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MATHEMATICAL SIMULATION OF STRESS-STRAIN STATE OF RIB-REINFORCED MULTILAYER CYLINDRICAL SHELLS UNDER LONGITUDINAL IMPULSE LOADING

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Abstract- The SSS (stress-strain state) of elastic multilayer composite cylindrical shells reinforced with ribs is studied. A longitudinal impulse load acts on the shell. A refined mathematical model of nonstationary dynamic processes and a method for calculating the dynamic deformation of the shell system in the process of non-stationary vibrations are developed on the basis of a geometrically nonlinear theory of shells and rods, taking into account transverse shears and normal deformations. The mathematical model is a system of nonlinear partial differential equations of hyperbolic type. The problem is solved by the finite difference method. The results of a numerical study of vibrations and dynamic deformation of discretely reinforced shells with an inhomogeneous structure and initial shape imperfections are presented. The analysis of the adequacy of the proposed solution method and the reliability of the results obtained is carried out. The influence of reinforcing ribs, material structure and shape errors of the outer surface in the form of initial deflections on dynamic stability is considered. A criterion for the dynamic stability of shells under impulse loads is formulated. The analysis of numerical results of solving specific problems has shown that the use of the criterion of loss of stability from the condition of occurrence of plastic deformations in comparison with the application of the criterion of the condition of the onset of a sharp increase in deflection leads to qualitatively different results. The problem under consideration is an important scientific problem of structural mechanics, namely, increasing the bearing capacity of complex shell structures and are of great scientific and practical importance.

Keywords: Cylindrical Shell, Composite Materials, Reinforcing Ribs, Nonlinear Deformation, Dynamic Stability, Impulse Loads, Mathematical Models, Numerical Methods.

1. INTRODUCTION

Multilayer thin shells made of composite materials reinforced by ribs are complex inhomogeneous elastic structural elements that are used in modern technology. The teness of the reinforcing ribs and the structure of the material, has a particularly strong effect on non-stationary vibrations under the action of impulsed loads. With such dynamic load large surface deflections occur in a structurally inhomogeneous shell, there is a significant redistribution of deformation and stress fields and loss of structural stability. This leads to the need to apply the wave theory of shells, which more adequately reflects the distribution of deformation and stress fields over the thickness of the object under study. Thus, when solving dynamic problems, along with classical ones, refined solution methods based on a geometrically nonlinear model are used, taking into account shear deformations and inertia of rotation of a normal element.

Thus, when solving nonstationary dynamic problems, it is necessary to use refined computational models and solution methods based on a geometrically nonlinear model that takes into account shear deformations and inertia of rotation of a normal element. The advantage of the vibration equations obtained on their basis is that they are hyperbolic type equations. Such equations can be used to describe the processes of wave propagation and the features of deformation in a thin-walled reinforced shell and its individual layers under the action of impulsed loads.

When considering the processes of contact interaction in problems of dynamics, the wave equations more adequately describe the behavior of structures with spatial discontinuities on the contact surfaces of the constituent layers and ribs. The analysis of the papers devoted to this problem shows that the wave processes are most deeply studied for elastic smooth isotropic shells. At the same time, complex inhomogeneous structures with inclusions of various stiffness in the form of multilayering, reinforcing ribs and composite materials are widely used in practice. From the review papers [1-13] and the results of theoretical and experimental studies [4, 7, 12, 14-27], [31] for inhomogeneous composite shells it follows that the problem of their dynamic stability and strength under the action of impulse loads is currently insufficiently studied.

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Also, insufficient attention is paid to the influence of the initial imperfections of the geometric shape of the shell in the SSS analysis. This makes it necessary to clarify the applied calculation models. Thus, the development of refined calculation methods and the study of non-stationary vibrations and dynamic deformation of multilayer structurally inhomogeneous shells under impulsed loading is an urgent research problem.

In the refined model, the vibration equations of discretely reinforced shells, taking into account the discrete location of the reinforcing ribs, represent a system of hyperbolic equations with terms that contain derivatives in spatial coordinates no higher than the second order and discontinuous coefficients in the form of Dirac delta functions. They make it possible to study wave processes more correctly in the presence of spatial discontinuities. To solve them, it is convenient to use the integro-interpolation method of constructing difference schemes proposed in [29, 30].

