presenting a question to which teachers could respond with either yes or no, while also giving them the opportunity to add their remarks using the "other" option.

The question was as follows: According to you, is it important to develop modeling skills in students at the beginning of secondary school? Most teachers answered either yes or (88.7%) or no (7.5%), while two teachers expressed their opinion by stating that it is important to develop this skill starting from middle school or even from primary school. The responses are distributed as shown in Table 6.

Table 6. Percentage of responses related to the second question

Types of responses	Percentage
Yes	88.7 %
No	7.5 %
At the beginning of middle school	1.9 %
Primary school	1.9 %

The functional dimension was approached through the diverse responses of teachers to the question: In your opinion, what roles are attributed to mathematical modeling in the first year of secondary school?

Table 7. Percentage of responses related to the third question

The roles attributed to mathematical modeling in the first year of secondary school are as follows	Percentage
Allows for the development of certain reasoning	35.2 %
Facilitates the creation of a model	11.1 %
Establishes a connection between mathematics and reality (everyday life)	68.5%
Provides a framework for the problem	20.4%
Involves performing operations on a preconstructed model	14.8%
There is no specific role	3.7%

The majority of teachers are aware of the importance of modeling, and the evidence shows that they agree to incorporate it into mathematics education. By analyzing the received responses, we found that the most chosen role by teachers is to establish a connection between mathematics and reality (everyday life), with a percentage of 68.5%. Additionally, modeling is also seen as facilitating the development of reasoning, with a percentage of 35.2%. However, only 3.7% of responses indicated that no specific role was attributed to modeling in the first year of secondary qualifying education.

On the organizational side: This point was addressed by obtaining responses regarding the challenges and difficulties encountered by teachers in implementing this approach in their classrooms. These responses were collected through two questions: The first question is: Do you encounter obstacles in implementing this type of approach in your classes? For this question, we provided two types of responses, either "No" or "Yes", while allowing teachers the freedom to express their opinions or add remarks and proposals using the "other" option. The second question is: What difficulties do you find in implementing this type of approach in your classes? For this question, we presented several choices as response options, always including the "other" option to allow teachers to add suggestions. The responses provided by teachers to these questions have been classified in tables.

Table 8. Percentage of responses related to the fourth question

Response Types	Percentage
Yes	94.3 %
No	5.7 %

The majority of the responses agree that there are obstacles in implementing modeling in the classroom (94.3%).

Table 9. Percentage of Reponses Related to the fifth Question

The difficulties encountered by teachers in implementing modeling in the classroom.	Percentage
Time constraints	59.3%
Program constraints (lesson allocation, organization of chapters in textbooks, overloaded curriculum...)	61.1%
Language barrier (students not proficient in French language)	59.3%
Lack of resources and training	51.9%
Lack of general motivation among students to engage with contextualized problems	57.4%

The most striking observation in these results is that the majority of teachers selected all the options, and some of them added additional constraints based on their practical experience with the modeling approach. The percentages for each option are similar, confirming that all the proposed choices are genuine challenges for teachers. Additionally, some teachers mentioned other obstacles:

- Abdlkbir: Lack of necessary prerequisites to tackle problem situations; most learners are unable to adapt their skills to find possible solutions to a given situation.
- Abdnebi: Low level of mathematical proficiency among students.
- Mohamed: Obstacles related to students' knowledge of key mathematical tools at the secondary level.
- Nourddine: Lack of mathematical tools (prerequisites).

Several teachers emphasized the constraints faced by students when they have to model a situation due to the lack of prerequisites and the necessary mathematical tools to solve problems. Other teachers mentioned the lack of motivation among students to solve non-mathematical and unfamiliar situations as an additional reason for not addressing this type of problem in class.

The preliminary analysis of our survey, based on the teachers' statements within the scope of our research, has allowed us to address our research questions and confirmed the validity of our research hypotheses. The nature of the difficulties and constraints reported by teachers is more institutional than personal. These challenges depend on the educational institution rather than the teachers themselves. They are related to time constraints [20], understanding problem statements, lack of training [20], lack of prerequisites among students, curriculum overload, and the absence of support for these approaches. All these reasons can explain the resistance of certain applications that explicitly hinder the implementation of modeling. However, the opinions of teachers align with integrating this approach as a skill to be included in mathematics education programs, while still preserving the technical aspect of mathematics by concealing horizontal mathematization. This promotes working with a model already provided by the teacher, thereby limiting the student's activity to vertical mathematization of the problem.

7. CONCLUSION AND PERSPECTIVES

The study undertaken focused on the difficulties encountered in integrating the mathematical modeling approach into secondary education in Morocco. This study is motivated by the recognition of the fundamental role of mathematics as a service discipline, as well as the importance of transmitting and acquiring this approach to shape students as critical and constructive future citizens (in accordance with the guidelines of PISA). The results of the analysis, conducted on both institutional and professional levels, highlight a general prevalence of horizontal mathematization in our curriculum, which does not align with the official incentives that actively promote the practice of modeling.

Furthermore, these findings indicate a limited epistemological understanding of this approach. Secondary school teachers tend to regard models as exact representations of reality, often overlooking their heuristic function as tools for exploring phenomena and their predictive function. This study highlights the non-conformity between institutional reports and those of teachers, revealing the challenges associated with modeling for the latter. It is crucial not to confine the implementation of this approach in education to mere adherence to official incentives. It is time to support, train, and enhance the competencies and knowledge of teachers. Additionally, curriculum developers should incorporate problem-solving examples that illustrate this approach in textbooks. Developing a genuine culture of modeling among teachers from their initial training is of paramount importance, while organizing situations that align more closely with institutional and professional requirements to facilitate the integration of this approach in diverse contexts.

In the context of future scientific research, our objective is to analyze teachers' instructional plans before their actual implementation and to characterize the tasks redefined by teachers in relation to the prescribed tasks. We aim to understand if the issues related to algebraic modeling, as they manifest through different types of proposed tasks, are still perceptible to teachers and to explore the approaches they envision addressing them. Concurrently, we plan to investigate the methods and reasoning used by students when solving modeling problems, both in mathematics and other domains, across different types of educational institutions in Morocco. This broader research aims to deepen our understanding of the processes and cognitive strategies employed by students when faced with modeling problems, as well as to examine potential differences among various educational institutions. By integrating these two complementary approaches, our goal is to contribute to the improvement of mathematics modeling instruction and the development of effective pedagogical strategies to enhance students' modeling skills, while considering the perspectives and practices of teachers.

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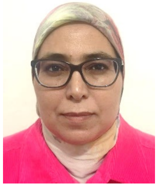

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