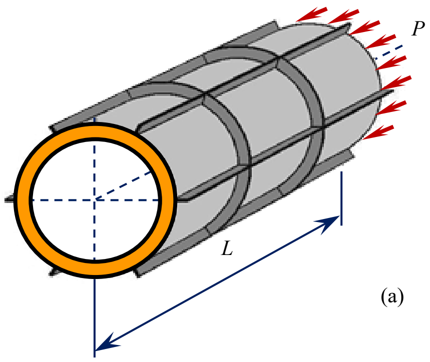
It should be noted that there is still no single approach to choosing the criterion of dynamic stability of the structure. A limiting displacement equal to one or two shell thicknesses, used in [12], is usually used as a stability criterion for smooth shells. However, this criterion does not reflect the potential capabilities of the structure and may be very far from the appropriate exhaustion of its stability. As a rule, the general loss of stability of the ribbed shell under dynamic loading is preceded by a local loss of its stability between the ribs [13], and the general case of deformation is realized only with very small bending stiffness of the reinforcing ribs. Therefore, this issue requires further research, which will expand the scope of applicability of the design scheme in a wider range of geometric, structural and physico-mechanical design parameters.

The purpose of this work is to construct a refined mathematical model of dynamic deformation and to study the effect of longitudinal impulse loading on vibrations, SSS and the correspondence of a small deformation to a small change in force of composite cylindrical shells with structural inhomogeneities

2. PROBLEM STATEMENT

The shell is considered as an elastic inhomogeneous mechanical system consisting of external load-bearing layers and an inner layer of composite materials. From the outside, it is reinforced with longitudinal and transverse ribs, taking into account the discrete placement. Figure 1 shows the calculation scheme of the problem. The SSS of the shell and reinforcing ribs is determined on the basis of the geometrically nonlinear theory of one-dimensional and two-dimensional thin bodies, taking into account transverse shear deformations [25]. In this case, the physical relations of composite layers are formulated on the basis of the the famous Hooke's law for orthotropic material using independent hypotheses of the applied theory of shells for each layer. The applied coordinates x and y are related to the curved coordinate system by the dependencies $\alpha_1 = x$ and $\alpha_2 = y/R$, where R is the radius of the shell.

The problem of vibrations that change over time and dynamic deformation of the shell structure under longitudinal impulse loading is solved by the finite difference method. The deformed state of the mating layers is determined through the components of the generalized displacement vector of the shell $\overline{U} = (u_1, u_2, u_3, \varphi_1, \varphi_2)$, and deformations of the reinforcing ribs are determined through the components of similar generalized vectors $\overline{U}_i$ and $\overline{U}_j$ displacements of the centers of their cross sections.



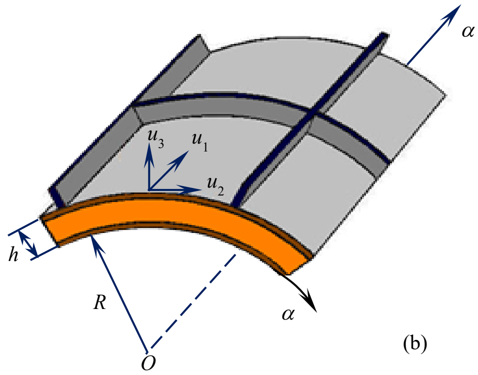


Figure 1. a) The design scheme of an inhomogeneous shell structure, b) Its coordinate system

The deviation of the shape of the median surface from the nominal one is given by the field of normal displacements $w^0 = w^0(x, y)$. For this reason the initial parameter (deformations) of the shell $\varepsilon_{11}^0, \varepsilon_{22}^0, \varepsilon_{12}^0, \varepsilon_{13}^0, \varepsilon_{23}^0$ are determined through the initial dents w^0. It is assumed that the initial imperfections are purely flexural in nature. The components of deformations of the shell and ribs with initial deflections are determined by the dependencies:

$$\varepsilon_{11} = \frac{\partial u_1}{\partial x} + \frac{1}{2}\left(\frac{\partial u_3}{\partial x}\right)^2 - \frac{1}{2}\left(\frac{\partial w^0}{\partial x}\right)^2$$

$$\varepsilon_{22} = \frac{\partial u_2}{\partial y} + \frac{u_3 - w^0}{R} + \frac{1}{2}\left(\frac{\partial u_3}{\partial y}\right)^2 - \frac{1}{2}\left(\frac{\partial w^0}{\partial y}\right)^2$$

$$\varepsilon_{12} = \frac{\partial u_1}{\partial y} + \frac{\partial u_2}{\partial x} + \frac{\partial u_3}{\partial x}\left(\frac{\partial u_3}{\partial y} - \frac{u_2}{R}\right) - \theta_1^0\theta_2^0, \ \theta_1^0 = \frac{\partial w^0}{\partial x}$$

$$\theta_2^0 = \frac{\partial w^0}{\partial y}, \ \varepsilon_{13} = \varphi_1 + \frac{\partial}{\partial x}(u_3 + w^0), \ k_{11} = \frac{\partial \varphi_1}{\partial x}$$

$$\varepsilon_{23} = \varphi_2 + \frac{\partial}{\partial y}(u_3 + w^0) - \frac{u_2}{R}, \ k_{22} = \frac{\partial \varphi_2}{\partial y} \quad (1)$$

$$k_{12}=\frac{\partial\varphi_1}{\partial y}+\frac{\partial\varphi_2}{\partial x},\varepsilon_{11i}=\frac{\partial u_{1i}}{\partial x}\pm z_{1i}\frac{\partial\phi_{1i}}{\partial x}+\frac{1}{2}\left(\theta_{1i}^2+\theta_{2i}^2\right)-\frac{1}{2}\left(\theta_{1i}^0\right)^2$$

$$\varepsilon_{22i}=\theta_{2i}\,,\ \varepsilon_{13i}=\varphi_{1i}+\theta_{1i}-\theta_{1i}^0,,\ \theta_{1i}=\frac{\partial u_{3i}}{\partial x},\ \theta_{1i}^0=\frac{\partial w^0}{\partial x}$$

$$\theta_{2i}=\frac{\partial u_{2i}}{\partial x}\pm z_{1i}\frac{\partial\varphi_{2i}}{\partial x}\ ,\ k_{11i}=\frac{\partial\varphi_{1i}}{\partial x}\,,\ k_{12i}=\frac{\partial\varphi_{2i}}{\partial x}$$

$$\varepsilon_{22j}=\frac{\partial u_{2j}}{\partial y}\pm z_{2j}\frac{\partial\phi_{2j}}{\partial y}+\frac{u_{3j}-w^0}{R_j}+\frac{1}{2}\left(\theta_{1j}^2+\theta_{2j}^2\right)-\frac{1}{2}\left(\theta_{2j}^0\right)^2$$

$$\varepsilon_{21j}=\theta_{1j}\,,\ \ \varepsilon_{23j}=\varphi_{2j}+\theta_{2j}-\theta_{2j}^0$$

$$\theta_{1j}=\frac{\partial u_{1j}}{\partial y}\pm z_{2j}\frac{\partial\varphi_{1j}}{\partial y}\,,\ \theta_{2j}=\frac{\partial u_{3j}}{\partial y}-\frac{1}{R_j}\left(u_{2j}\pm z_{2j}\,\varphi_{2j}\right)$$

$$\theta_{2j}^0=\frac{\partial w^0}{\partial y}\,,\ k_{22j}=\frac{\partial\varphi_{2j}}{\partial y}\,,\ k_{21j}=\frac{\partial\varphi_{1j}}{\partial y}$$

where, R, R_j are the radii radii of the shell and circumferential ribs; z_{1i}, z_{2j} are the eccentricities of the ribs; h is the thickness of the coating; φ_1, φ_2 are the angles of rotation of the normal to the surface of the shell relative to the coordinate axes.

3. METHOD FOR SOLUTION

The vibration equations of a discretely reinforced shell, taking into account the contact conditions of the ribs, are represented as:

$$\begin{aligned}
&\frac{\partial N_{11}}{\partial x}+\frac{\partial S}{\partial y}+\sum_{i=1}^{I}\frac{\partial N_{11i}}{\partial x}\delta(y-y_i)+\sum_{j=1}^{J}\frac{\partial\overline{N}_{21j}}{\partial y}\delta(x-x_j)+P_1=\\
&=\rho h\frac{\partial^2u_1}{\partial t^2}+\sum_{i=1}^{I}\rho_{1i}F_{1i}\left(\frac{\partial^2u_{1i}}{\partial t^2}\pm z_{1i}\frac{\partial^2\varphi_{1i}}{\partial t^2}\right)\delta(y-y_i)+\\
&+\sum_{j=1}^{J}\rho_{2j}F_{2j}\left(\frac{\partial^2u_{1j}}{\partial t^2}\pm z_{2j}\frac{\partial^2\varphi_{1j}}{\partial t^2}\right)\delta(x-x_j)\\
&\frac{\partial S}{\partial x}+\frac{\partial N_{22}}{\partial y}+\frac{\overline{N}_{23}}{R}+\sum_{i=1}^{I}\frac{\partial\overline{N}_{12i}}{\partial x}\delta(y-y_i)+\\
&+\sum_{j=1}^{J}\left(\frac{\partial N_{22j}}{\partial y}+\frac{\overline{N}_{23j}}{R_j}\right)\delta(x-x_j)+P_2=\rho h\frac{\partial^2u_2}{\partial t^2}+\\
&+\sum_{i=1}^{I}\rho_{1i}F_{1i}\left(\frac{\partial^2u_{2i}}{\partial t^2}\pm z_{1i}\frac{\partial^2\varphi_{2i}}{\partial t^2}\right)\delta(y-y_i)+\\
&+\sum_{j=1}^{J}\rho_{2j}F_{2j}\left(\frac{\partial^2u_{2j}}{\partial t^2}\pm z_{2j}\frac{\partial^2\varphi_{2j}}{\partial t^2}\right)\delta(x-x_j),\\
&\frac{\partial\overline{N}_{13}}{\partial x}+\frac{\partial\overline{N}_{23}}{\partial y}-\frac{N_{22}}{R}+\sum_{i=1}^{I}\frac{\partial\overline{N}_{13i}}{\partial x}\delta(y-y_i)+\\
&+\sum_{j=1}^{J}\left(\frac{\partial\overline{N}_{23j}}{\partial y}-\frac{N_{22j}}{R_j}\right)\delta(x-x_j)+P_3=\rho h\frac{\partial^2u_3}{\partial t^2}+\\
&+\sum_{i=1}^{I}\rho_iF_i\frac{\partial^2u_{3i}}{\partial t^2}\delta(y-y_i)+\sum_{j=1}^{J}\rho_{2j}F_{2j}\frac{\partial^2u_{3j}}{\partial t^2}\delta(x-x_j),\\
&\frac{\partial M_{11}}{\partial x}+\frac{\partial H}{\partial y}-N_{13}+\sum_{i=1}^{I}\left(\frac{\partial M_{11i}}{\partial x}\pm z_{1i}\frac{\partial N_{11i}}{\partial x}-N_{13i}\right)\delta(y-y_i)+\\
&+\sum_{j=1}^{J}\left(\frac{\partial M_{21j}}{\partial y}\pm z_{2j}\frac{\overline{N}_{21j}}{\partial y}\right)\delta(x-x_j)+m_1=\rho\frac{h^3}{12}\frac{\partial^2\varphi_1}{\partial t^2}+\\
&+\sum_{i=1}^{I}\rho_{1i}F_{1i}\left[\left(z_{1i}^2+\frac{I_{1i}}{F_{1i}}\right)\frac{\partial^2\varphi_{1i}}{\partial t^2}\pm z_{1i}\frac{\partial^2u_{1i}}{\partial t^2}\right]\delta(y-y_i)+\\
&+\sum_{i=1}^{I}\rho_{1i}F_{1i}\left[\left(z_{1i}^2+\frac{I_{1i}}{F_{1i}}\right)\frac{\partial^2\varphi_{1i}}{\partial t^2}\pm z_{1i}\frac{\partial^2u_{1i}}{\partial t^2}\right]\delta(y-y_i),\\
&\frac{\partial H}{\partial x}+\frac{\partial M_{22}}{\partial y}-N_{23}+\sum_{i=1}^{I}\left(\frac{\partial M_{22i}}{\partial x}\pm z_{1i}\frac{\partial\overline{N}_{12i}}{\partial x}\right)\delta(y-y_i)+\\
&+\sum_{j=1}^{J}\left(\frac{\partial M_{22j}}{\partial y}\pm z_{2j}\frac{N_{22j}}{\partial y}-N_{23i}\right)\delta(x-x_j)+m_2=\rho\frac{h^3}{12}\frac{\partial^2\varphi_2}{\partial t^2}+\\
&+\sum_{i=1}^{I}\rho_{1i}F_{1i}\left[\left(z_{1i}^2+\frac{I_{kp1i}}{F_{1i}}\right)\frac{\partial^2\varphi_{2i}}{\partial t^2}\pm z_{1i}\frac{\partial^2u_{2i}}{\partial t^2}\right]\delta(y-y_i)+\\
&+\sum_{j=1}^{J}\rho_{2j}F_{2j}\left[\left(z_{2j}^2+\frac{I_{2j}}{F_{2j}}\right)\frac{\partial^2\varphi_{2j}}{\partial t^2}\pm z_{2j}\frac{\partial^2u_{2j}}{\partial t^2}\right]\delta(x-x_j),
\end{aligned}\tag{2}$$

$$\overline{N}_{12i}=N_{12i}+N_{11i}\theta_{2i},\overline{N}_{13}=N_{13}+N_{11}\theta_1+S\theta_2,\overline{N}_{13i}=N_{13i}+N_{11i}\theta_{1i}$$

$$\overline{N}_{21j}=N_{21j}+N_{22j}\theta_{1j},\overline{N}_{23}=N_{23}+N_{22}\theta_2+S\theta_1,\ \overline{N}_{23j}=N_{23j}+N_{22}\theta_{2j}$$

The forces and bending moments acting in the shell and ribs are expressed through deformations by dependencies:

$$\begin{aligned}
&N_{11}=B_{11}(\varepsilon_{11}+\nu_{21}\varepsilon_{22}),N_{13}=B_{13}\varepsilon_{13},N_{23}=B_{23}\varepsilon_{23}\\
&N_{22}=B_{22}(\varepsilon_{22}+\nu_{12}\varepsilon_{11}),S=B_{12}\varepsilon_{12},H=D_{12}k_{12}\\
&M_{11}=D_{11}(k_{11}+\nu_{21}k_{22}),M_{11i}=E_{1i}I_{1i}k_{11i},\\
&M_{12i}=G_{1i}I_{kp1i}k_{12i},M_{21j}=G_{2j}I_{kp2j}k_{21j},\\
&M_{22}=D_{22}(k_{22}+\nu_{12}k_{11}),M_{22j}=E_{2j}I_{2j}k_{22j},N_{11i}=E_{1i}F_{1i}\varepsilon_{11i}\\
&N_{12i}=G_{1i}F_{1i}\varepsilon_{12i},N_{13i}=G_{1i}F_{1i}k_{1i}^2\varepsilon_{13i},N_{21j}=G_{2j}F_{2j}\varepsilon_{21j}\\
&N_{22j}=E_{2j}F_{2j}\varepsilon_{22j},N_{23j}=G_{2j}F_{2j}k_{2j}^2\varepsilon_{23j}
\end{aligned}\tag{3}$$

where, $B_{11}=\dfrac{E_1h}{1-\nu_{12}\nu_{21}}$, $B_{22}=\dfrac{E_2h}{1-\nu_{12}\nu_{21}}$, $B_{12}=G_{12}h$,

$B_{13}=G_{13}hk^2$, $B_{23}=G_{23}hk^2$, $D_{11}=\dfrac{E_1h^3}{12(1-\nu_{12}\nu_{21})}$,

$D_{22}=\dfrac{E_2h^3}{12(1-\nu_{12}\nu_{21})}$, $D_{12}=G_{12}\dfrac{h^3}{12}$.

where, k^2,k_{1i}^2,k_{2j}^2 are the coefficients that take into account the shift by thickness; $E_1,E_2,G_{12},G_{13},G_{23},\nu_{12},\nu_{21},\rho$ are the mechanical characteristics and density of the shell material; $I_{1i},I_{2j},I_{kp1i},I_{kp2j},F_{1i},F_{2j}$ are the moments of inertia and the cross–sectional areas of the ribs; ρ_{1i}, ρ_{2j} is the density of the rib material; z_{1i}, z_{2j},I,J are the ribs eccentricity and number; P_1, P_2, P_3, m_1, m_2 are the the intensity of the external forces and bending moments acting on the structure; x_j, y_i are the coordinate axes of the reinforcing ribs; δ is the Dirac's unit impulse function. The system of Equation (2) is supplemented by the given boundary and initial conditions of the problem.

We assume that the initial dents of the shell do not cause initial stresses. Therefore, at the moment of time t=0, under the initial conditions of displacement and the absence of external influences on the shell, all stresses should be zero. In this case, the equilibrium equations are satisfied identically. The resulting mathematical structure of the process of unsteady oscillations is represented by a system of differential Equations (1)-(3) of hyperbolic type. The main feature of these equations is geometric nonlinearity and the presence of discontinuous coefficients along spatial coordinates. This is due to the discreteness of the arrangement of the longitudinal and transverse reinforcing ribs and their variable stiffness of the spasmodic character.

The numerical solution method is based on a finite-difference approximation of the vibration Equation (2) by spatial coordinates using an explicit difference scheme of integration in time coordinate [14, 28, 29]. This imposes certain restrictions on the construction of the difference grid [30]. Therefore, the solution is sought on a smooth surface between the ribs of the region and on the lines of spatial discontinuities. In matrix-vector form, the difference equations are represented by the dependence

$$[C]\,\overline{U} - [M]\frac{\partial^2 \overline{U}}{\partial t^2} = \overline{P}(t) \tag{4}$$

where, $[M]$ and $[C]$ are the mass and stiffness matrices of a discrete difference system of Equation (4); $\overline{U}$ and $\overline{P}(t)$ are the generalized displacement and axial load vectors. The applied explicit finite difference scheme leads to a limitation of the discrete time step of the difference grid.

The condition for the dynamic strength and stability of the shell structure is the criterion of material fluidity [4]. It was obtained using the theory of small elastic-plastic deformations of Mises [23] and is characterized by the intensity of stresses σ_i at the time of the appearance of plastic deformations and differs from the alternative criterion, which is associated with the limiting value of the transverse deflection of the structure [12]. For an elastic-plastic material, the condition for the occurrence of plastic deformations is $\sigma_i \geq \sigma_T$. Here σ_T is the yield strength of the material under one-dimensional strain of tension-compression. The intensity of stresses in the shell is determined by the dependence [23]:

$$\sigma_i = \sqrt{\sigma_{11}^2 + \sigma_{22}^2 - \sigma_{11}\sigma_{22} + 3(\sigma_{12}^2 + \sigma_{13}^2 + \sigma_{23}^2)}$$

where, $\sigma_{11}, \sigma_{22}, \sigma_{12}, \sigma_{13}, \sigma_{23}$ are the components of the stress tensor of the constituent layers of the shell. To substantiate the reliability of the solutions obtained, the calculation results were compared with experimental data [16] and a numerical study of the convergence of the computational process for the problem under consideration was performed.

4. ANALYSIS OF NUMERICAL RESULTS

The effect of the axial edge load $P_1(y,t)$ on a three-layer cylindrical shell of a finite length L=0.4 m with initial deflections is studied by the example of a shell pivotally supported at x=0 and rigidly clamped at x=L. The outer layers are made of duralumin, and the inner layer consists of a composite filler [5] with a density of $\rho_2 = 4.2\times10^2$ kg/m^3. The shell is externally reinforced with six longitudinal and three transverse ribs, which are evenly spaced along the coordinate axes $y = y_i$ and $x = x_j$ ($x_j = 0.25jL$, $j = \overline{1.3}$; $y_i = 0.25(2\pi R i)$, $i = \overline{1.6}$). Geometric and physico-mechanical parameters of the shell and ribs:

$L/R = 4; R/h = 20; H_{1i} = H_{2j} = 4h; h_{1i} = h_{2j} = h;$

$F_{1i} = F_{2j} = H_{1i}h_{1i}$; $E_1 = E_2 = 70\,\text{GPa}$; $\nu_1 = \nu_3 = 0.3$;

$\rho_1 = \rho_3 = 2.7\times10^3\,\text{kg/m}^3$; $E_{1i} = E_{2j} = E_1; G_{1i} = G_{2j} = G$;

$\rho_{1i} = \rho_{2j} = \rho_1$.

where, R, h is the radius of the median surface and the thickness of the shell; $h_{1i}, h_{2j}, F_{1i}, F_{2j}, H_{1i}, H_{2j}$ is geometric characteristics of the edges; h_1, h_3 is thickness of the outer layers. The material of the shell and ribs is considered elastic-plastic with a yield strength $\sigma_T = 310\,\text{MPa}$.

The value of the initial dents is set by the dependence $w^0 = h_0 \sin(4\pi x/L)$. Its amplitude is equal to $h_0 = 0.02$h. The boundary conditions of the hinge support at $x = 0$ have the form $u_2 = u_3 = \varphi_2 = 0$:

$$N_{11} + \sum_{i=1}^{I} N_{11i}\,\delta(y - y_i) = P_1(y,t)$$

$$M_{11} + \sum_{i=1}^{I} \left(M_{11i} \pm \eta_{1i} N_{11i}\right)\delta(y - y_i) = 0$$

for rigidly fixed end at $x = L : u_1 = u_2 = u_3 = \varphi_1 = \varphi_2 = 0$.

The time-dependent conditions for $t = 0$ are taken as:

$u_1 = u_2 = u_3 = \varphi_1 = \varphi_2 = 0$,

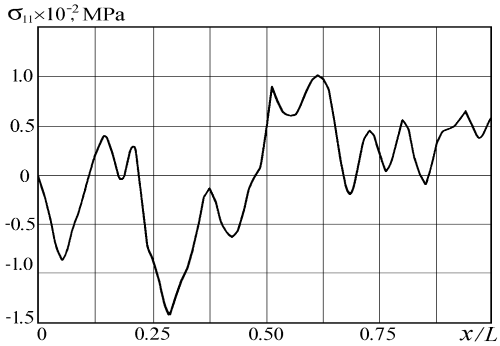
$\partial u_1/\partial t = \partial u_2/\partial t = \partial u_3/\partial t = \partial \varphi_1/\partial t = \partial \varphi_2/\partial t = 0$.

The law of loading action is given by a step function

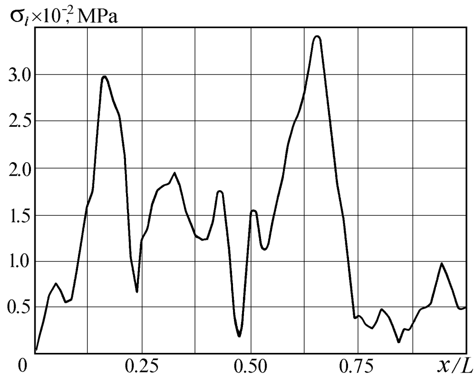
$P_1(y,t) = -A_0 \sin(\pi t/T)\left[\eta(t) - \eta(t-T)\right]$.

where, A_0 is the the the amplitude caused by external influence; T is the load duration; $\eta(t)$ is the Heaviside's function; t is the current time. The load parameters are $A_0 = 0.1$ MPa×m , $T = 50\times10^{-6}$ s. As a result of solving the problem of in fluctuations depending on time and momentum, the parameters of the stress-strain state of the shell and ribs at discrete time points in the interval $0 \leq t \leq 30$T are studied. It is established that under the action of a longitudinal edge load $P_1(y,t)$, the values of normal displacements u_3, longitudinal deformations ε_{11} and stresses σ_{11} are decisive. The maximum stress intensity values and plastic deformations act between the ribs, and the minimum stress intensity values are located in the area of the reinforcing ribs. This is confirmed by numerous theoretical and experimental studies [4]. The distribution of the stress magnitude σ_{11} between the stringers along the longitudinal coordinate at time t=11 T is shown in Figure 2a. The deformation parameter ε_{11} has the same character.

Figure 2b shows the dependence of the distribution of the magnitude of the stress intensity σ_i along the longitudinal coordinate along the line of symmetry between the stringers in the time period t=11 T. The maximum stresses in the $x \approx 5L/8$ section correspond to the beginning of plastic deformations ($\sigma_i \geq \sigma_T$).



(a)

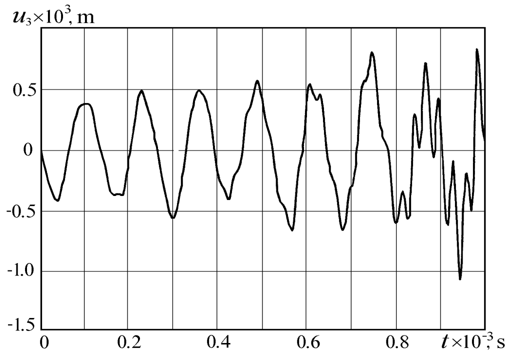


(b)

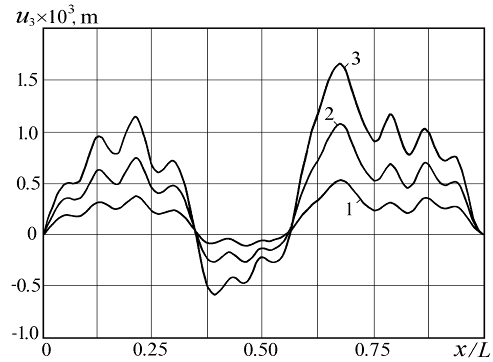
Figure 2. Distribution of longitudinal stresses σ_{11} and stress intensity σ_i in the longitudinal direction between the ribs at time t=11 T

Starting from this point in time, plastic deformations develop throughout the study area. Therefore, this moment in time can be considered the beginning of the loss of dynamic stability of the structure. Before this point in time, the shell had been operating in an elastic region. Thus, the general loss of structural stability of ribbed shells under impulsed loading is usually preceded by a local loss of structural stability of the outer layer of the shell between the ribs, which is confirmed by the results of the paper [13]. The general case of deformation is realized only with very small bending stiffness of the reinforcing ribs.

In the presence of an initial deflection w^0 centered at point $x=2L/3$, the intensity of local stresses and deformations in this area increases significantly. This is caused by an increase in the transverse displacement of the surface. The time dependence of the deflection value u_3 in the cross section $x=2L/3$ is shown in Figure 3a. If the beginning of a sharp increase in deflection is taken as the criterion of loss of stability [12], which, according to calculations, has the highest growth rate, then the time of loss of stability is $t \approx 0.8 \times 10^{-3}$ s (Figure 3a). This corresponds to the time of t=16 T. However, according to the Mises criterion of material fluidity, plastic deformations occur starting from the moment of time t=10 T. Thus, the use of different criteria for the loss of dynamic stability of discretely reinforced shells can lead to different results. Based on the results obtained, it can be concluded that the loss of stability of inhomogeneous shells largely corresponds to the time of occurrence of plastic deformations.



(a)



(b)

Figure 3. The dependence of the deflection value u_3 between the ribs on time and on the longitudinal coordinate

When considering this problem, calculations were performed for different values of the amplitude h_0 of initial deflections. For example, in Figure 3b, lines 1, 2 and 3 show the dependences of the magnitude of the u_3 displacement for amplitudes that are 0.01, 0.02 and 0.03 h. It can be seen from the presented dependencies that with an increase in the amplitudes of the initial deflections, the displacement value u_3 also increases. It should be noted that for the amplitudes 0.02 and 0.03 h, plastic deformations occur much earlier than for 0.01 h. At the same time, local deformations and stresses increase significantly. The time period before the loss of stability is shortened. Destructions prevails in the area of the initial deflection location. Therefore, even relatively small initial deflections significantly affect the dynamic stability of discretely reinforced shells.

5. CONCLUSIONS

A refined mathematical model and a method for calculating vibrations that change over time and the process of deformation from sudden action of discretely reinforced by ribs elastic multilayer composite shells with imperfections in the shape of the surface in the form of initial dents under the action of longitudinal the force that creates pressure have been developed. The problem is solved in a geometrically nonlinear theory of one-dimensional and two-dimensional thin bodies, taking into account transverse shear and normal deformations in a wide wave range. New numerical solutions have been obtained for a reinforced three-layer composite cylindrical shell. The results of the SSS study showed that reinforcing ribs and structural shape errors under impulsed loading have a significant effect on deformations and stresses. The general complete loss of stability of the structure of the shell under the action of an non-stationary load is usually preceded by a local loss of resilience loss of structural stability between the ribs due to the residual deformations that occur.

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